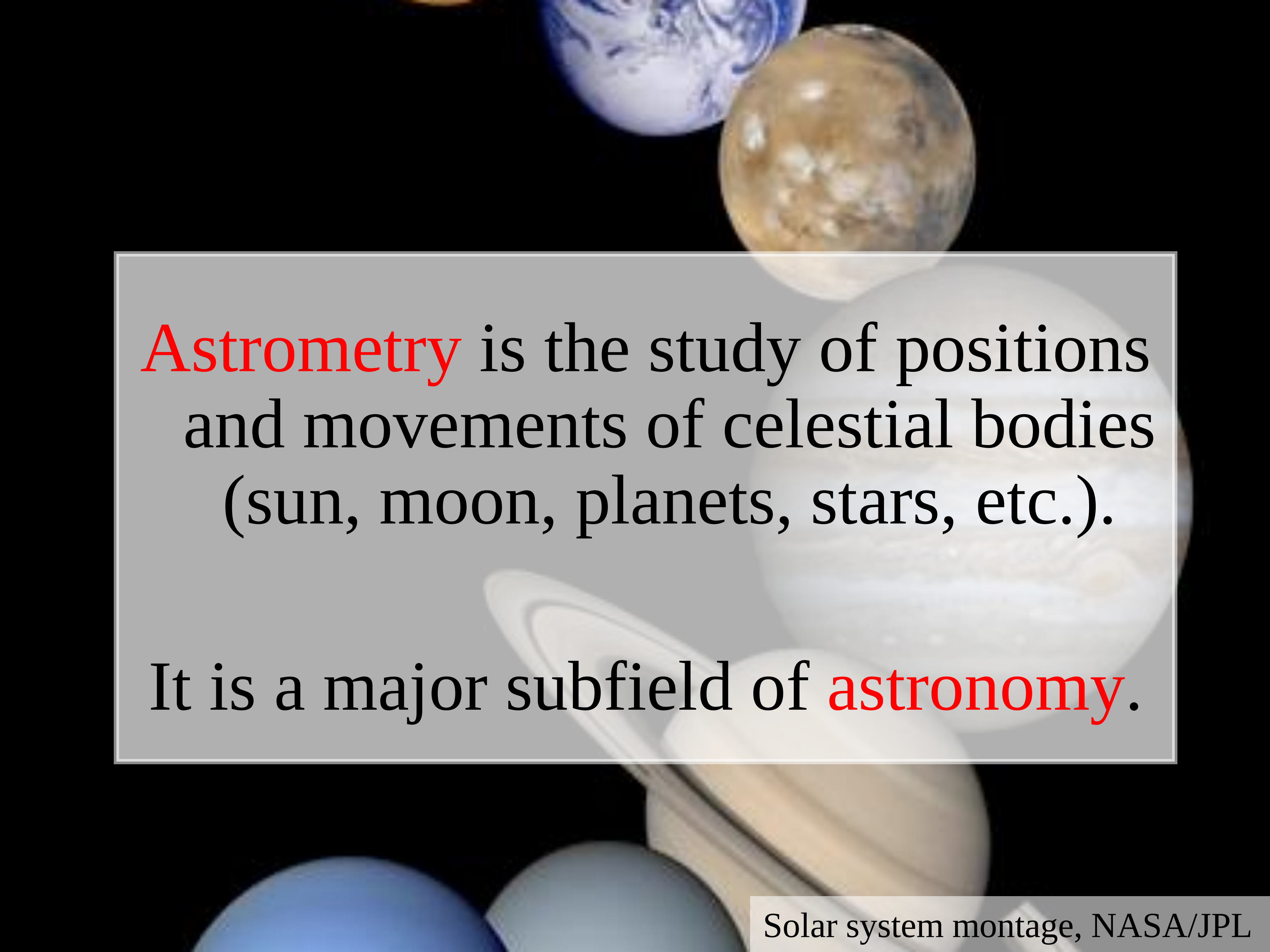
The Cosmic Distance Ladder

Terence Tao (UCLA)

Orion nebula, Hubble & Spitzer telescopes, composite image, NASA/JPL

Astrometry

Solar system montage, NASA/JPL



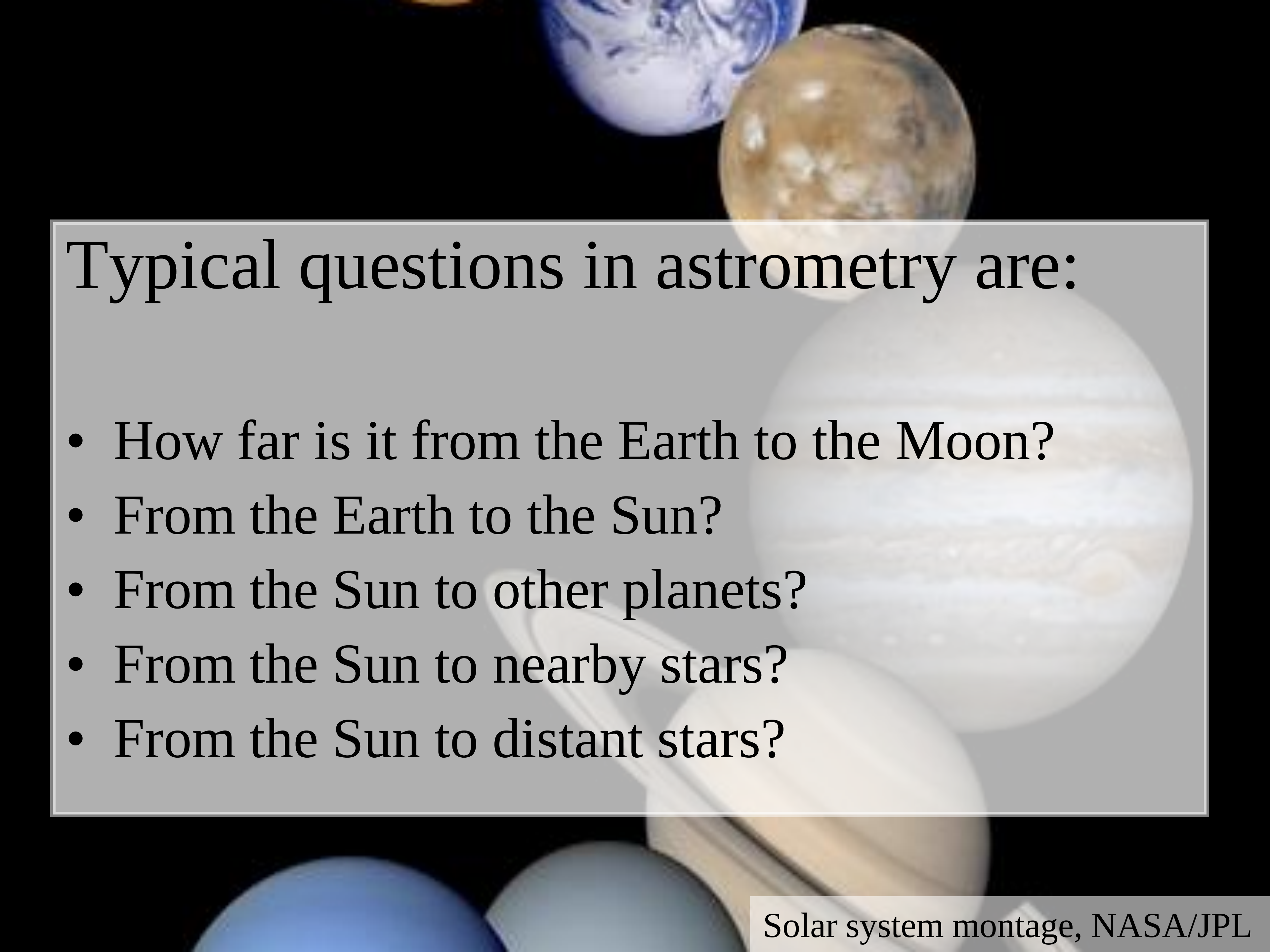
Astrometry is the study of positions and movements of celestial bodies (sun, moon, planets, stars, etc.).

It is a major subfield of astronomy.

Solar system montage, NASA/JPL

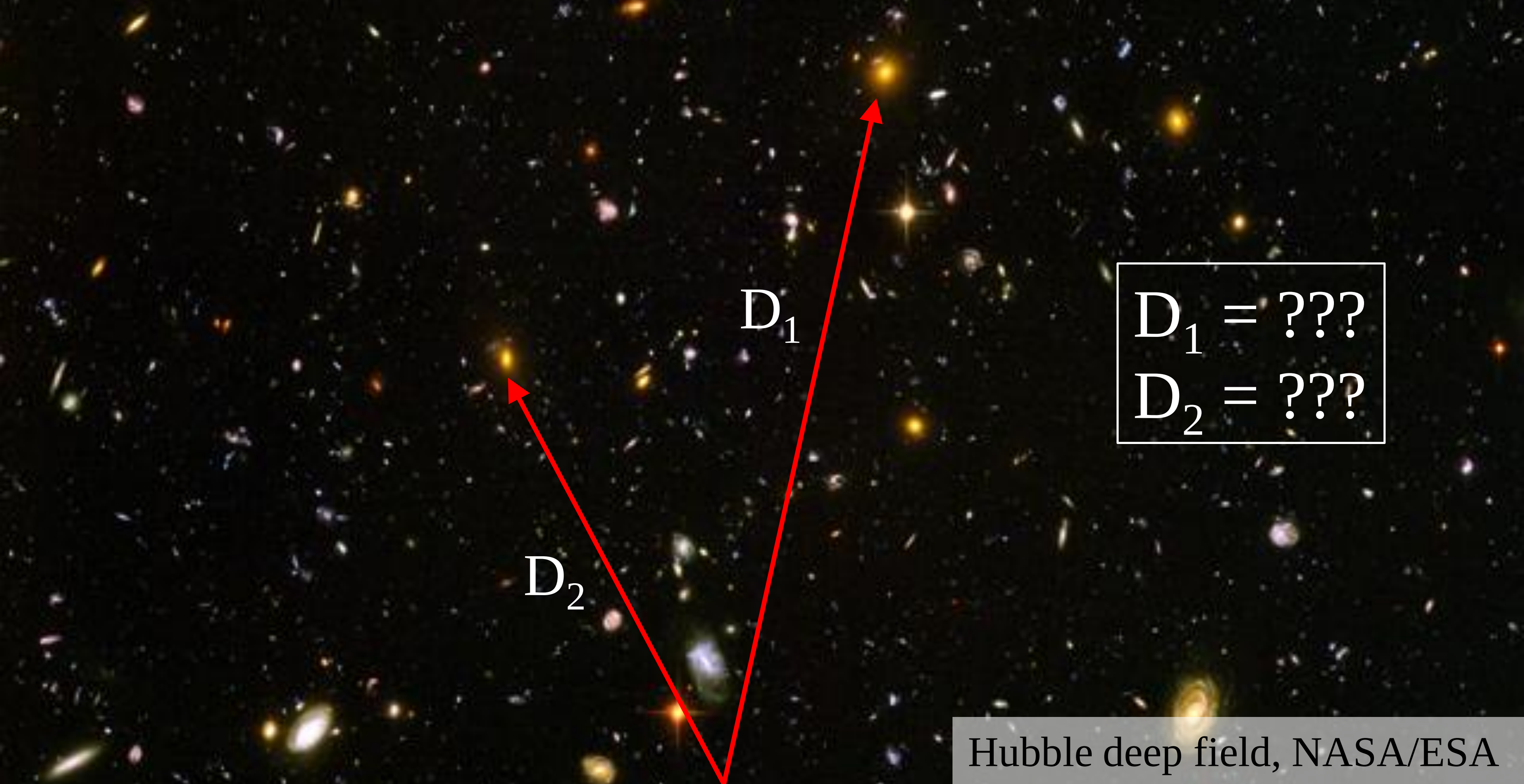
Typical questions in astrometry are:

- How far is it from the Earth to the Moon?
- From the Earth to the Sun?
- From the Sun to other planets?
- From the Sun to nearby stars?
- From the Sun to distant stars?



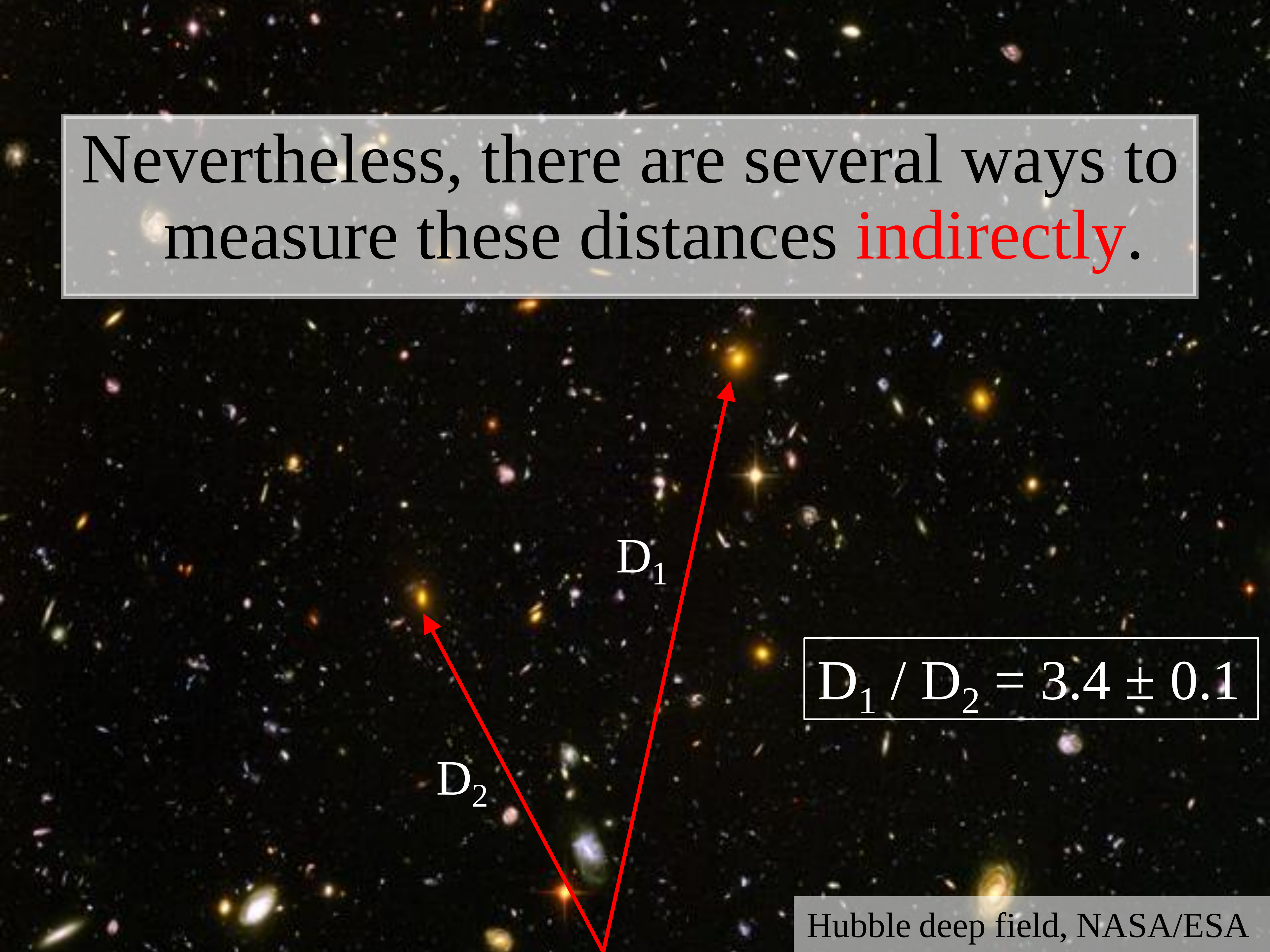
Solar system montage, NASA/JPL

These distances are far too vast to be measured directly.



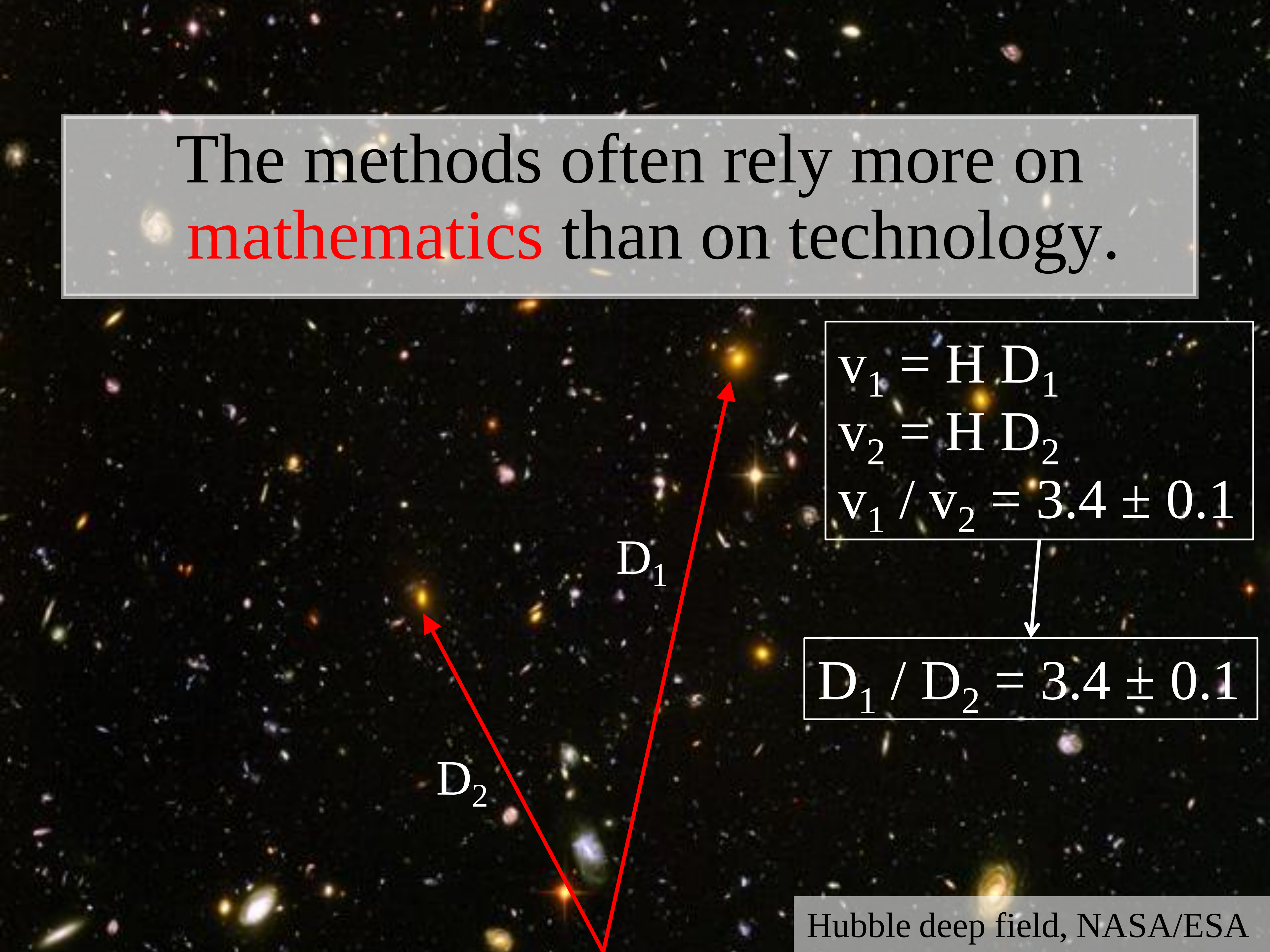
Hubble deep field, NASA/ESA

Nevertheless, there are several ways to measure these distances indirectly.



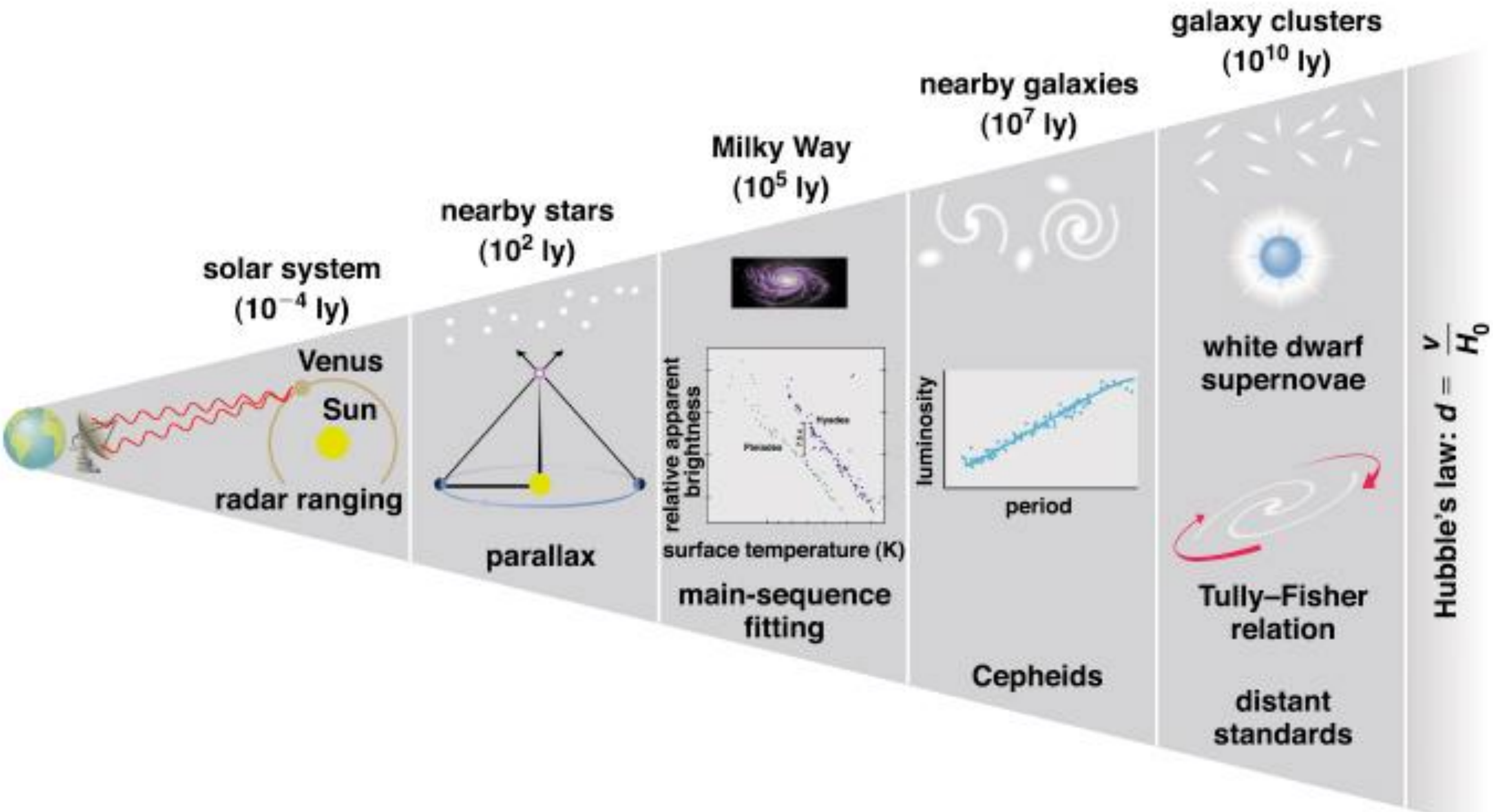
Hubble deep field, NASA/ESA

The methods often rely more on mathematics than on technology.



