VIII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
THE CORRESPONDENCE ROUND

Below is the list of problems for the first (correspondence) round of the VIII Sharygin Geometrical Olympiad.

The olympiad is intended for high-school students of 8–11 grades (these are four elder grades in Russian school). In the list below each problem is indicated by the numbers of school grades, for which it is intended. However, the participants may solve problems for elder grades as well (solutions for younger grades will not be considered).

Your work containing the solutions for the problems (in Russian or in English) should be sent not later than April 1, 2012, by e-mail to geomolymp@mccme.ru in pdf, doc or jpg files. Please, follow several simple rules:

1. Each student sends his work in a separate message (with delivery notification). The size of the message must not exceed 10 Mb.

2. If your work consists of several files, send it as an archive.

3. If the size of your message exceeds 10 Mb divide it into several messages.

4. In the subject of the message write "The work for Sharygin olympiad", and present the following personal information in the body of your message:

- last name;

- all other names;

- E-mail, post address, phone number;

- the current number of your grade at school;

- the number and/or the name and the mail address of your school;

- full names of your teachers in mathematics at school and/or of instructors of your extra math classes (if you attend additional math classes after school).

If you have no e-mail access, please, send your work by regular mail to the following address: *Russia, 119002, Moscow, Bolshoy Vlasyevsky per., 11. Olympiad in honour of Sharygin.* In the title page write your personal information indicated in the item 4 above.

In your work you should start writing the solution to each problem in a new page. First write down the statement of the problem, and then the solution. Present your solutions in detail, including all significant arguments and calculations. Provide all necessary figures. Solutions of computational problems have to be completed with a distinctly presented answer. Please, be accurate to provide good understanding and correct estimating of your work !

If your solution depends on some well-known theorems from standard textbooks, you may simply refer to them instead of providing their proofs. However, any fact not from the standard curriculum should be either proved or properly referred (with an indication of the source).

You may (this isn't necessary) note the problems which you liked. Your opinion is interesting for the Jury.

Winners of the correspondence round will be invited to take part in the final round in Summer 2012 in Dubna town (near Moscow). If you want to know your detailed results, please use e-mail.

1. (8) In triangle ABC point M is the midpoint of side AB, and point D is the foot of altitude CD. Prove that $\angle A = 2\angle B$ if and only if $AC = 2MD$.

2. (8) A cyclic n-gon is divided by non-intersecting (inside the n-gon) diagonals to $n-2$ triangles. Each of these triangles is similar to at least one of the remaining ones.

For what n this is possible?

3. (8) A circle with center I touches sides AB, BC, CA of triangle ABC in points C_1, A_1, B_1. Lines AI, CI, B_1I meet A_1C_1 in points X, Y, Z respectively. Prove that $\angle YB_1Z = \angle XB_1Z$

4. (8) Given triangle ABC. Point M is the midpoint of side BC, and point P is the projection of B to the perpendicular bisector of segment AC. Line PM meets AB in point Q. Prove that triangle QPB is isosceles.

5. (8) On side AC of triangle ABC an arbitrary point is selected D. The tangent in D to the circumcircle of triangle BDC meets AB in point C_1; point A_1 is defined similarly. Prove that $A_1C_1||AC$.

6. (8–9) Point C_1 of hypothenuse AC of a right-angled triangle ABC is such that $BC = CC_1$. Point C_2 on cathetus AB is such that $AC_2 = AC_1$; point A_2 is defined similarly. Find angle AMC, where M is the midpoint of A_2C_2.

7. (8–9) In a non-isosceles triangle ABC the bisectors of angles A and B are inversely proportional to the respective sidelengths. Find angle C.

8. (8–9) Let BM be the median of right-angled triangle ABC ($\angle B = 90°$). The incircle of triangle ABM touches sides AB, AM in points A_1, A_2; points C_1, C_2 are defined similarly. Prove that lines A_1A_2 and C_1C_2 meet on the bisector of angle ABC.

9. (8–9) In triangle ABC, given lines l_b and l_c containing the bisectors of angles B and C, and the foot L_1 of the bisector of angle A. Restore triangle ABC.

