Year 2010-2011

Problemath Series 3

29 November 2010

(Deadline: Thursday 23 December 14:00)

Problemath 7

Two brilliant mathematicians, Alice and Bob have been informed that two distinct natural numbers x and y have been chosen from the set $\{2,3,4,...,69,70\}$. Alice then receives the sum $x+y$ and Bob receives the product xy. Here is the conversation which takes place:

Alice says: "I know that you cannot find x and y."

Bob answers: " In this case, I know x and y".

Alice replies: " And so, so do I!"

What are the values of x and y ?

Problemath 8

Let $f:\mathbb{R}^2 \to \mathbb{R}$ be a function which associates a real value to every point of $\mathbb{R}^2$.

If $f(a)+f(b)+f(c)+f(d)=0$ each time that a,b,c,d are the four vertices of a square, **can one conclude that $f(p)=0$ for every point p of $\mathbb{R}^2$?**

Problemath 9

Are there two functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ such that, for one specific real number a, the five expressions written below are two by two distinct real numbers?

$(1): g\left(f(a)\right)$

$(2): \lim\limits_{x\underset{\neq}{\to}a} g\left(f(x)\right)$

$(3): g\left(\lim\limits_{x\underset{\neq}{\to}a} f(x)\right)$

$(4): \lim\limits_{y\underset{\neq}{\to}f(a)} g(y)$

$(5): \lim\limits_{x\underset{\neq}{\to}a} \lim\limits_{y\underset{\neq}{\to}f(x)} g(y)$