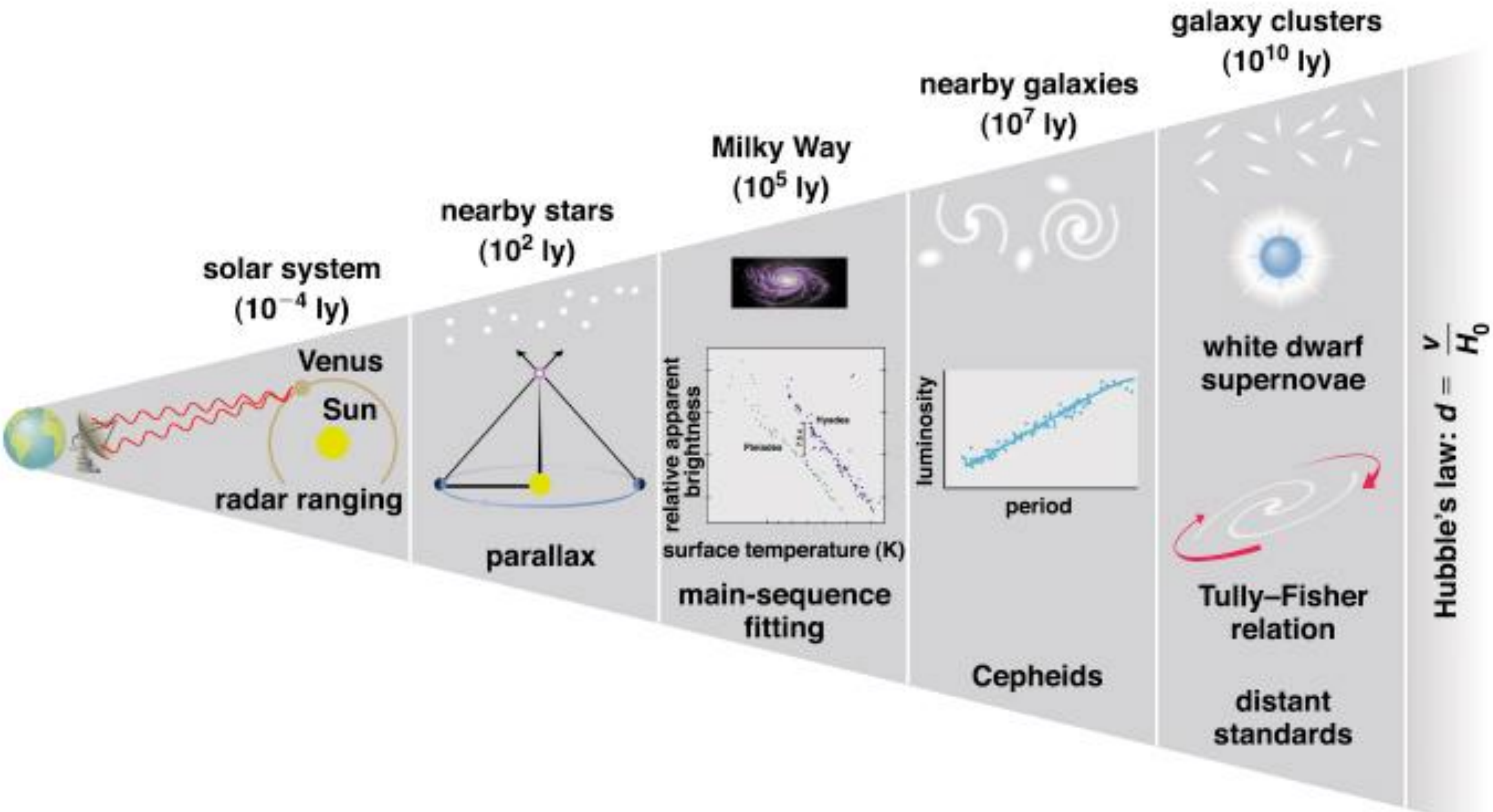
Hubble deep field, NASA/ESA

The indirect methods control large distances in terms of smaller distances.



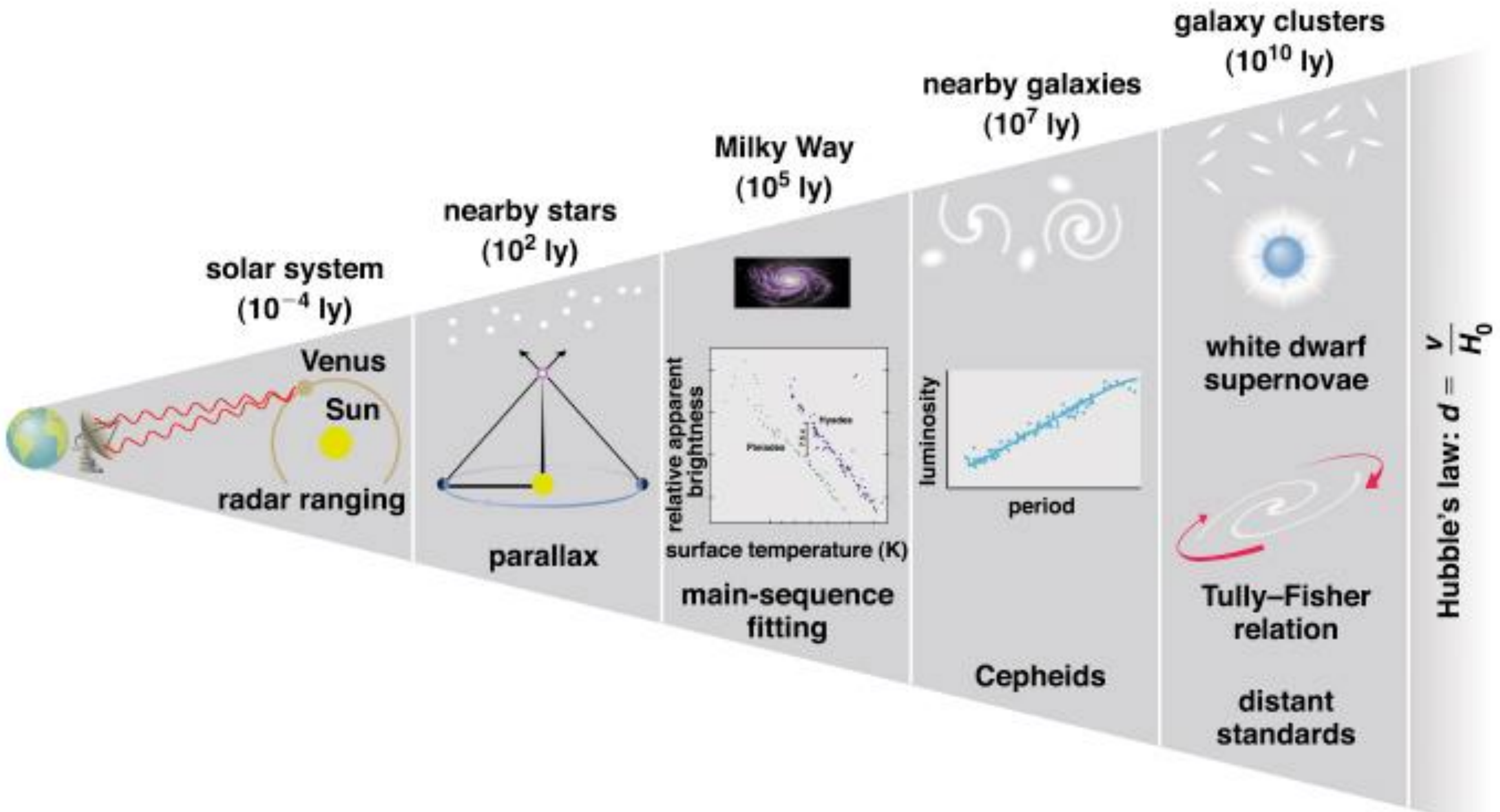
From "The Essential Cosmic Perspective", Bennett et al.

The smaller distances are controlled by even smaller distances...



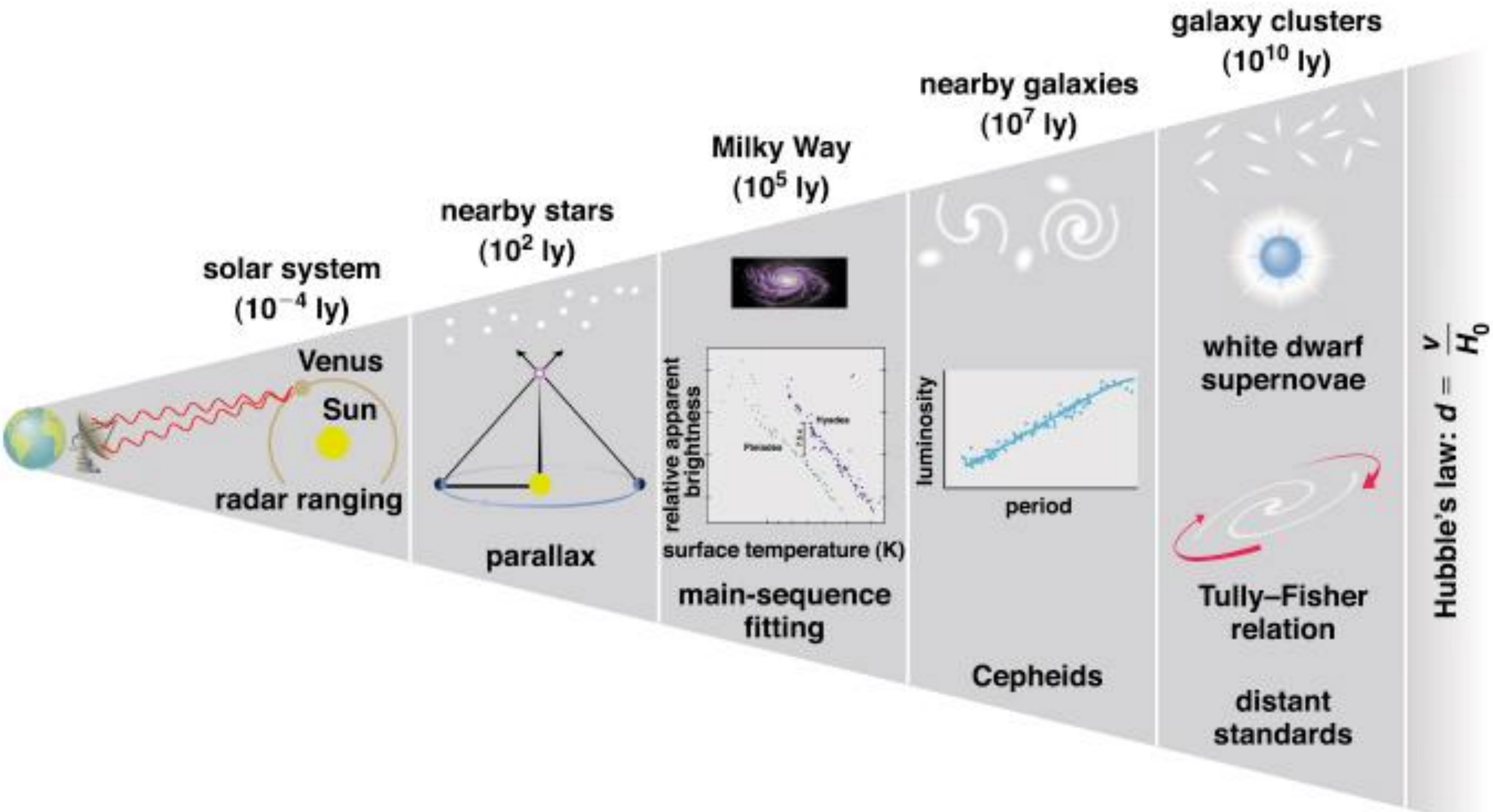
From "The Essential Cosmic Perspective", Bennett et al.

… and so on, until one reaches distances that one can measure directly.



From "The Essential Cosmic Perspective", Bennett et al.

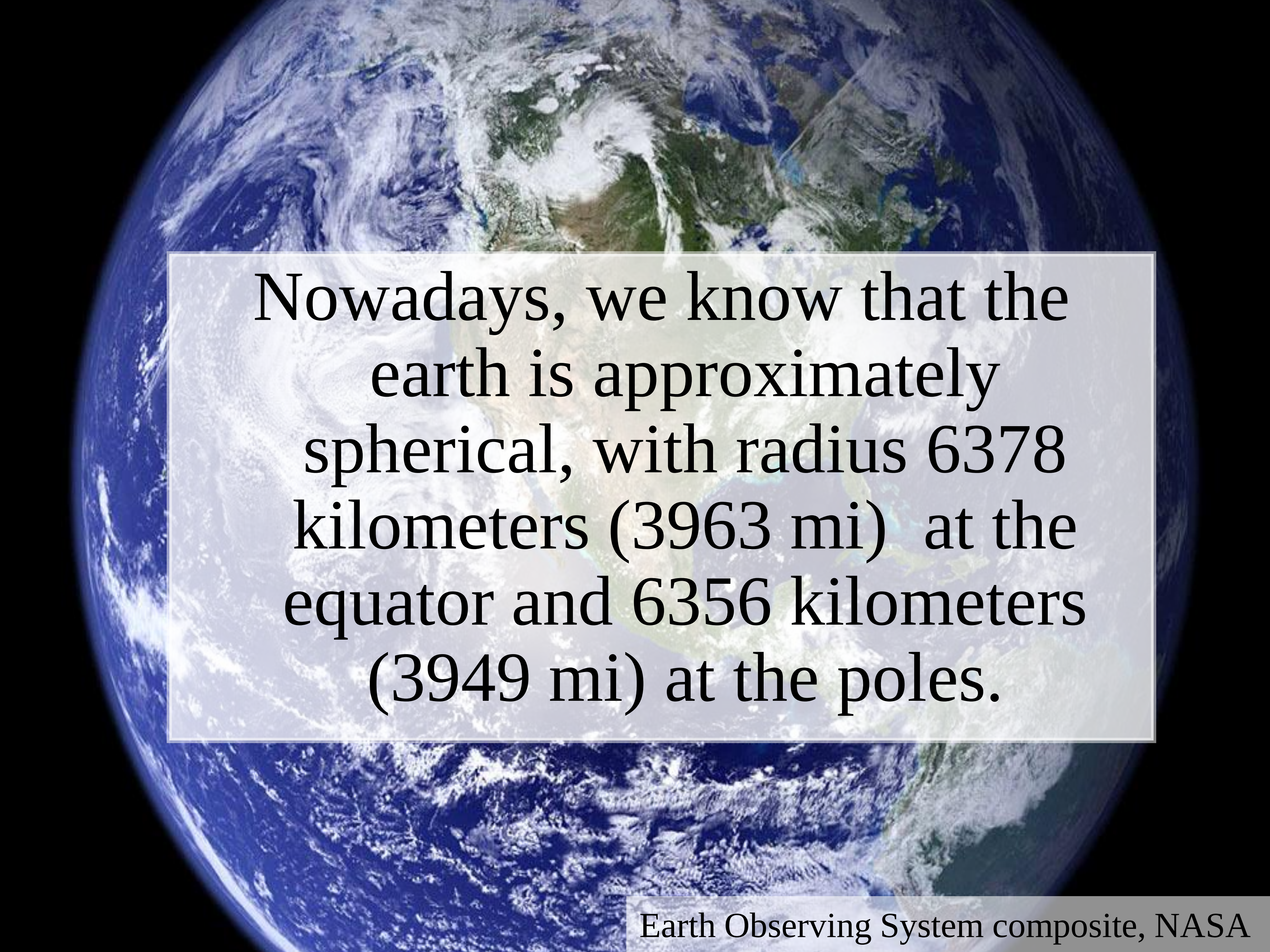
This is the cosmic distance ladder.



From "The Essential Cosmic Perspective", Bennett et al.

1st rung: the Earth

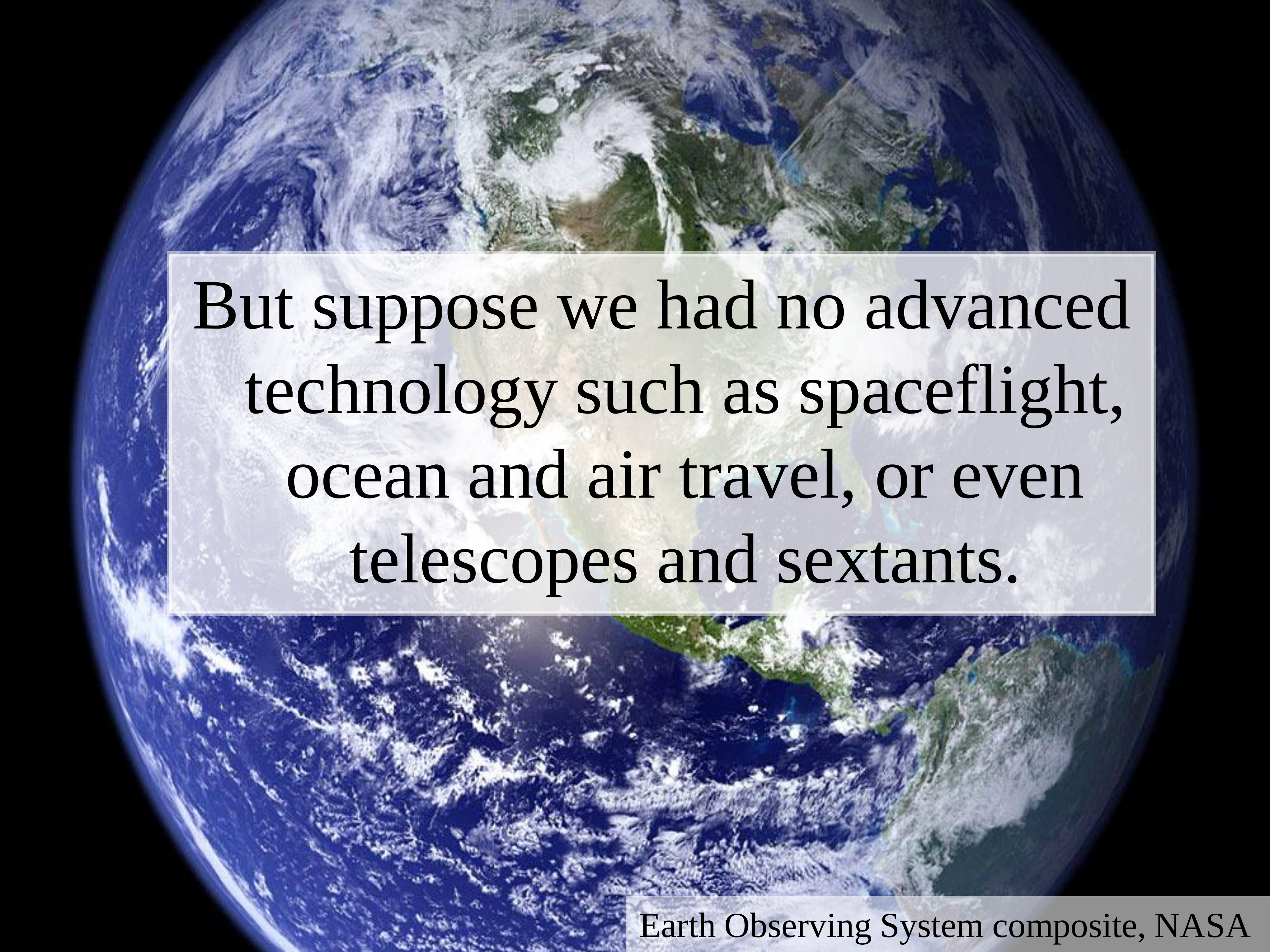
Earth Observing System composite, NASA



Earth Observing System composite, NASA

These values have now been verified to great precision by many means, including modern satellites.

