Communiqué de presse – english version

Human Societies and the Bruss-Duerinckx Theorem

Whatever the future will bring for mankind, certain things will probably never change: People need food and resources. They want to see safety and comfort, and in particular a future for their children.

If we agree on this, can we predict directions in which societies will move? In particular, will we see society forms which are more extreme than anything mankind has ever seen before?

Remarkably, new society forms *cannot* possibly be more extreme than what we have seen before. We have already tested the limits. This result, recently proved at the ULB by Bruss and Duerinckx (2013), is the strongest result in a series of findings in a long-term research project.

It began around 1982. Bruss, at that time at FUNDP Namur, had been motivated to think about societies when following numerous discussions in the media about the pros and contras of different political systems. The hypotheses of the discussants gave too much room for personal interpretations, he thought, and no conclusions of substance could be drawn.

Could one say something definite in the mathematical sense of theorem and proof, by confining oneself on those hypotheses upon which everybody seems to agree? Two hypotheses passed this screening:

Hypothesis (H1) Individuals want to survive and to see a future for their descendants.

Hypothesis (H2) Individuals prefer in general a higher standard of living to a lower one.

If necessary, (H1) is supposed to have the higher priority. To deal with these hypotheses, the problem was to find a model for a human society, i.e., featuring births and deaths, production, consumption and heritage of resources, policies to distribute resources, and interactions between individuals and society. After many trials Bruss proposed new *resource dependent branching processes* (RDBPs) as a model. Here he chose individuals to interact with society by emigration (for instance), if their minimum resource claims are not respected.

Realistic RDBPs for the development of a society are complicated, however, and so the results were modest in the beginning. Then came the idea to re-interpret the development of the society as a sequence of control questions leading, each time, to a new short-horizon RDBP. Society would ask itself: If, all things staying the same, we go on like this, can we survive, and if so, is the standard of living acceptable for us? If not, let us correct, be it by distributing resources differently, be it by encouraging to have more children, consume less, increase productivity, etc. Each correction time is understood as the birth time of a *new* RDBP. So understanding the development of a society now means understanding easier RDBPs with fixed parameters and distributions.

After an intermediate result (Bruss and Robertson (1991)), real progress was made. It turned out that two special societies stood out: The first one, called the weakest-first society (wf-society), serves with priority the smallest random resource claims as long as resources suffice. It can be seen as an extreme form of communism. Since this policy keeps emigration at the minimum it seems it should grow more quickly than any other society. Similarly one would expect a society which, to the exact opposite, always serves the largest resource claims first (strongest-first society, reminding us of extreme capitalism) would linger in size behind any other society. Interestingly, only the first intuition is correct; the second one is strictly wrong.

The more remarkable is that, despite of this, Bruss and Duerinckx (2014) have proven that once population sizes are sufficiently large, the wf-society has the greatest probability of surviving, and the sf-society indeed the smallest one, and that these two societies form an *envelope* of all imaginable societies through which no society, whatever its policy, can escape for long. This is the new Bruss-Duerinckx theorem of the envelopment of societies.

Religions, ideologies, and other driving forces of societies have not been mentioned. They are not neglected but implicit: what counts for (H1) and (H2) is only the effective impact on the corresponding parameters, and not what explains the impact.

All societies following (H1) and (H2) are, a priori, on the same footing. This is why the notion of an extreme society has for such societies the unambiguous meaning of living on the envelope. The wf-society offers the largest survival probability but the lowest standard of living. People will try to get away from it according to (H2). The sf-society does everything for (H2) and the standard of living is highest possible but emigration jeopardizes

survival severely. The envelope is thus "attractive" and "repulsive" at the same time, and no society would like to stay on it.

Mankind has experienced capitalism and communism and thus tested the limits. With so many different societies we can imagine, is this not very surprising?

Not really. The wf-society is the most attractive society for (H1) and the sf-society the most attractive for (H2). *Attractive states* in a complex random system have typically a large probability of being visited. Since in our case these two states are mutually repulsive with respect to the other hypothesis, none of them is *absorbing* so that this fact is, indeed, not surprising.

Now, and in particular, why is the Envelopment Theorem of interest?

It is of great interest because we can *explicitly* compute the critical survival parameters for the sf-society and the wf-society. Every society observing (H1) and (H2) must accept these critical parameters as upper and lower bounds for policies on the longer horizon — be it in Cuba, in the US, in China, or anywhere else.

*

References:

F. Thomas Bruss and James B. Robertson, *"Wald's Lemma" for sums of order statistics of i.i.d. random variables,* Adv. Appl. Prob.,Vol 23, 612-623 (1991)

F. Thomas Bruss and Mitia Duerinckx, *Resource dependent branching processes and the envelope of societies*, to appear in Annals of Appl. Probab. (2014).

*

Thomas Bruss is Professor of Mathematics at the ULB, and Mitia Duerinckx a student in the second year of the Master in Mathematics at the ULB.