10. In a convex quadrilateral all sidelengths and all angles are pairwise different.

    a)(8–9) Can the greatest angle be adjacent to the greatest side and at the same time the smallest angle be adjacent to the smallest side?

    b)(9–11) Can the greatest angle be non-adjacent to the smallest side and at the same time the smallest angle be non-adjacent to the greatest side?

11. Given triangle ABC and point P. Points A', B', C' are the projections of P to BC, CA, AB. A line passing through P and parallel to AB meets the circumcircle of triangle $PA'B'$ for the second time in point C_1. Points A_1, B_1 are defined similarly. Prove that

    a) (8-10) lines AA_1, BB_1, CC_1 concur;

    b) (9–11) triangles ABC and $A_1B_1C_1$ are similar.

12. (9–10) Let O be the circumcenter of an acute-angled triangle ABC. A line passing through O and parallel to BC meets AB and AC in points P and Q respectively. The sum of distances from O to AB and AC is equal to OA. Prove that $PB + QC = PQ$.

13. (9–10) Points A, B are given. Find the locus of points C such that C, the midpoints of AC, BC and the centroid of triangle ABC are concyclic.

14. (9–10) In a convex quadrilateral $ABCD$ suppose $AC \cap BD = O$ and M is the midpoint of BC. Let $MO \cap AD = E$. Prove that $\frac{AE}{ED} = \frac{S_{\triangle ABO}}{S_{\triangle CDO}}$.

15. (9–11) Given triangle ABC. Consider lines l with the next property: the reflections of l in the sidelines of the triangle concur. Prove that all these lines have a common point.

16. (9–11) Given right-angled triangle ABC with hypothenuse AB. Let M be the midpoint of AB and O be the center of circumcircle ω of triangle CMB. Line AC meets ω for the second time in point K. Segment KO meets the circumcircle of triangle ABC in point L. Prove that segments AL and KM meet on the circumcircle of triangle ACM.

17. (9–11) A square $ABCD$ is inscribed into a circle. Point M lies on arc BC, AM meets BD in point P, DM meets AC in point Q. Prove that the area of quadrilateral $APQD$ is equal to the half of the area of the square.

18. (9–11) A triangle and two points inside it are marked. It is known that one of the triangle's angles is equal to 58°, one of two remaining angles is equal to 59°, one of two given points is the incenter of the triangle and the second one is its circumcenter. Using only the ruler without partitions determine where is each of the angles and where is each of the centers.

19. (10–11) Two circles with radii 1 meet in points X, Y, and the distance between these points also is equal to 1. Point C lies on the first circle, and lines CA, CB are tangents to the second one. These tangents meet the first circle for the second time in points B', A'. Lines AA' and BB' meet in point Z. Find angle XZY.

20. (10–11) Point D lies on side AB of triangle ABC. Let ω_1 and Ω_1, ω_2 and Ω_2 be the incirles and the excircles (touching segment AB) of triangles ACD and BCD. Prove that the common external tangents to ω_1 and ω_2, Ω_1 and Ω_2 meet on AB.

21. (10–11) Two perpendicular lines pass through the orthocenter of an acute-angled triangle. The sidelines of the triangle cut on each of these lines two segments: one lying inside the triangle and another one lying outside it. Prove that the product of two internal segments is equal to the product of two external segments.

22. (10–11) A circle ω with center I is inscribed into a segment of the disk, formed by an arc and a chord AB. Point M is the midpoint of this arc AB, and point N is the midpoint of the complementary arc. The tangents from N touch ω in points C and D. The opposite sidelines AC and BD of quadrilateral $ABCD$ meet in point X, and the diagonals of $ABCD$ meet in point Y. Prove that points X, Y, I and M are collinear.

23. (10–11) An arbitrary point is selected on each of twelve diagonals of the faces of a cube. The centroid of these twelve points is determined. Find the locus of all these centroids.

24. (10–11) Given are n $(n > 2)$ points on the plane such that no three of them aren't collinear. In how many ways this set of points can be divided into two non-empty subsets with non-intersecting convex envelops?