Earth Observing System composite, NASA

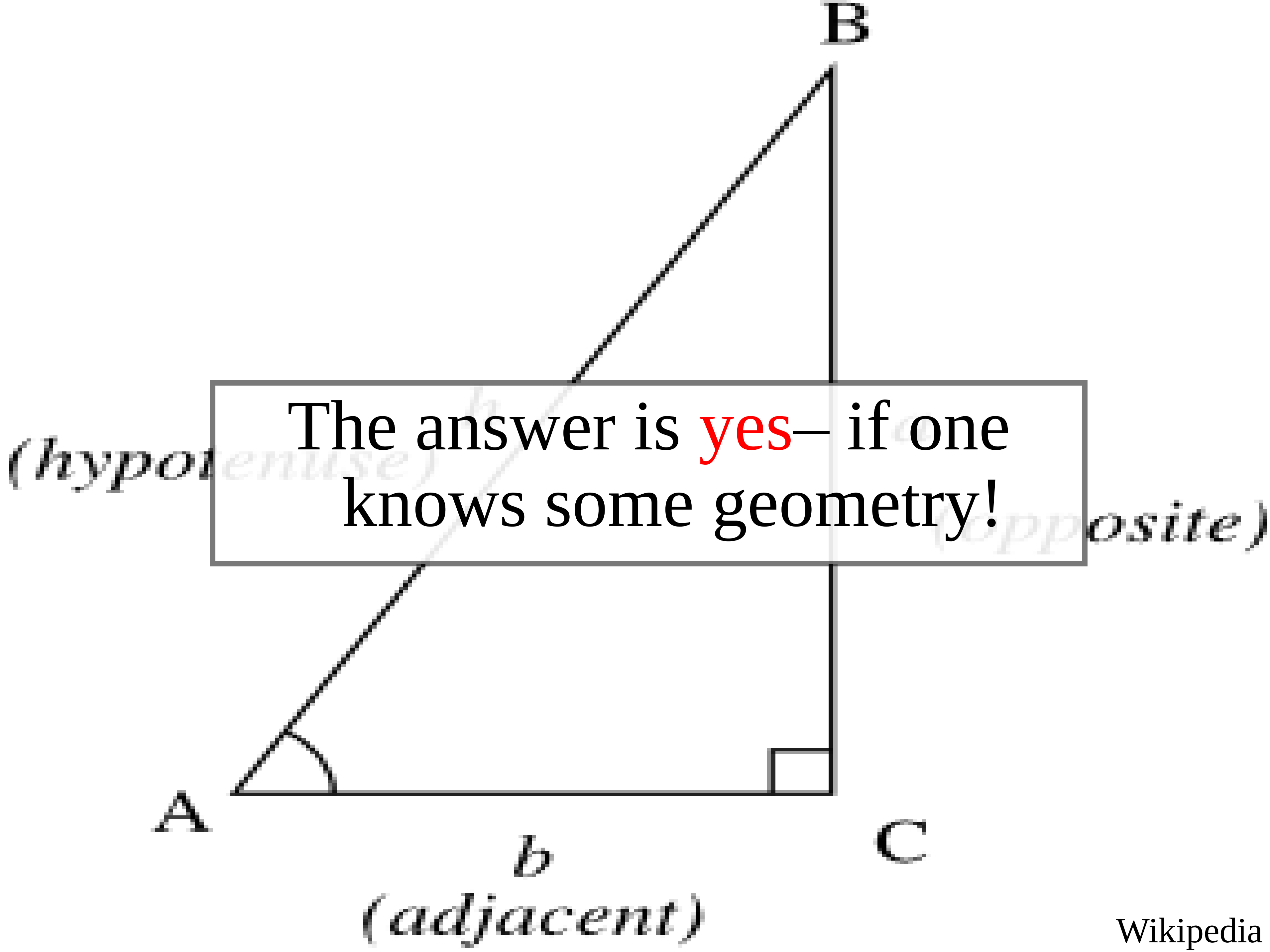


But suppose we had no advanced technology such as spaceflight, ocean and air travel, or even telescopes and sextants.

Earth Observing System composite, NASA

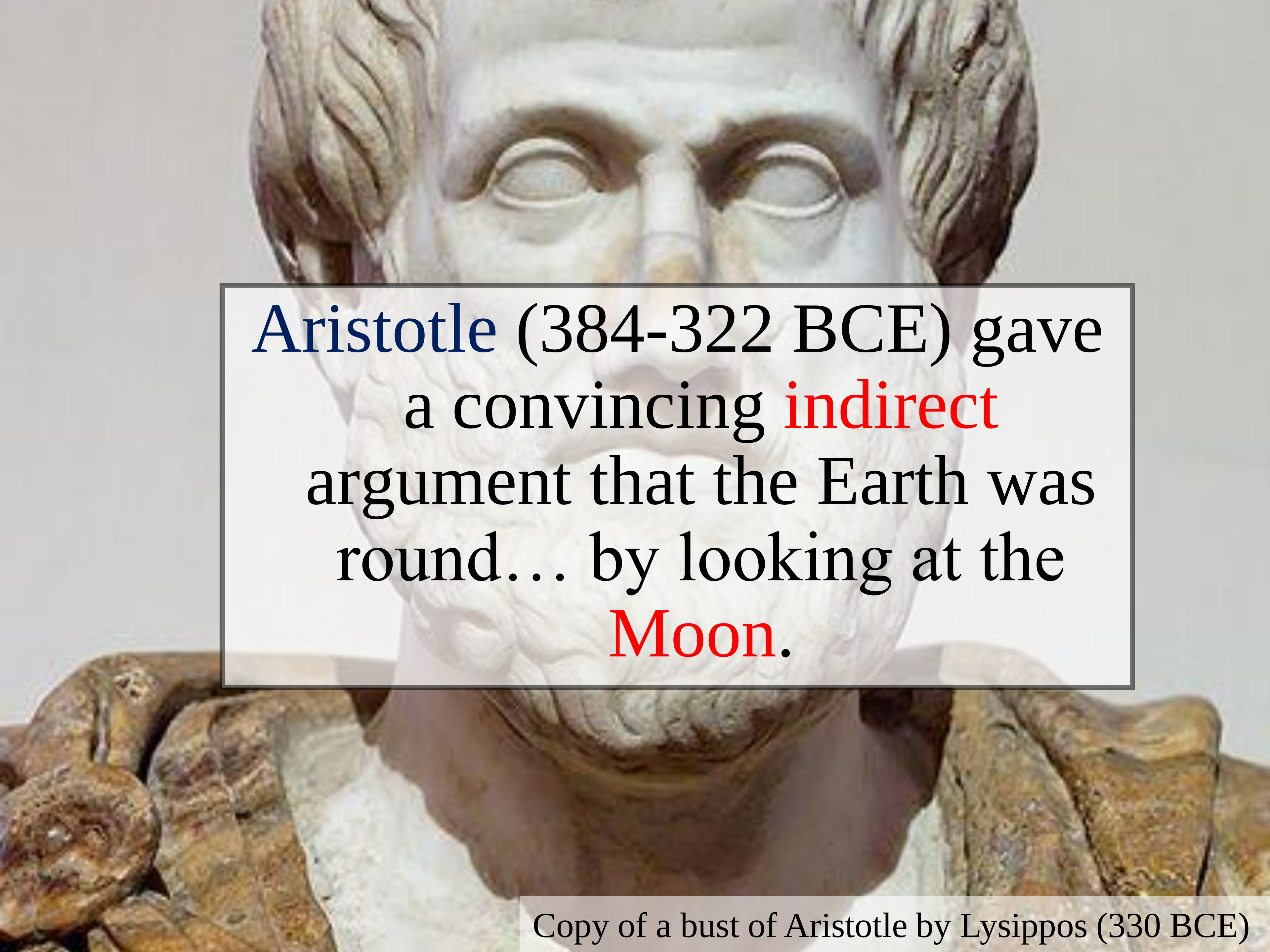
Earth Observing System composite, NASA

Earth Observing System composite, NASA



The answer is yes– if one knows some geometry!

Wikipedia



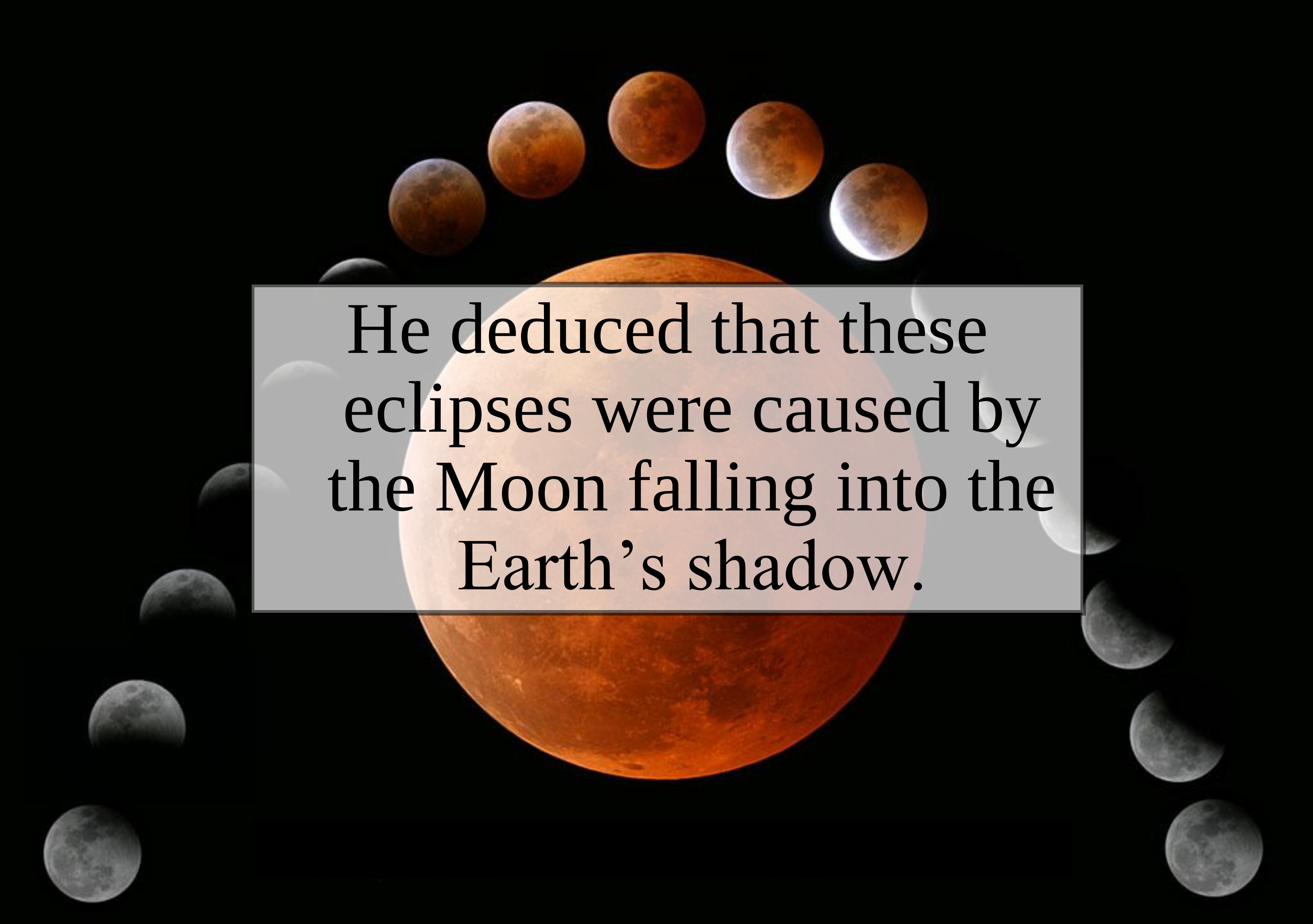
Aristotle (384-322 BCE) gave a convincing indirect argument that the Earth was round… by looking at the Moon.

Copy of a bust of Aristotle by Lysippos (330 BCE)

Aristotle knew that lunar eclipses only occurred when the Moon was directly opposite the Sun.

Lunar Eclipse Phases, Randy Brewer

He deduced that these eclipses were caused by the Moon falling into the Earth’s shadow.



Lunar Eclipse Phases, Randy Brewer

But the shadow of the Earth on the Moon in an eclipse was always a circular arc.

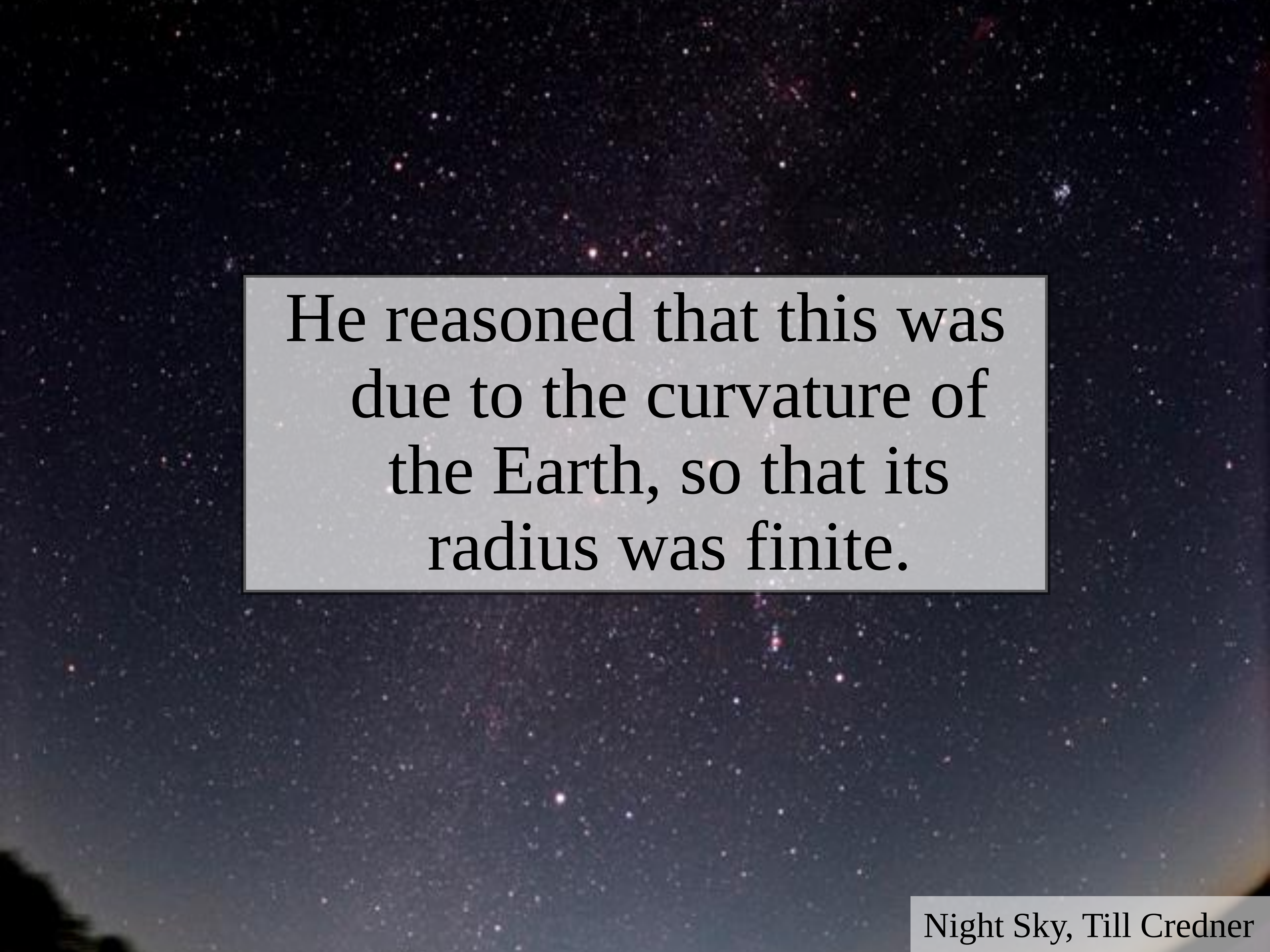
Lunar Eclipse Phases, Randy Brewer

In order for Earth’s shadows to always be circular, the Earth must be round.

Lunar Eclipse Phases, Randy Brewer

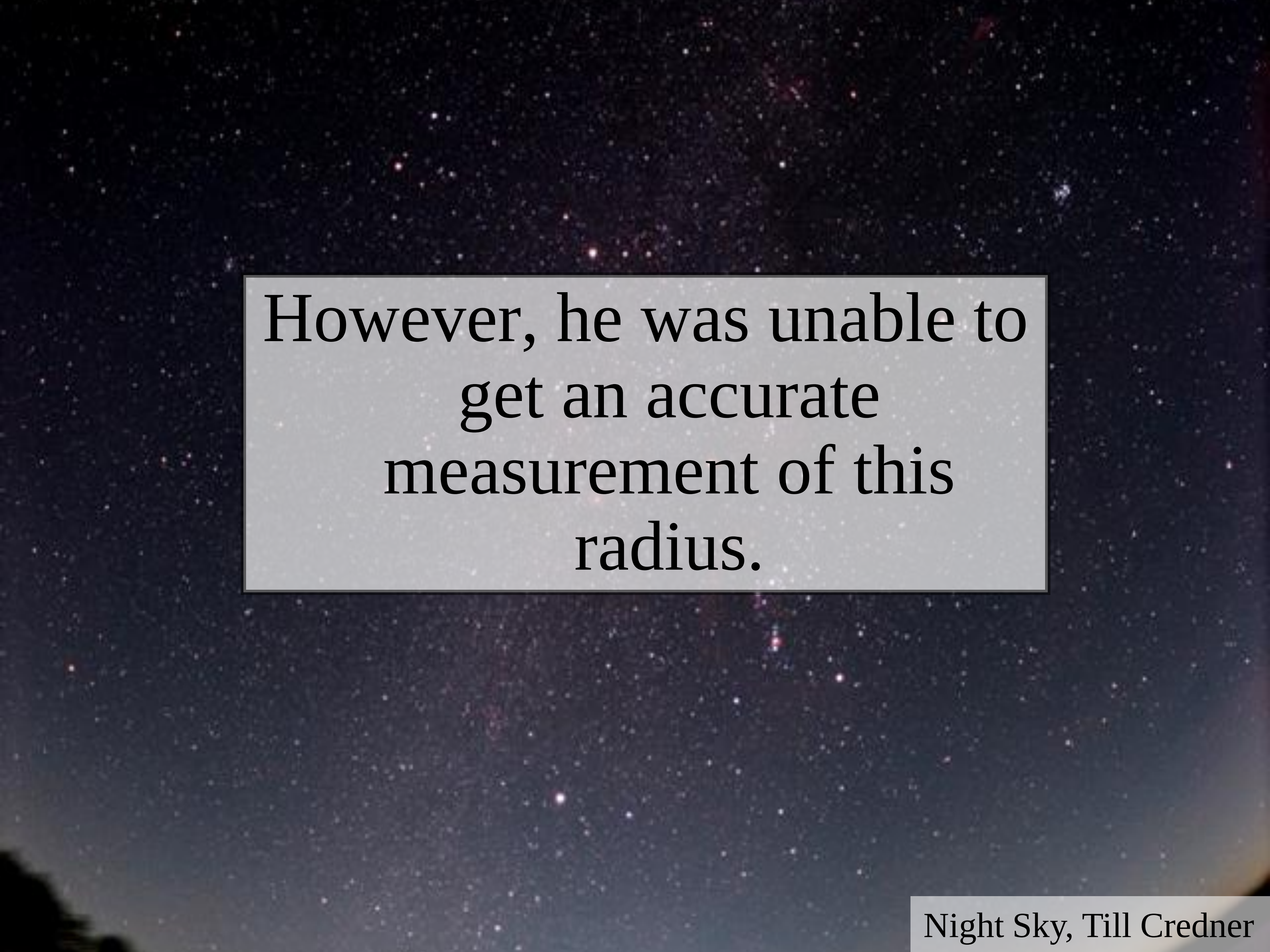
Aristotle also knew there were stars one could see in Egypt but not in Greece.

Night Sky, Till Credner



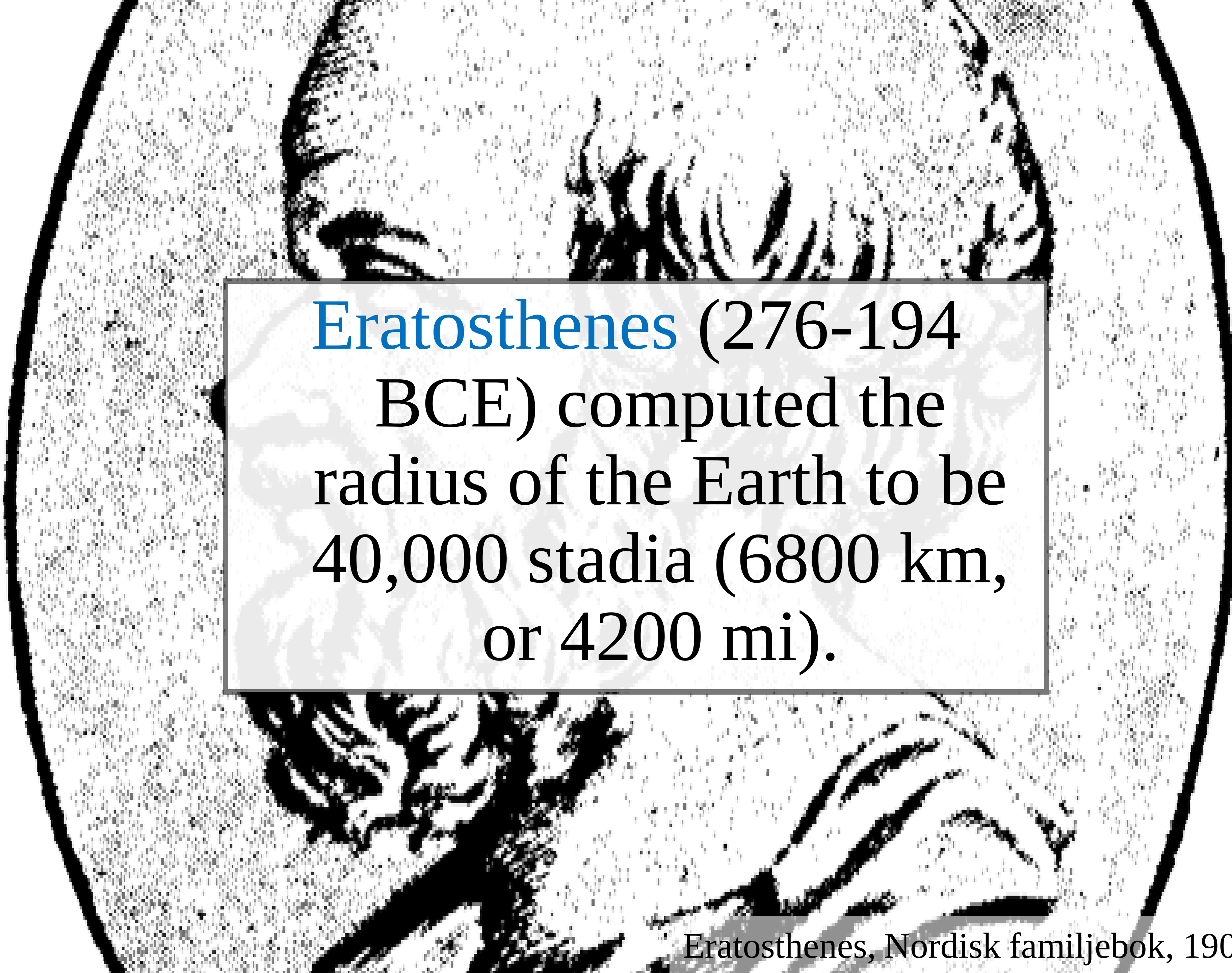
Night Sky, Till Credner

He reasoned that this was due to the curvature of the Earth, so that its radius was finite.



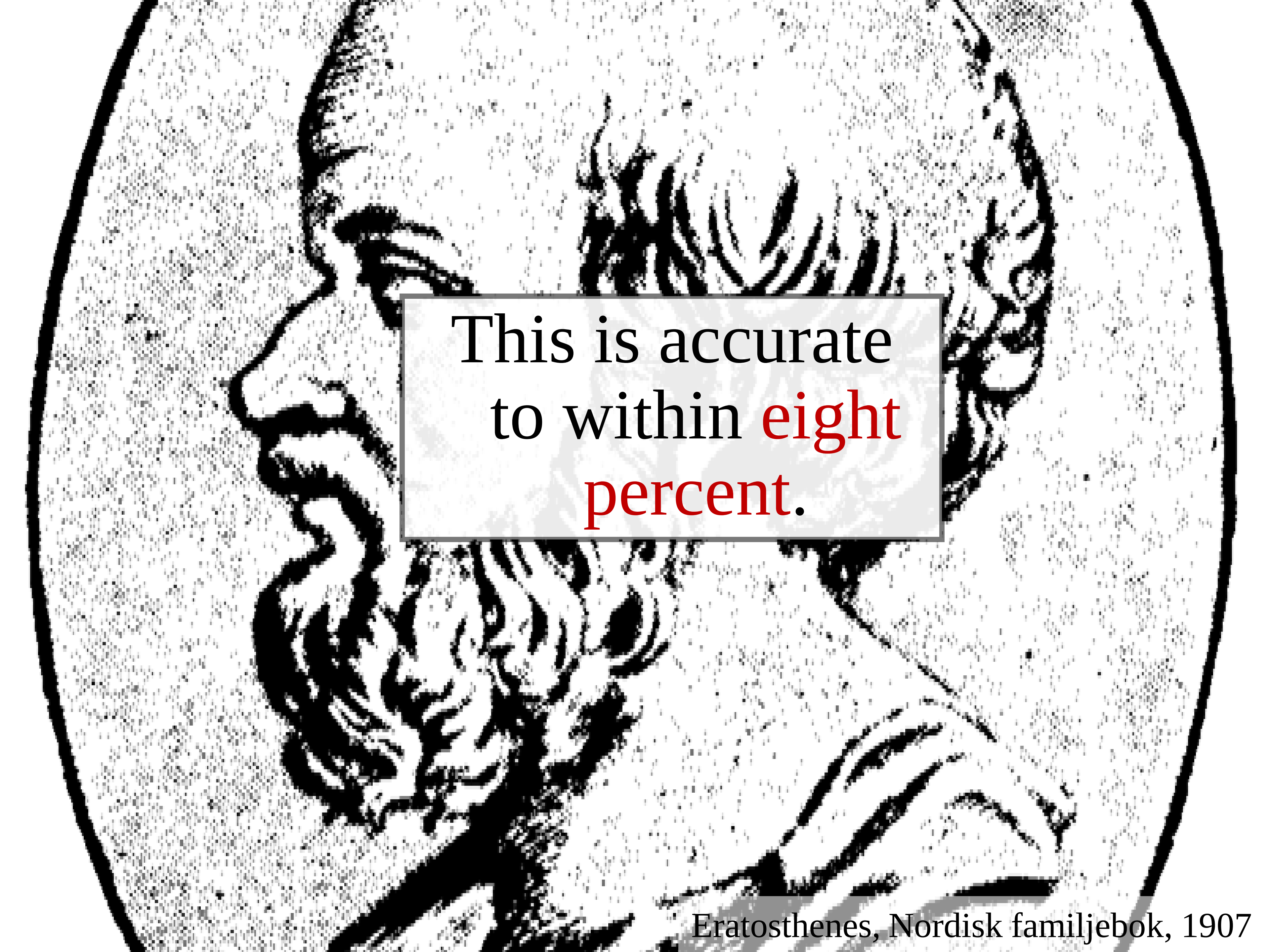
However, he was unable to get an accurate measurement of this radius.

Night Sky, Till Credner

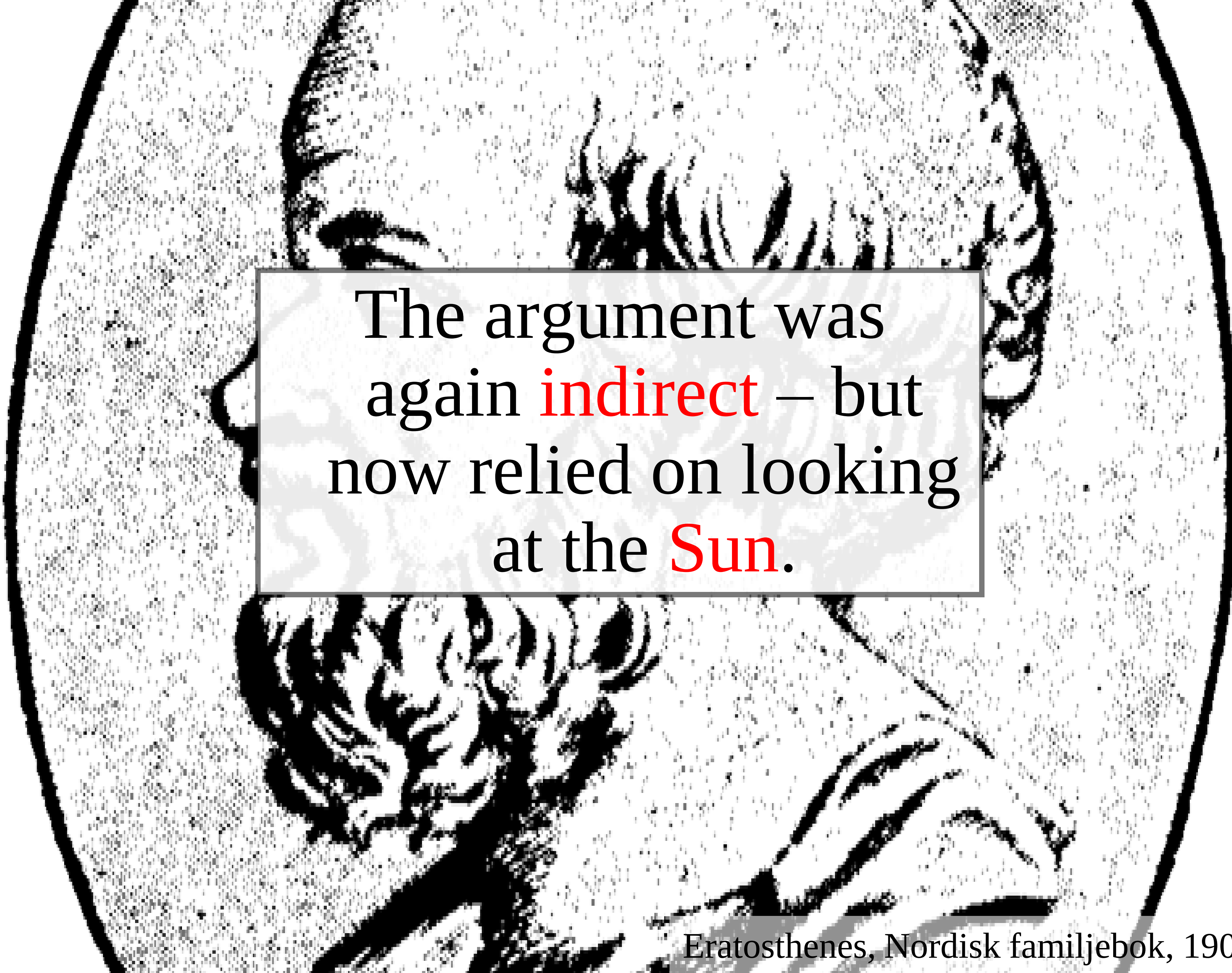


Eratosthenes (276-194 BCE) computed the radius of the Earth to be 40,000 stadia (6800 km, or 4200 mi).

Eratosthenes, Nordisk familjebok, 1907



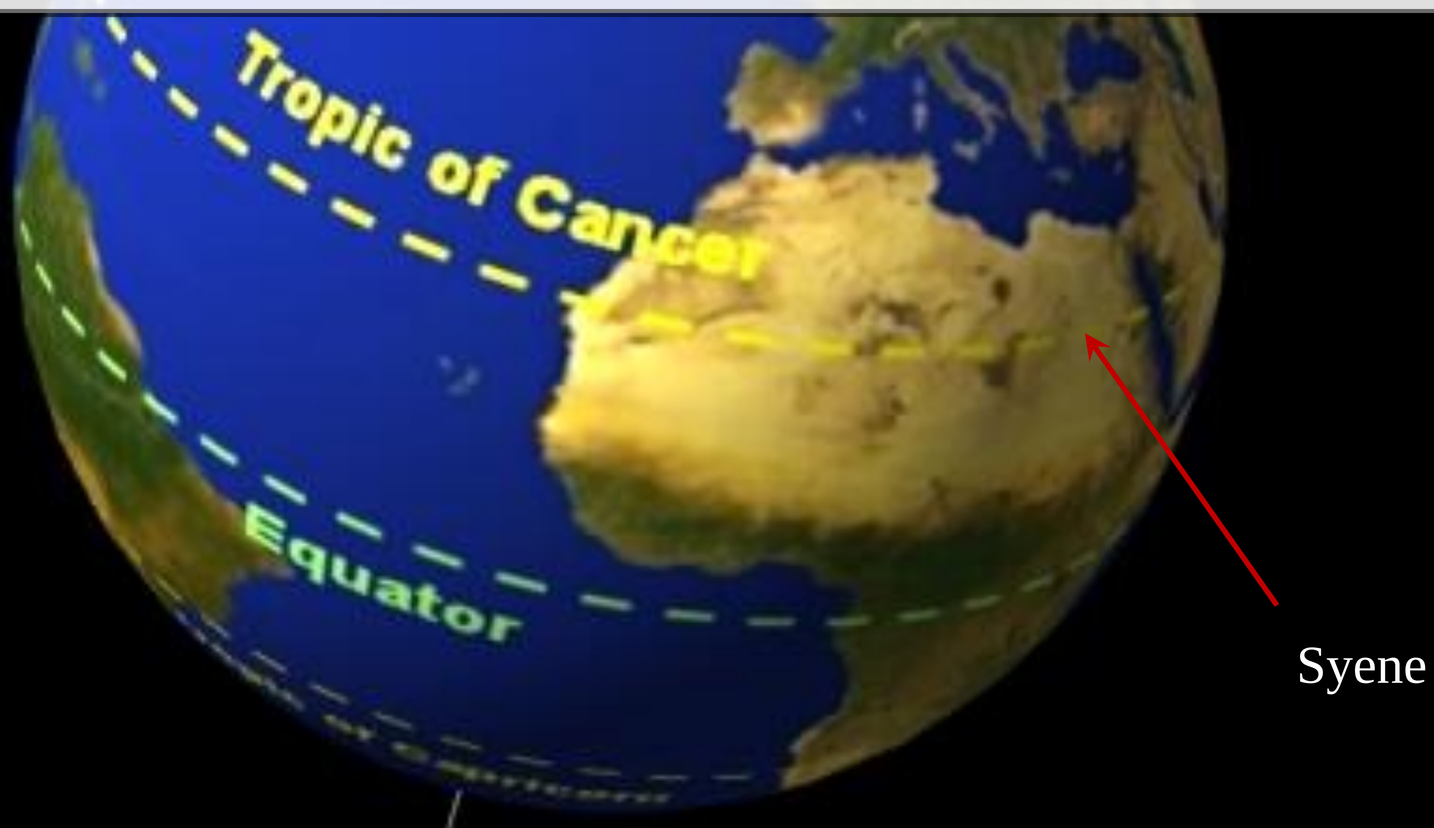
Eratosthenes, Nordisk familjebok, 1907



The argument was again indirect – but now relied on looking at the Sun.

Eratosthenes, Nordisk familjebok, 1907

Eratosthenes read of a well in Syene, Egypt which at noon on the summer solstice (June 21) would reflect the overhead sun.

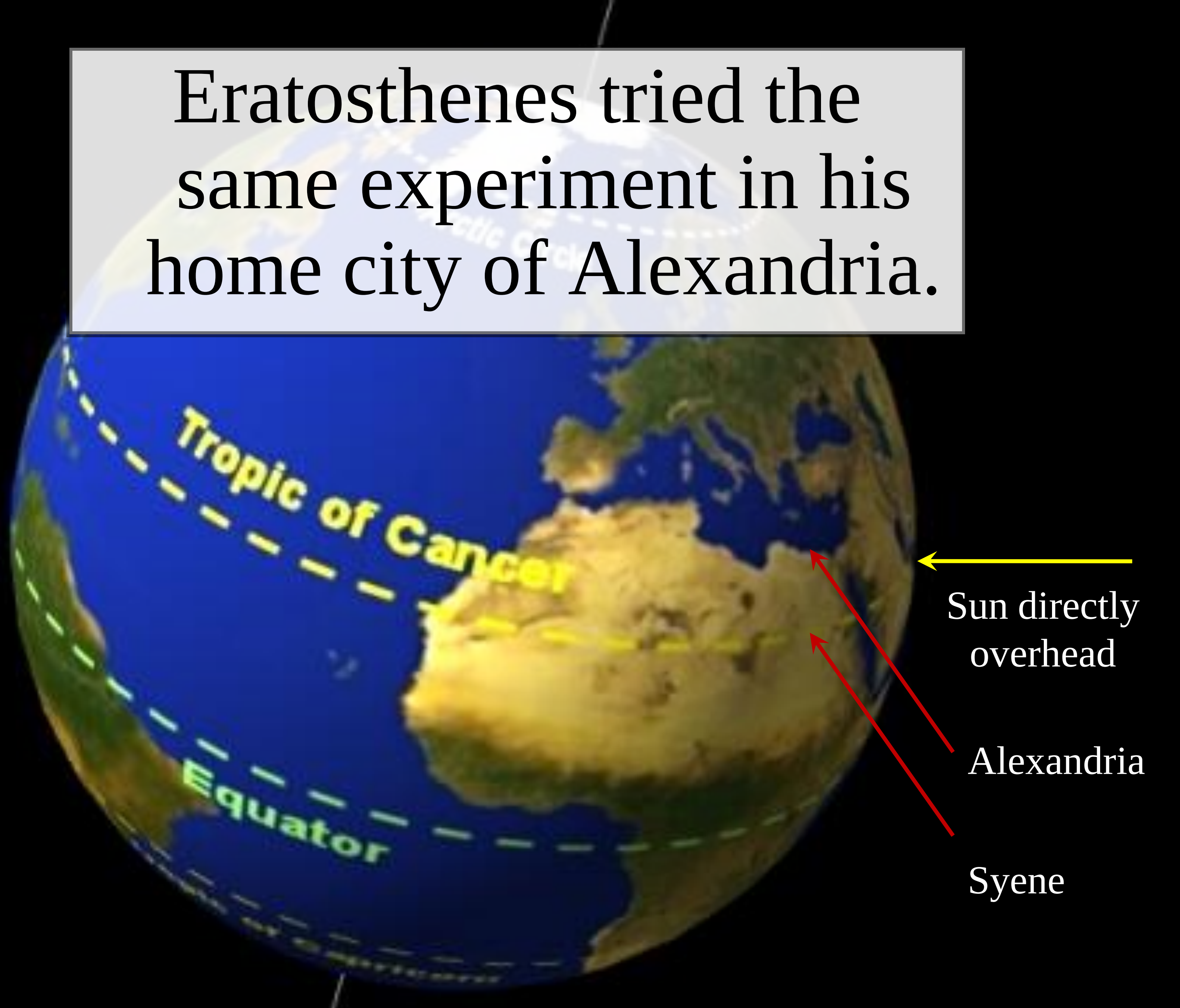


Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

[This is because Syene lies almost directly on the Tropic of Cancer.]

Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

Eratosthenes tried the same experiment in his home city of Alexandria.

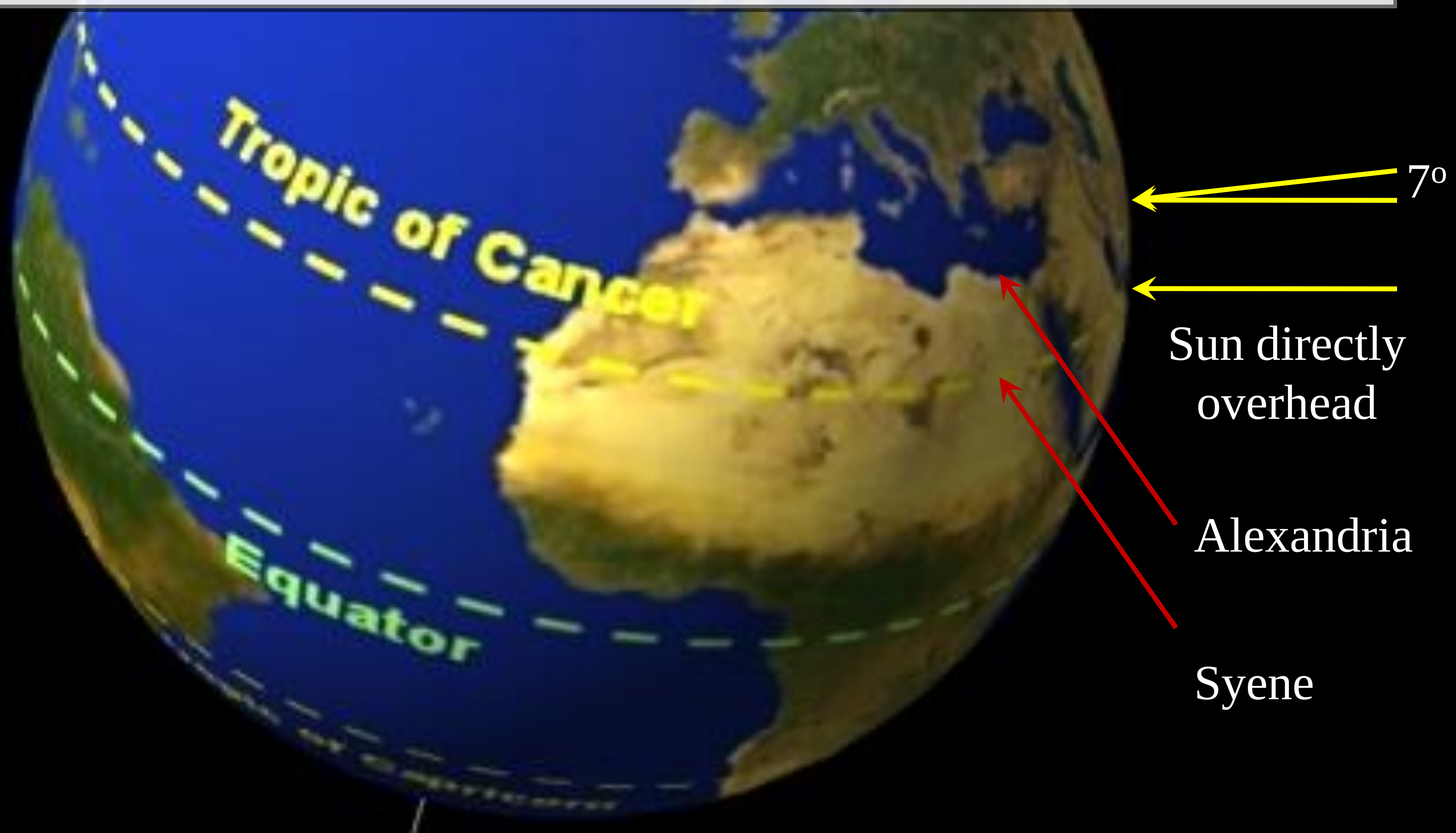


Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

But on the solstice, the sun was at an angle and did not reflect from the bottom of the well.

Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

Using a gnomon (measuring stick), Eratosthenes measured the deviation of the sun from the vertical as 7°.

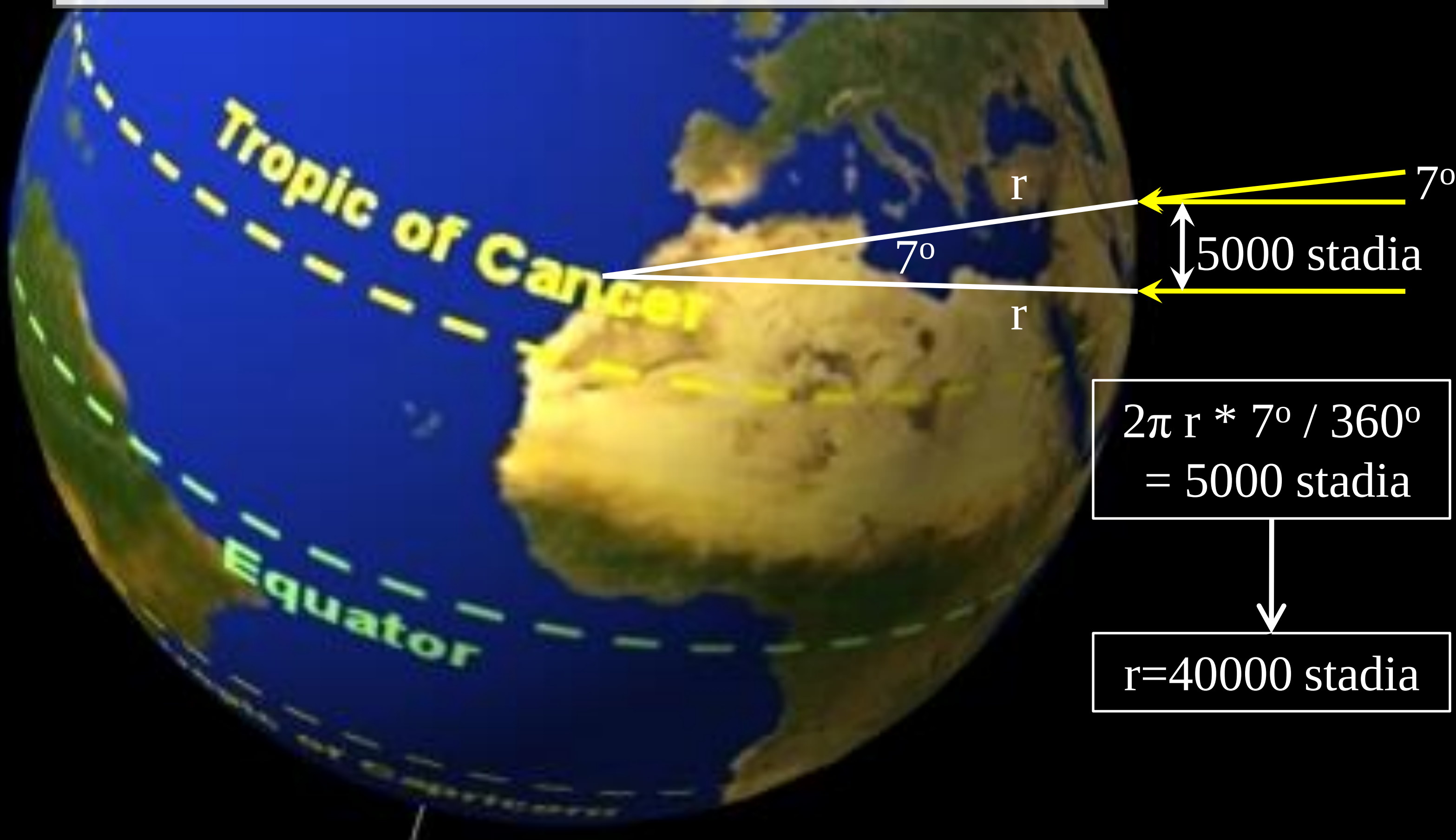


Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

From trade caravans and other sources, Eratosthenes knew Syene to be 5,000 stadia (740 km) south of Alexandria.

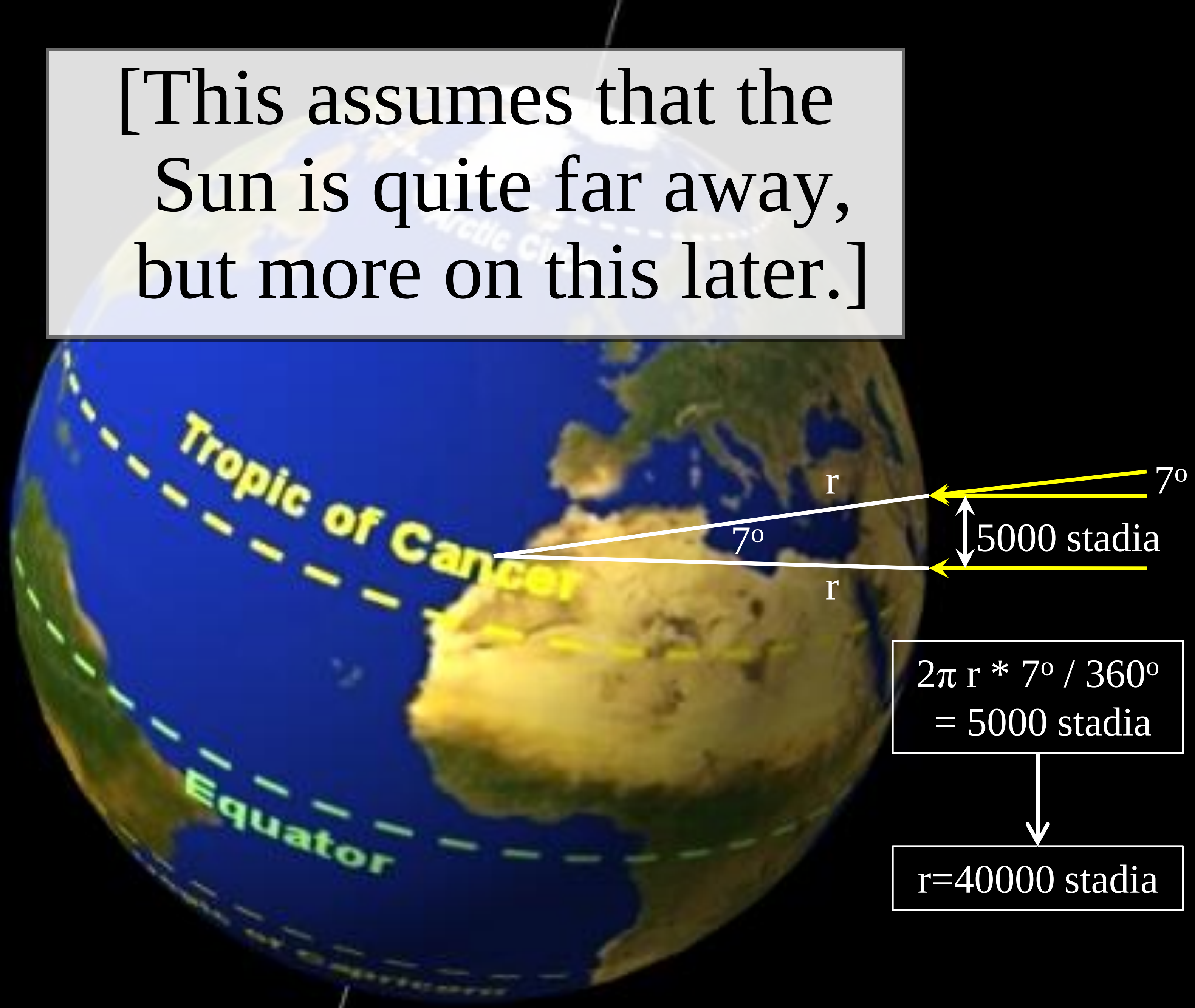
Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

This is enough information to compute the radius of the Earth.



Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

[This assumes that the Sun is quite far away, but more on this later.]

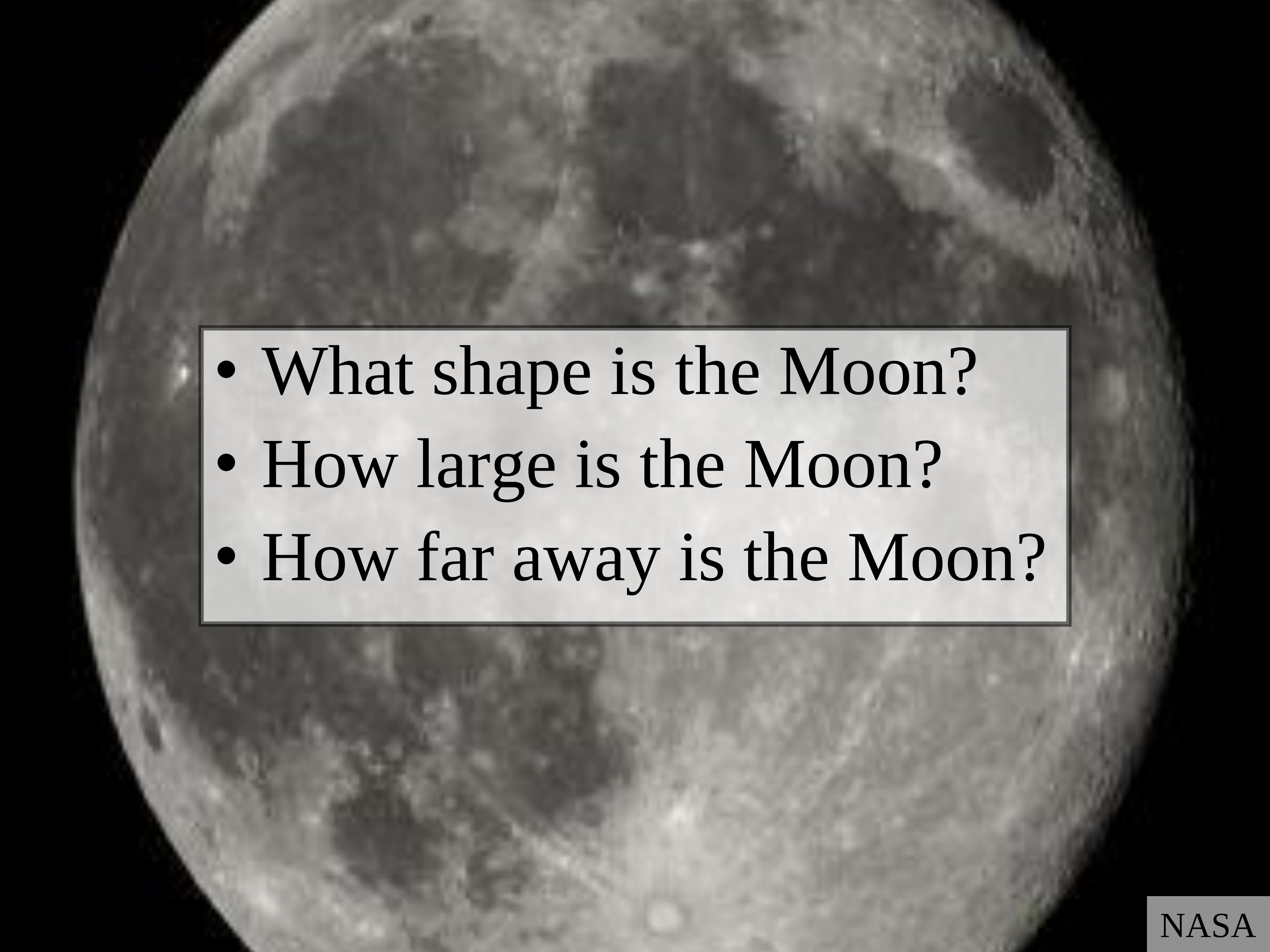


Tropic of Cancer: Swinburne University, COSMOS Encyclopedia of Astronomy

2nd rung: the Moon

NASA

- What shape is the Moon?
- How large is the Moon?
- How far away is the Moon?

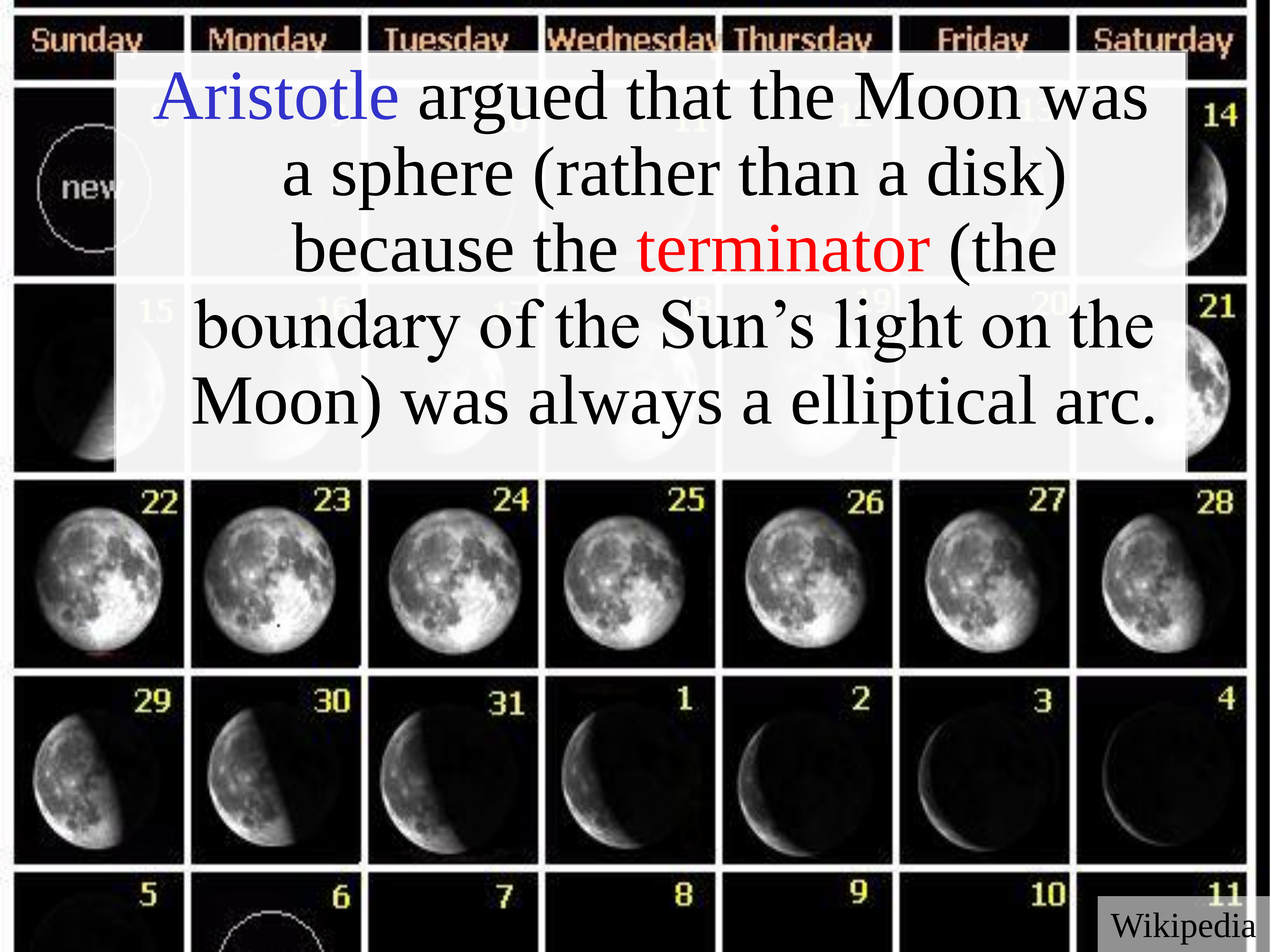


NASA

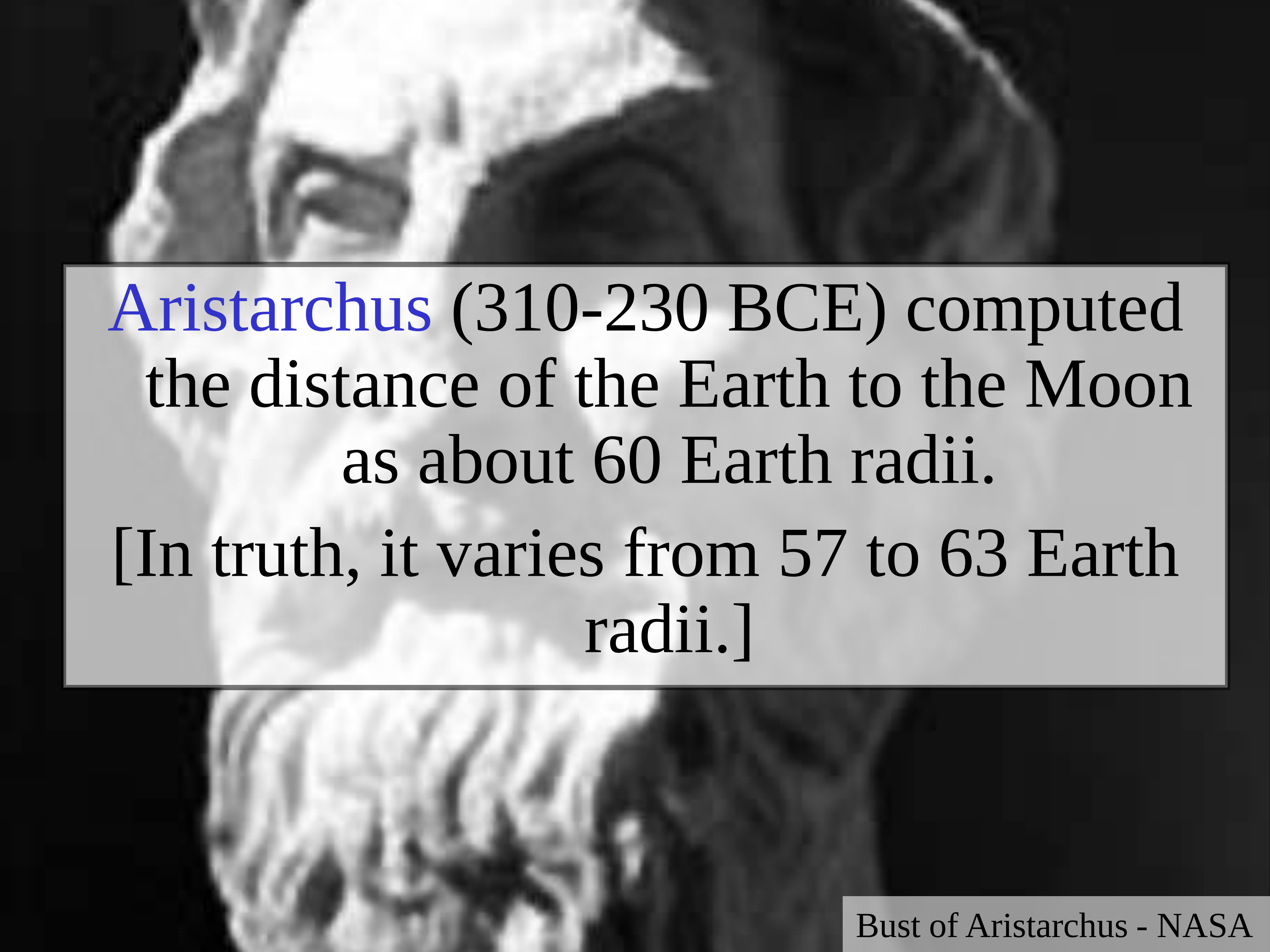
NASA

The ancient Greeks could answer these questions also.

Aristotle argued that the Moon was a sphere (rather than a disk) because the terminator (the boundary of the Sun's light on the Moon) was always a elliptical arc.



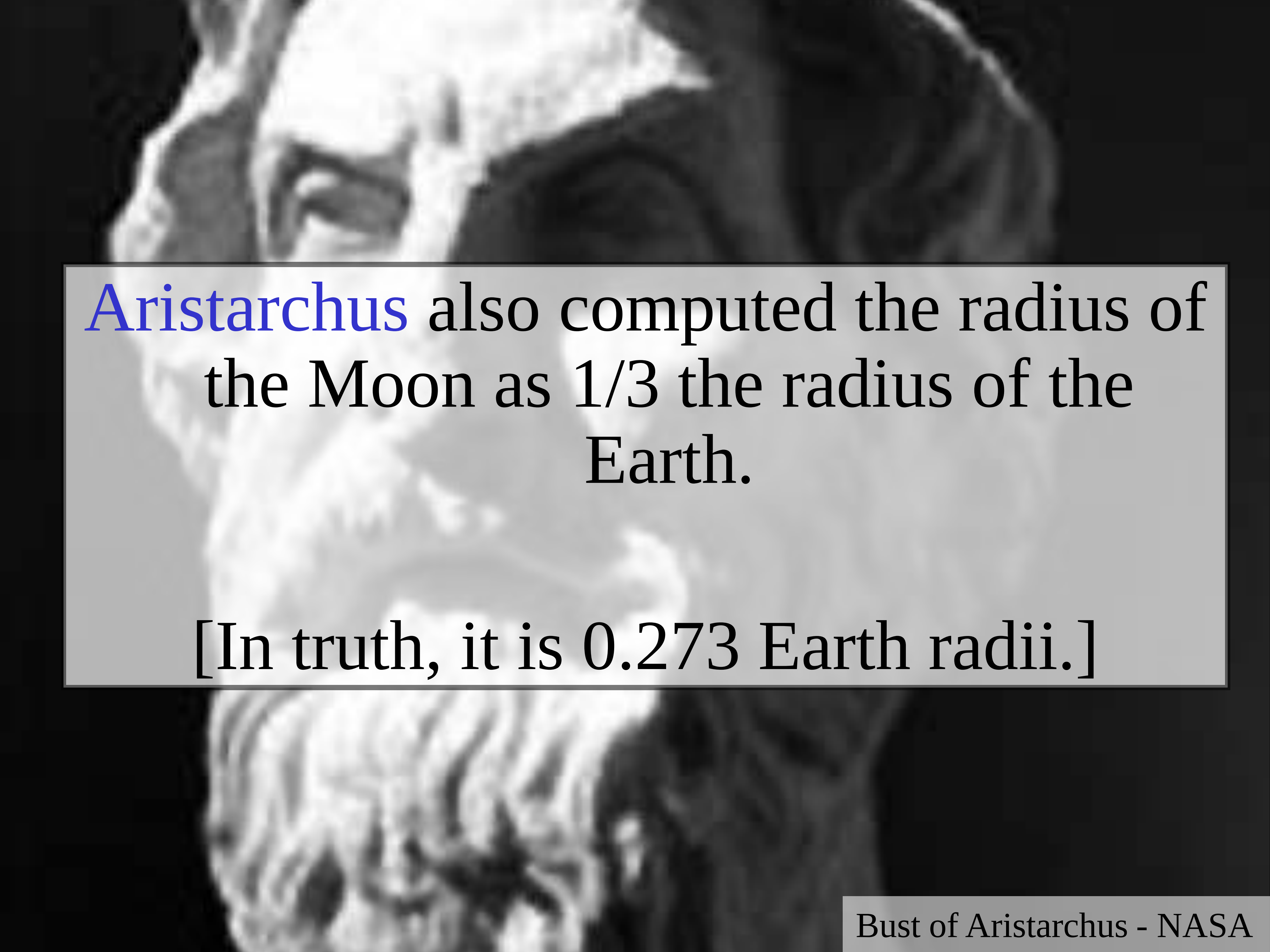
Wikipedia



Aristarchus (310-230 BCE) computed the distance of the Earth to the Moon as about 60 Earth radii.

[In truth, it varies from 57 to 63 Earth radii.]

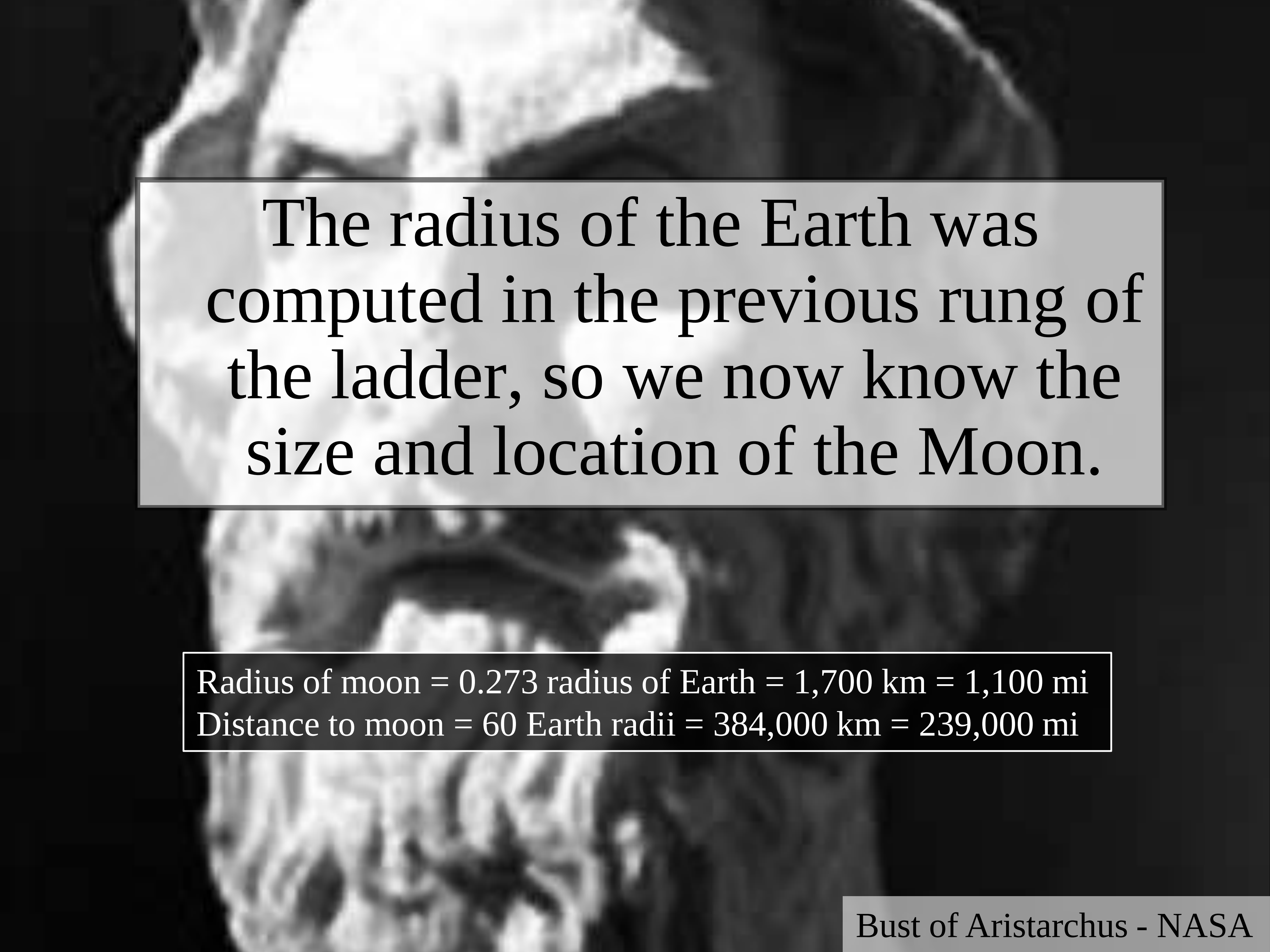
Bust of Aristarchus - NASA



Aristarchus also computed the radius of the Moon as 1/3 the radius of the Earth.

[In truth, it is 0.273 Earth radii.]

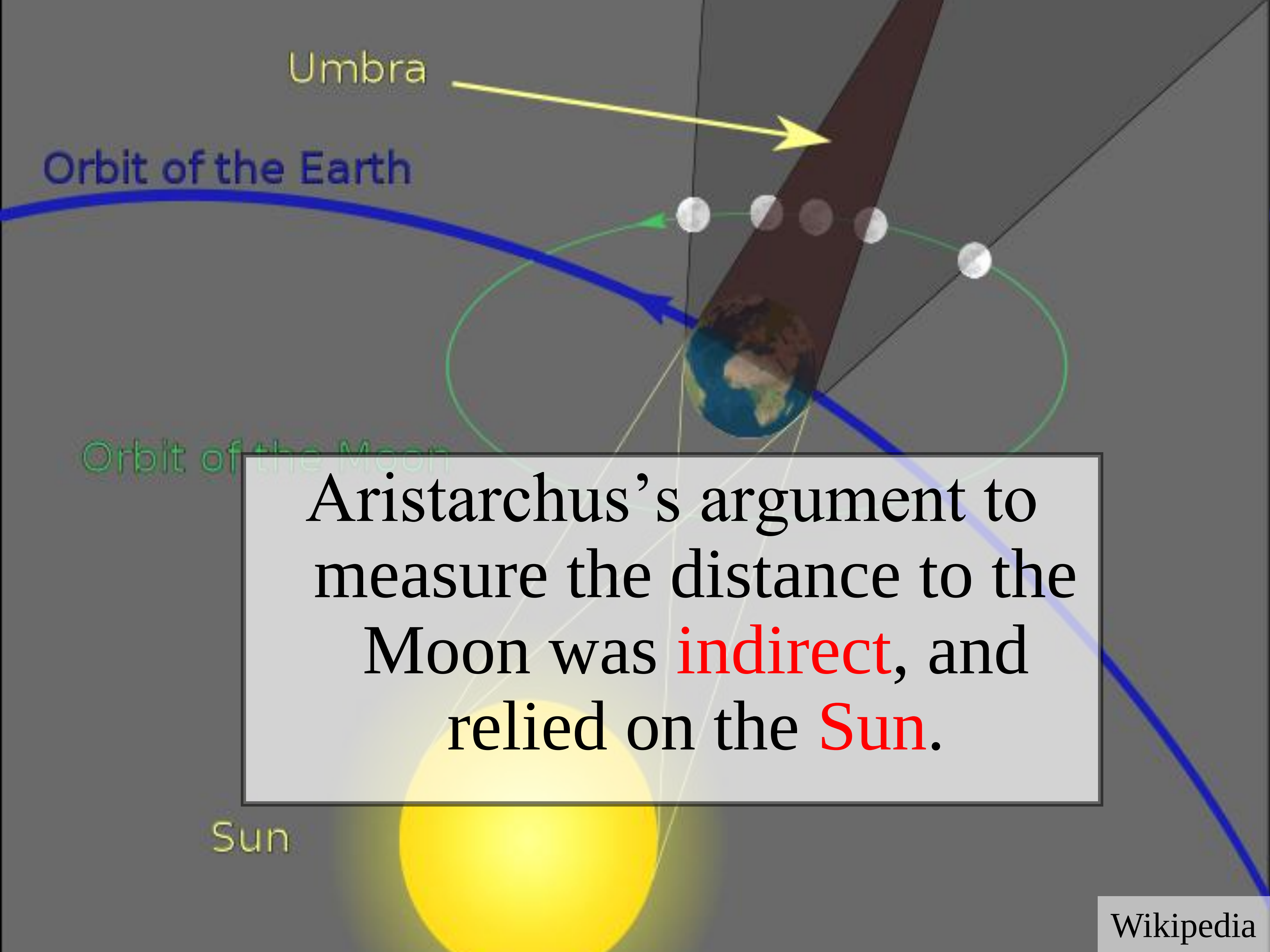
Bust of Aristarchus - NASA



The radius of the Earth was computed in the previous rung of the ladder, so we now know the size and location of the Moon.

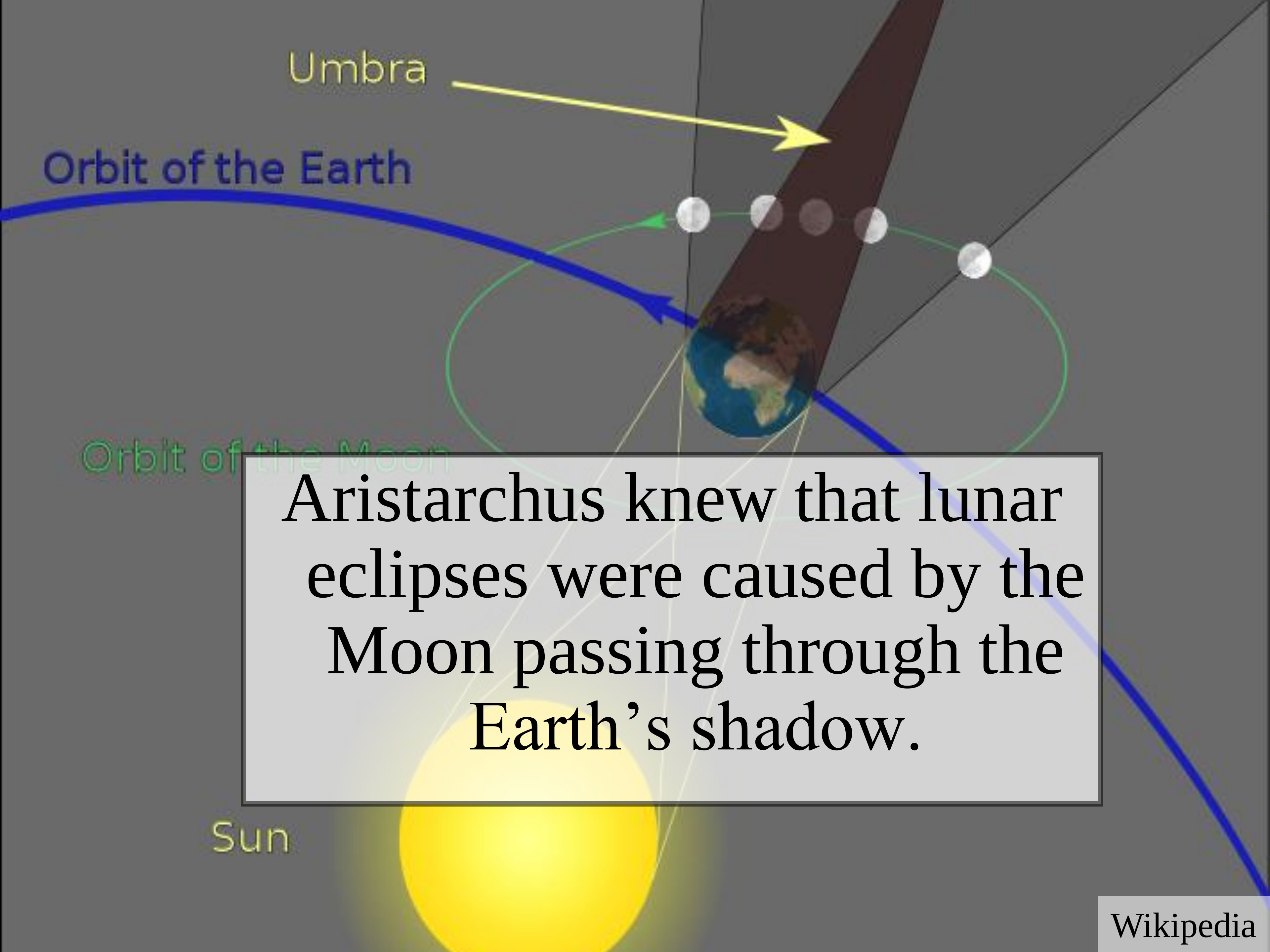
Radius of moon = 0.273 radius of Earth = 1,700 km = 1,100 mi
Distance to moon = 60 Earth radii = 384,000 km = 239,000 mi

Bust of Aristarchus - NASA



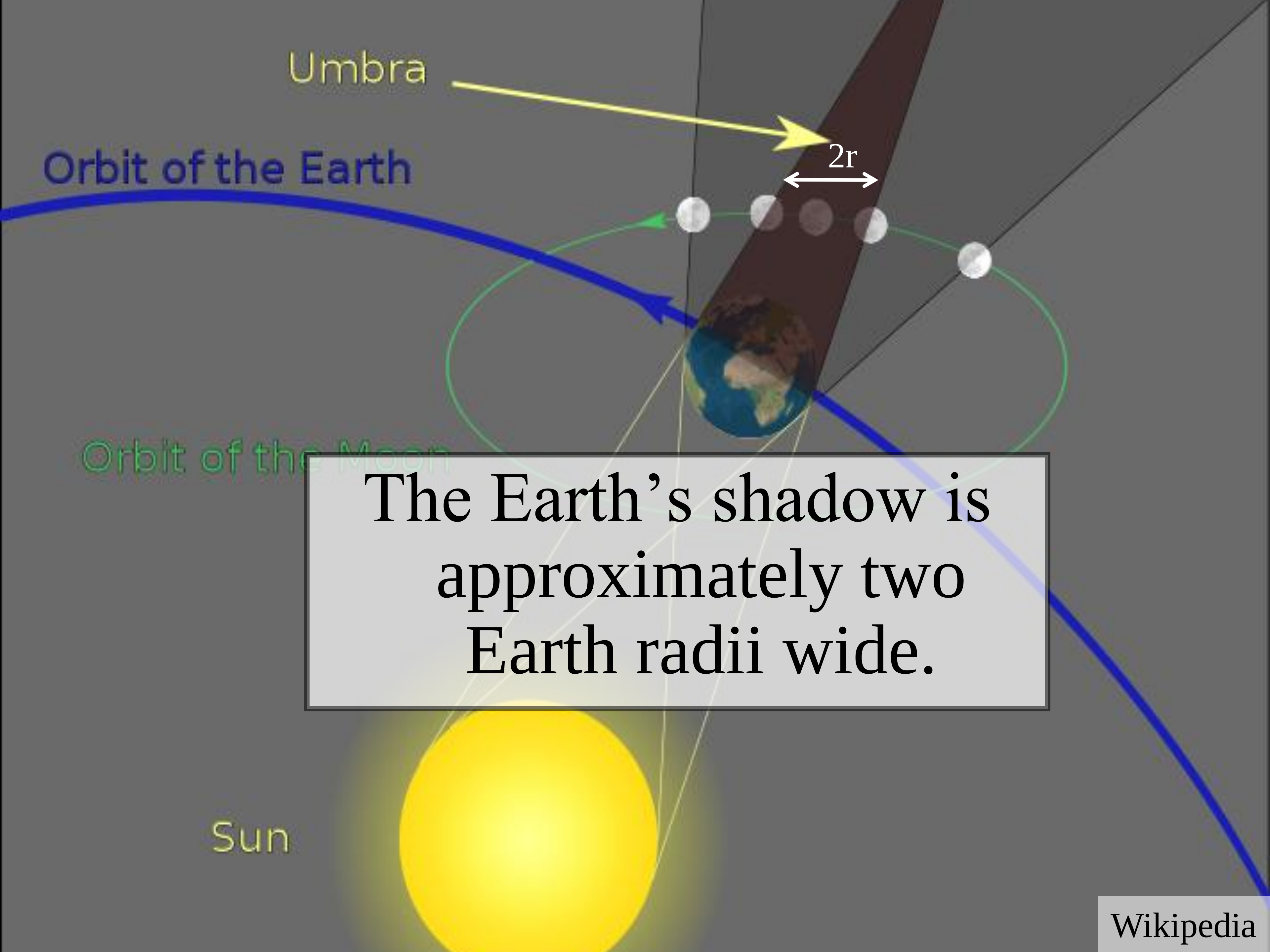
Aristarchus's argument to measure the distance to the Moon was indirect, and relied on the Sun.

Wikipedia

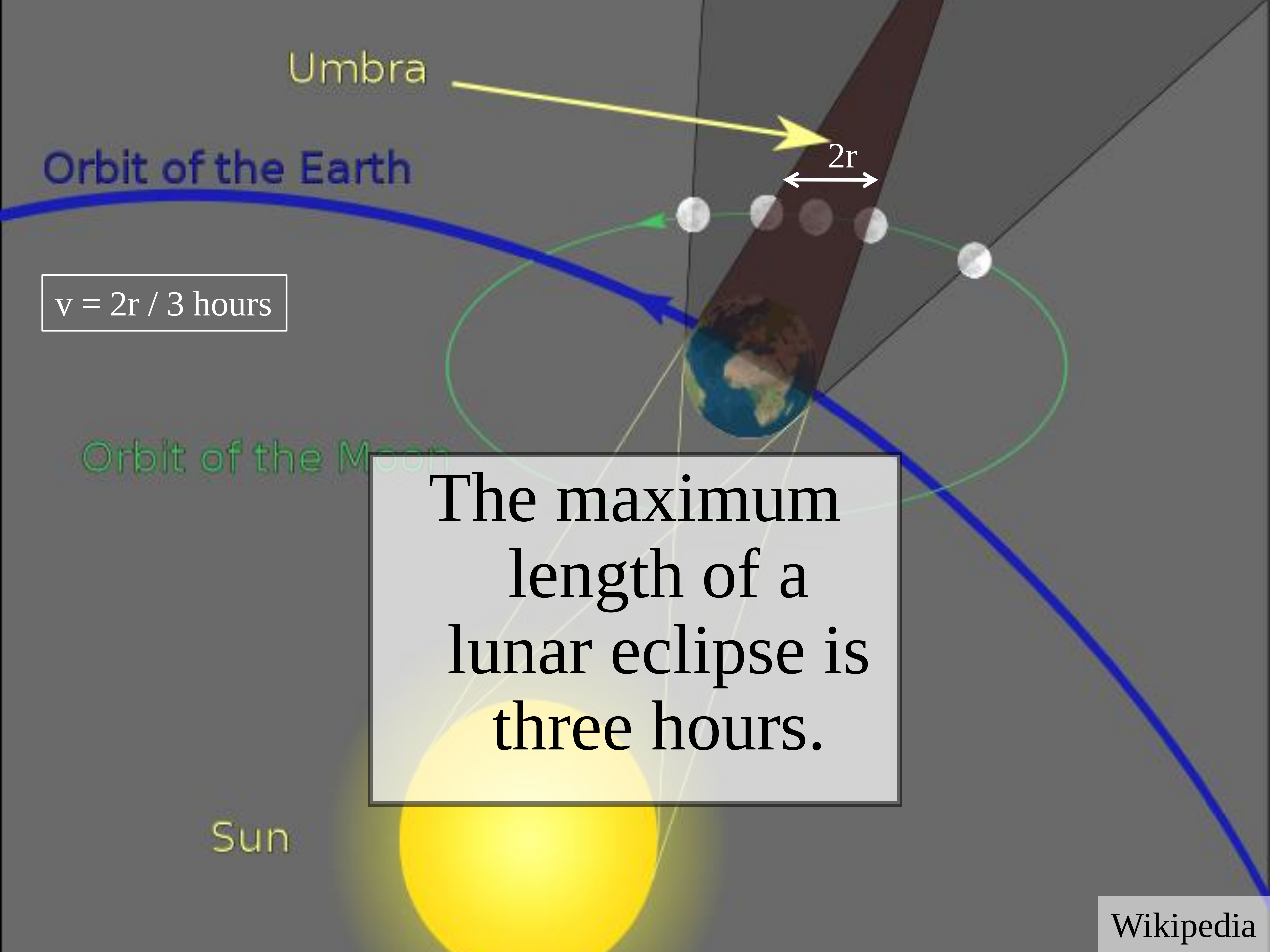


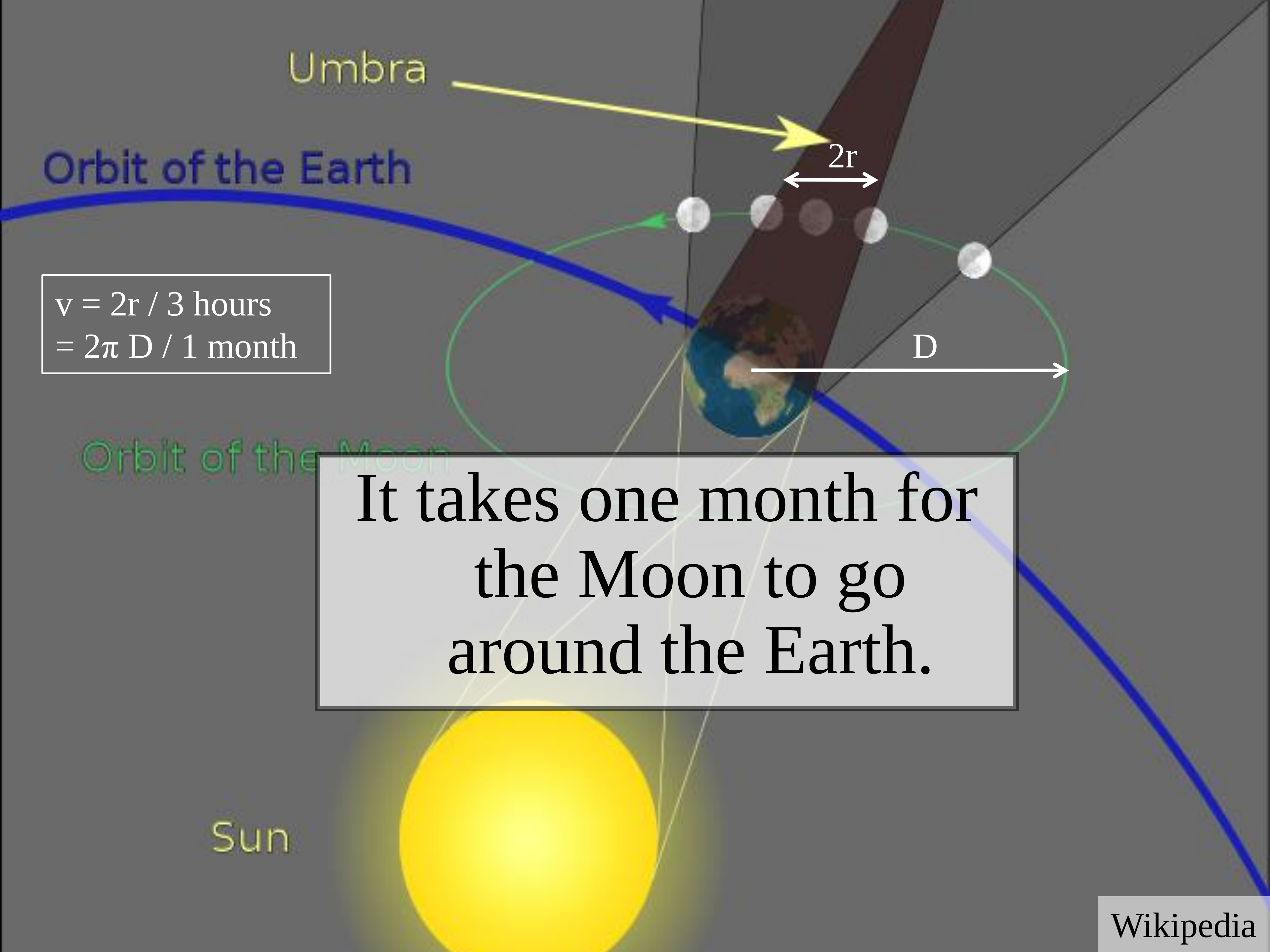
Aristarchus knew that lunar eclipses were caused by the Moon passing through the Earth’s shadow.

Wikipedia

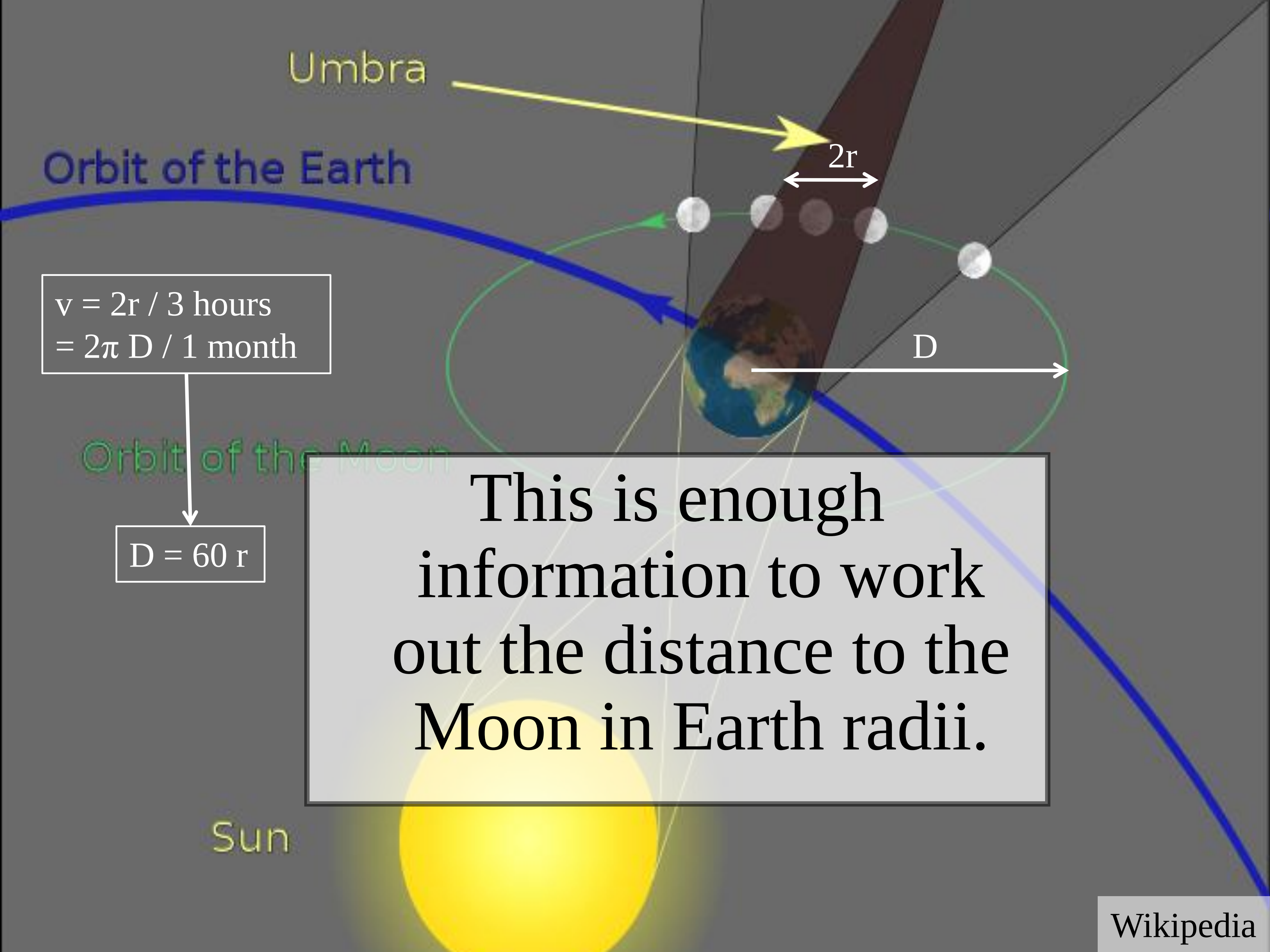


The Earth’s shadow is approximately two Earth radii wide.


Umbra
2r
Orbit of the Earth
v = 2r / 3 hours
Orbit of the Moon
The maximum length of a lunar eclipse is three hours.
Sun
Wikipedia



Umbra
2r
Orbit of the Earth
v = 2r / 3 hours
= 2π D / 1 month
D
Orbit of the Moon
It takes one month for the Moon to go around the Earth.
Sun
Wikipedia



This is enough information to work out the distance to the Moon in Earth radii.

Wikipedia

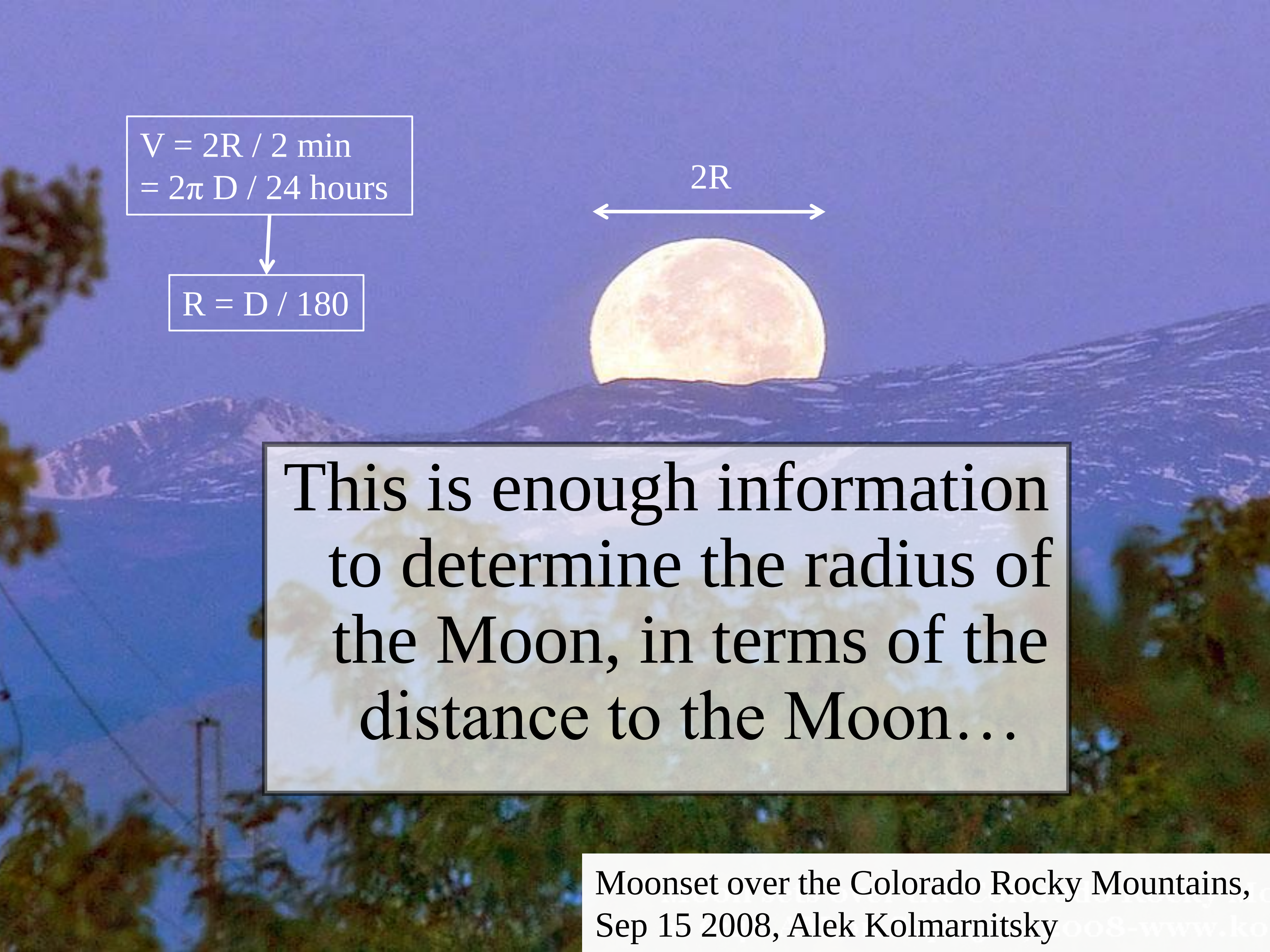
Also, the Moon takes about 2 minutes to set.

Moonset over the Colorado Rocky Mountains, Sep 15 2008, Alek Kolmarnitsky

$$V = 2R / 2\ \text{min} = 2\pi D / 24\ \text{hours}$$

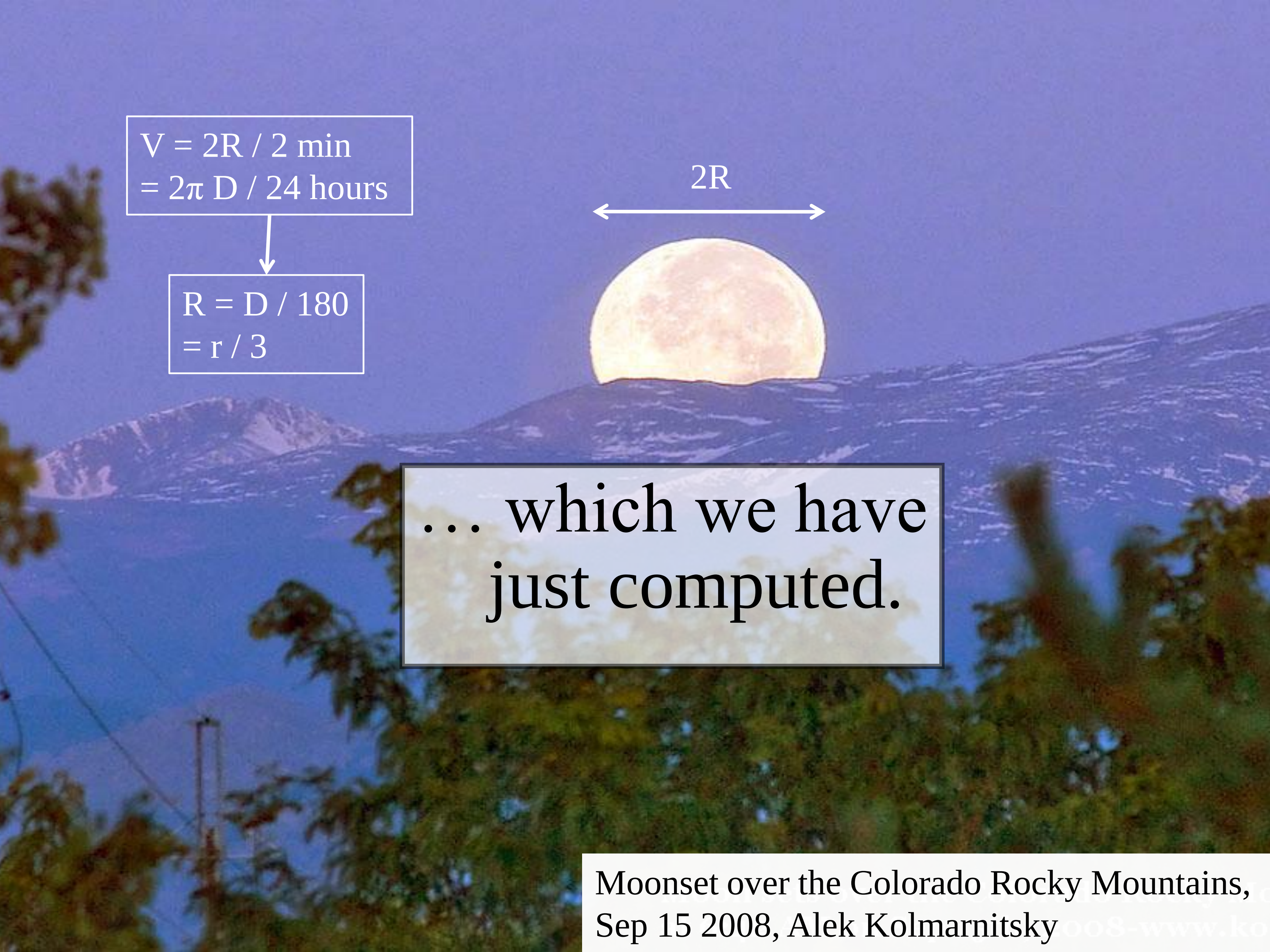
The Moon takes 24 hours to make a full (apparent) rotation around the Earth.

Moonset over the Colorado Rocky Mountains, Sep 15 2008, Alek Kolmarnitsky

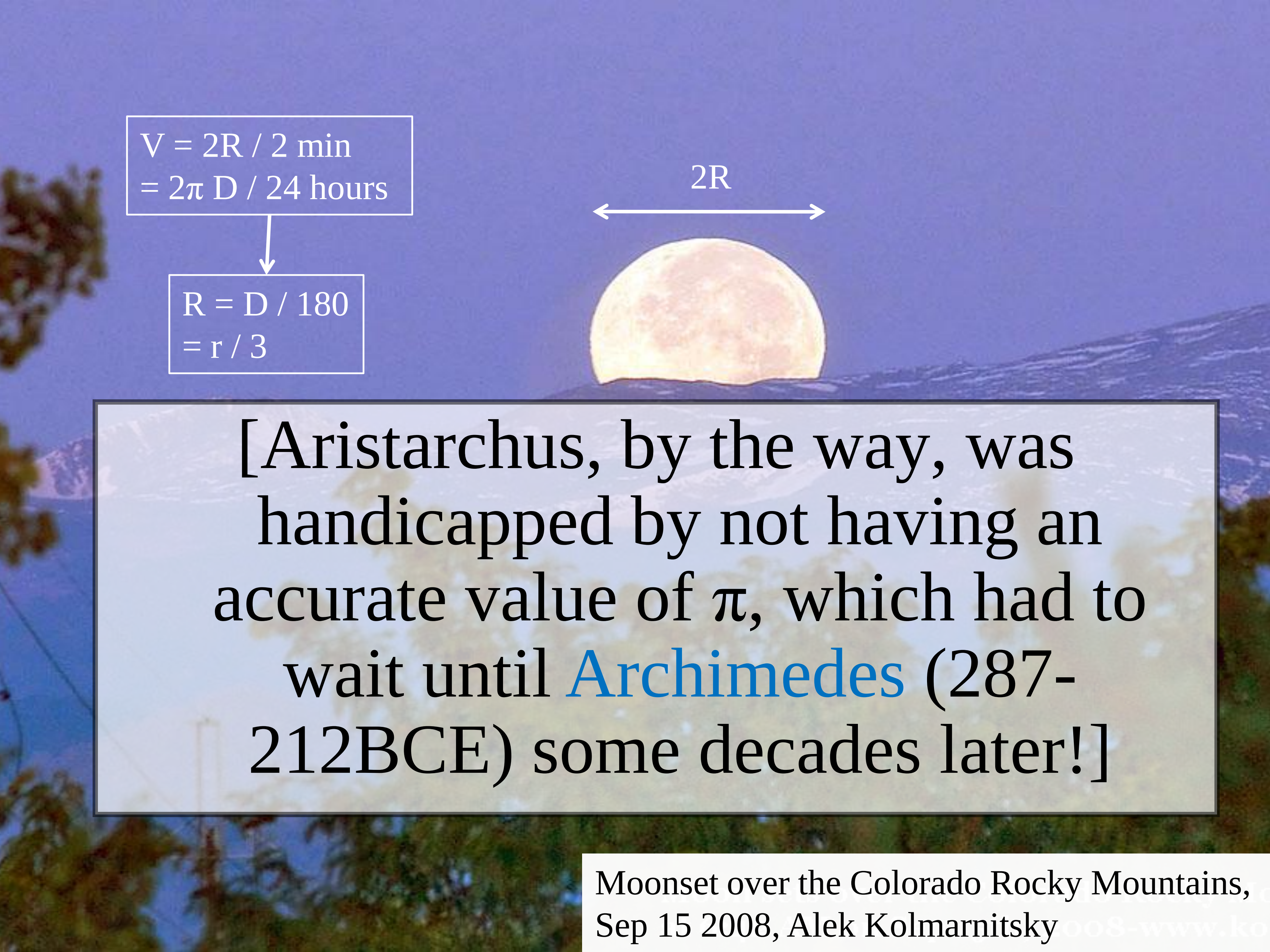


This is enough information to determine the radius of the Moon, in terms of the distance to the Moon…

Moonset over the Colorado Rocky Mountains, Sep 15 2008, Alek Kolmarnitsky



Moonset over the Colorado Rocky Mountains, Sep 15 2008, Alek Kolmarnitsky



[Aristarchus, by the way, was handicapped by not having an accurate value of π, which had to wait until Archimedes (287-212BCE) some decades later!]

Moonset over the Colorado Rocky Mountains, Sep 15 2008, Alek Kolmarnitsky

3rd rung: the Sun

EIT-SOHO Consortium, ESA, NASA

- How large is the Sun?
- How far away is the Sun?

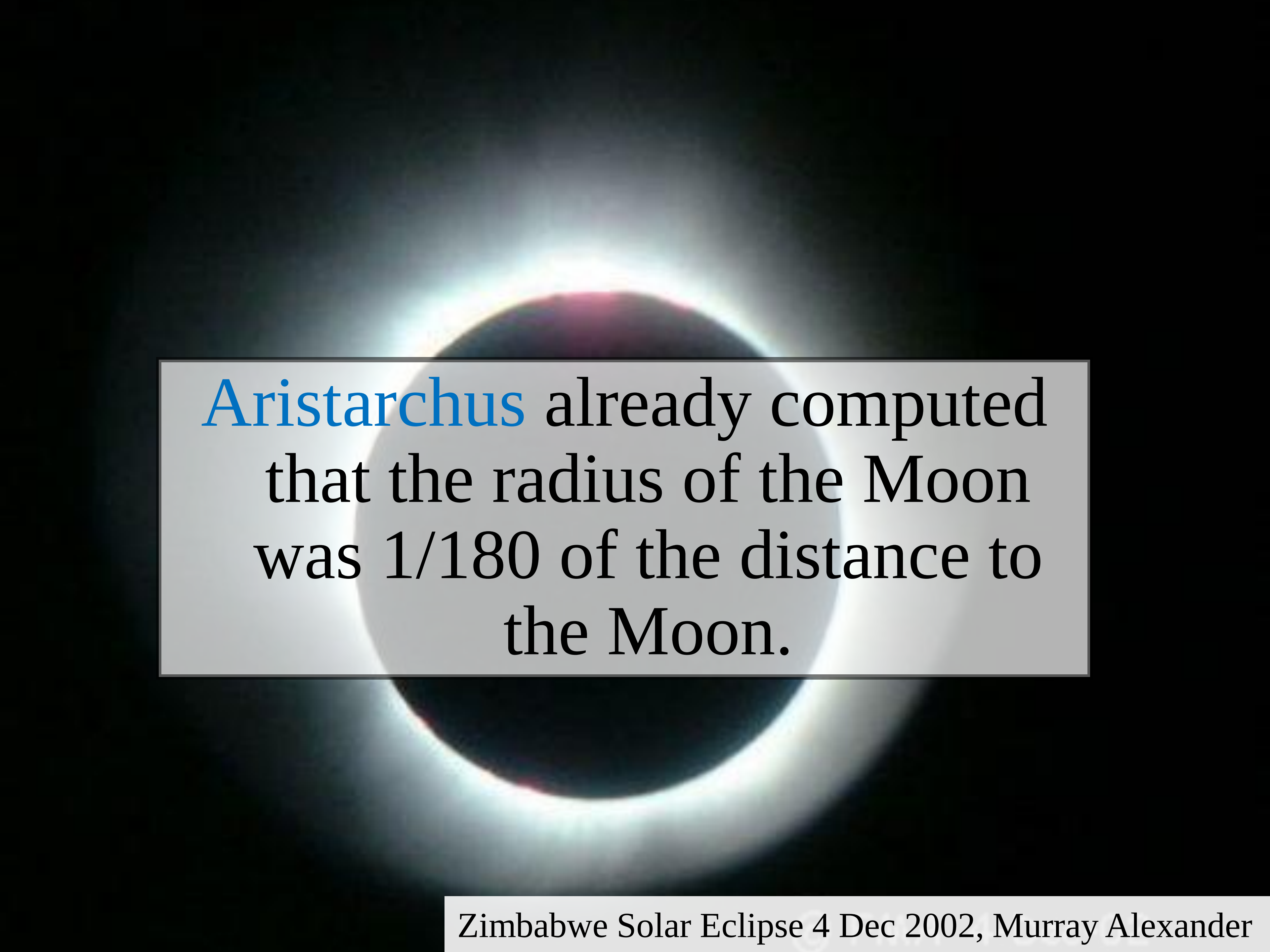
EIT-SOHO Consortium, ESA, NASA

Once again, the ancient Greeks could answer these questions (but with imperfect accuracy).

EIT-SOHO Consortium, ESA, NASA

Their methods were indirect, and relied on the Moon.

EIT-SOHO Consortium, ESA, NASA

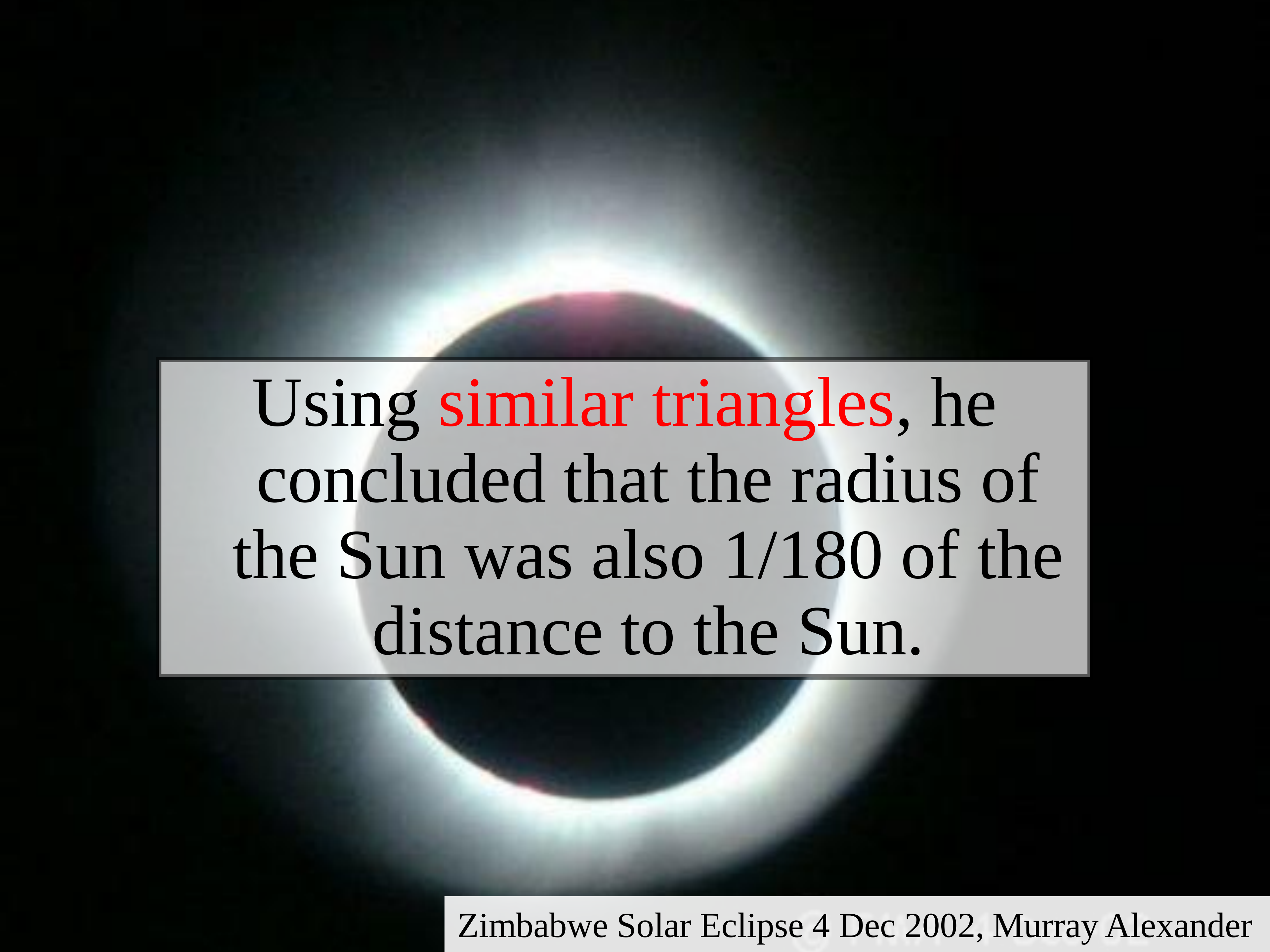


Aristarchus already computed that the radius of the Moon was 1/180 of the distance to the Moon.

Zimbabwe Solar Eclipse 4 Dec 2002, Murray Alexander

He also knew that during a solar eclipse, the Moon covered the Sun almost perfectly.

Zimbabwe Solar Eclipse 4 Dec 2002, Murray Alexander



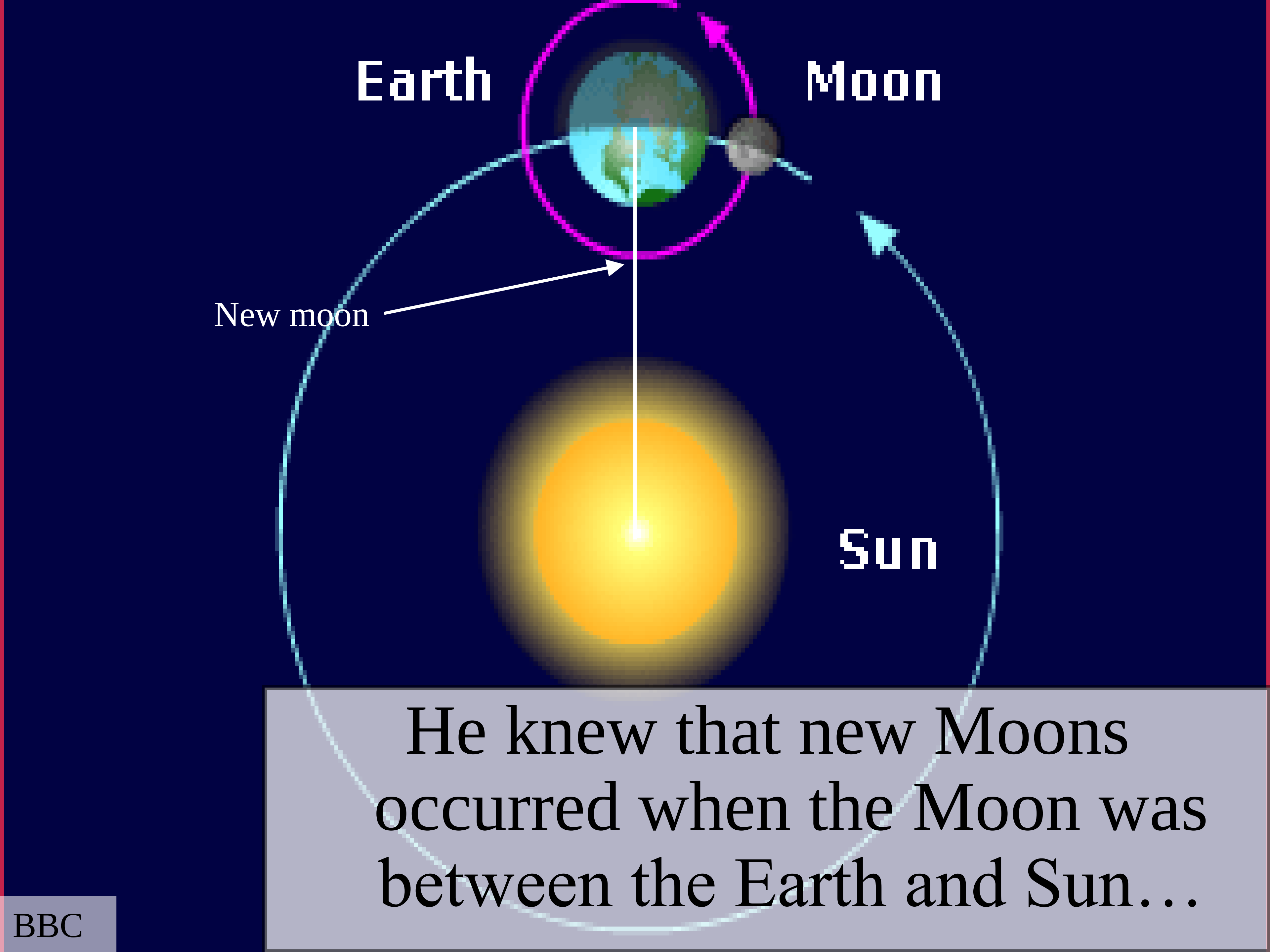
Using similar triangles, he concluded that the radius of the Sun was also 1/180 of the distance to the Sun.

Zimbabwe Solar Eclipse 4 Dec 2002, Murray Alexander

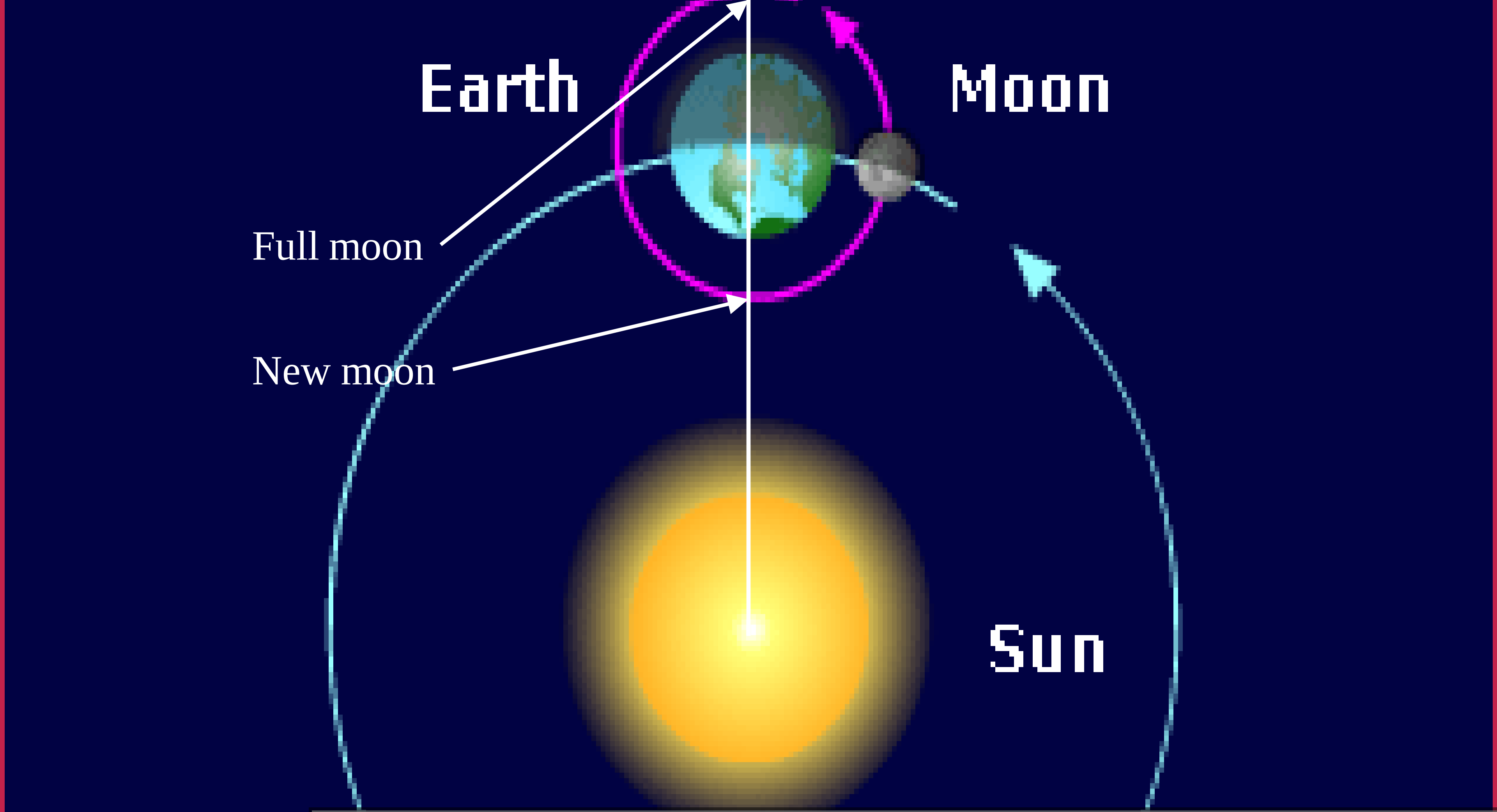
So his next task was to compute the distance to the Sun.

Zimbabwe Solar Eclipse 4 Dec 2002, Murray Alexander

Zimbabwe Solar Eclipse 4 Dec 2002, Murray Alexander

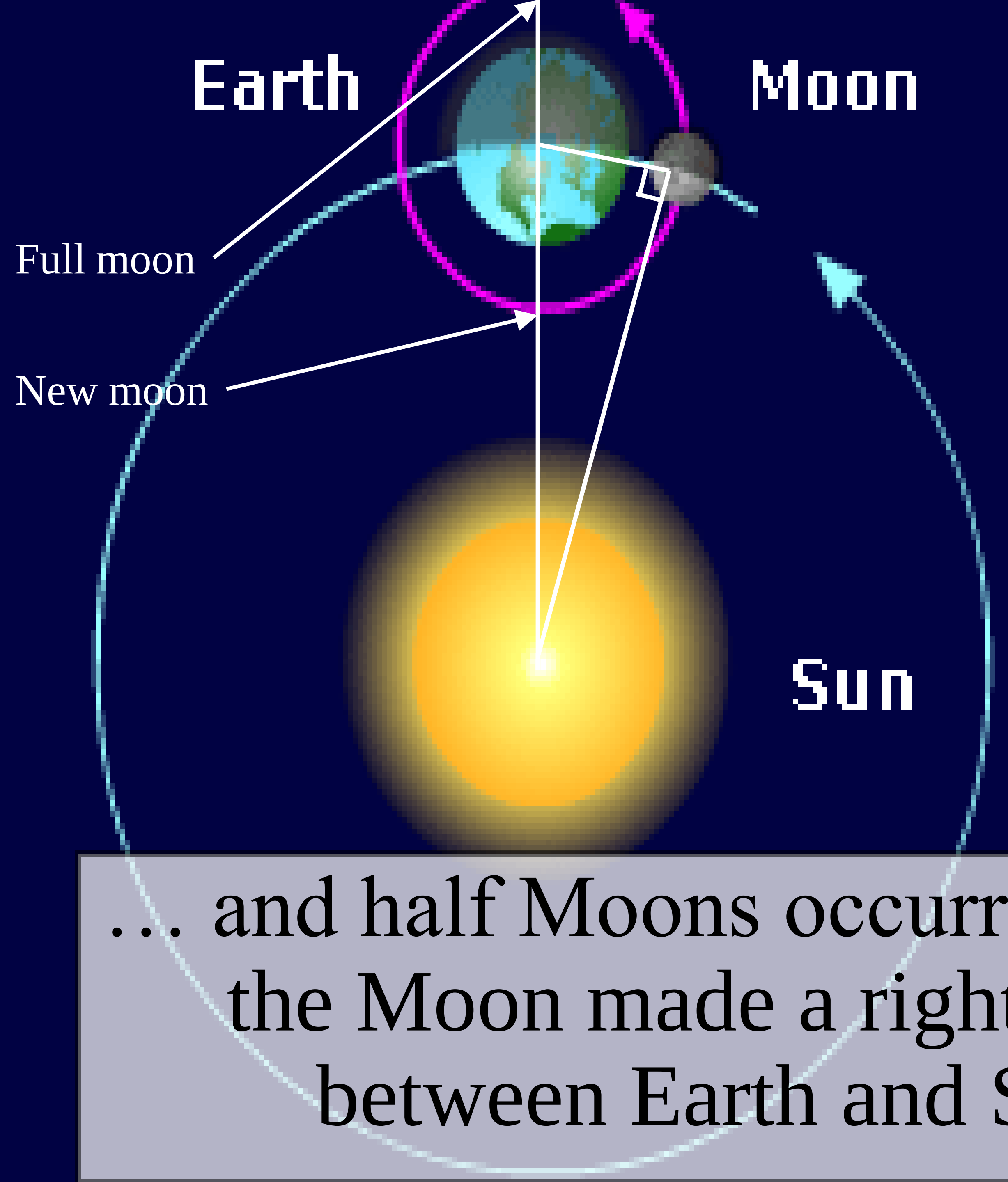


He knew that new Moons occurred when the Moon was between the Earth and Sun…



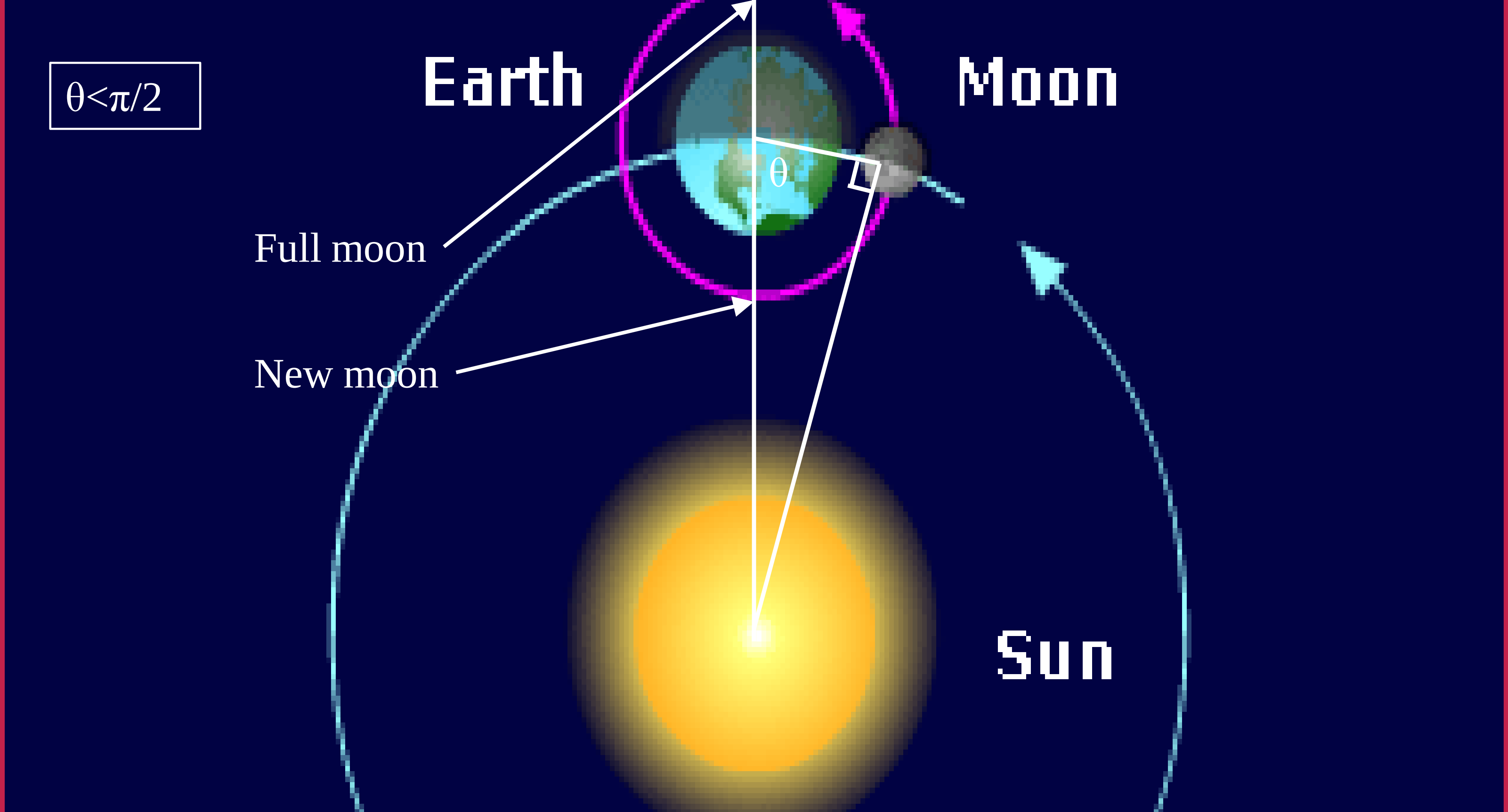
… full Moons occurred when the Moon was directly opposite the Sun…

BBC



… and half Moons occurred when the Moon made a right angle between Earth and Sun.

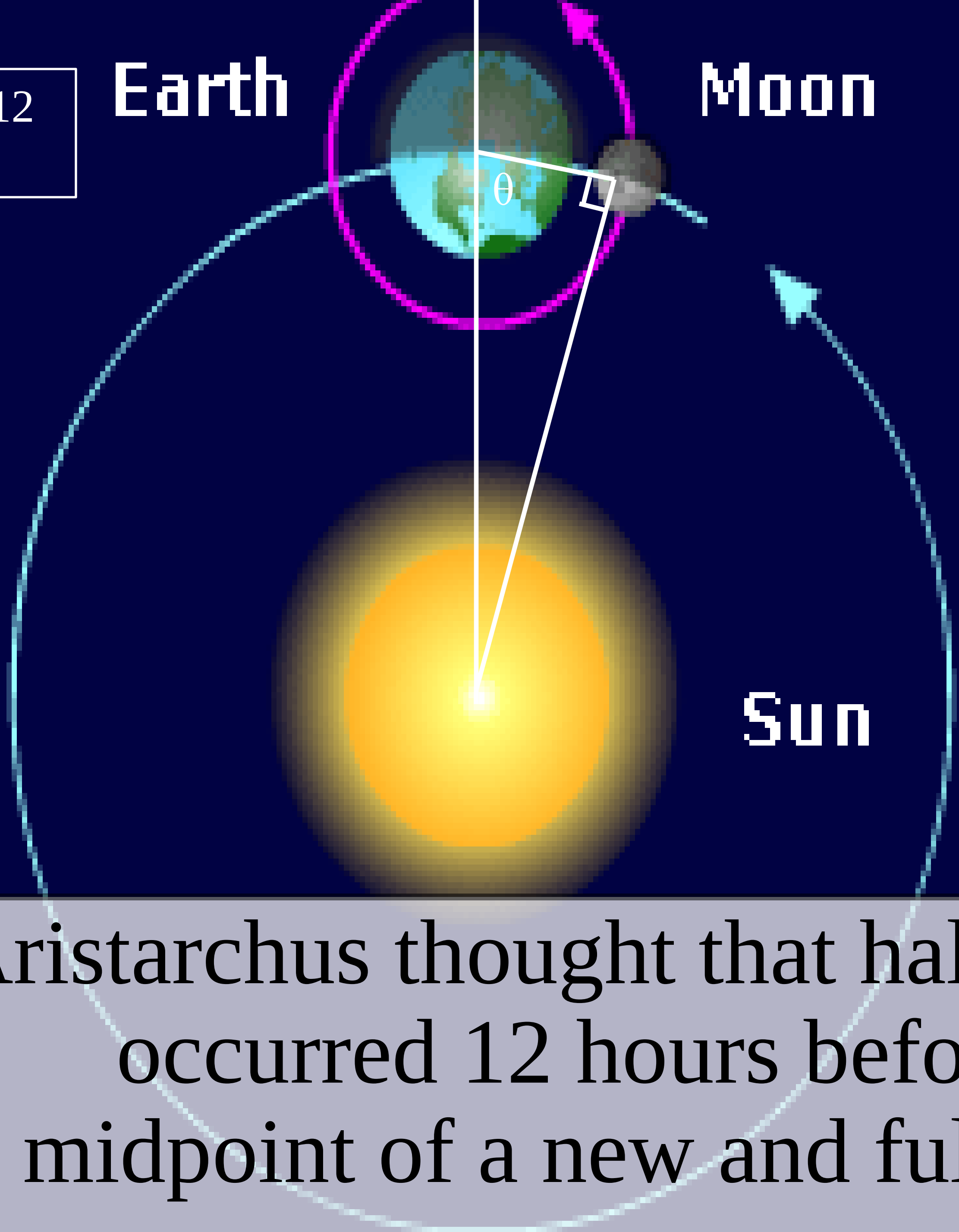
BBC



This implies that half Moons occur slightly closer to new Moons than to full Moons.

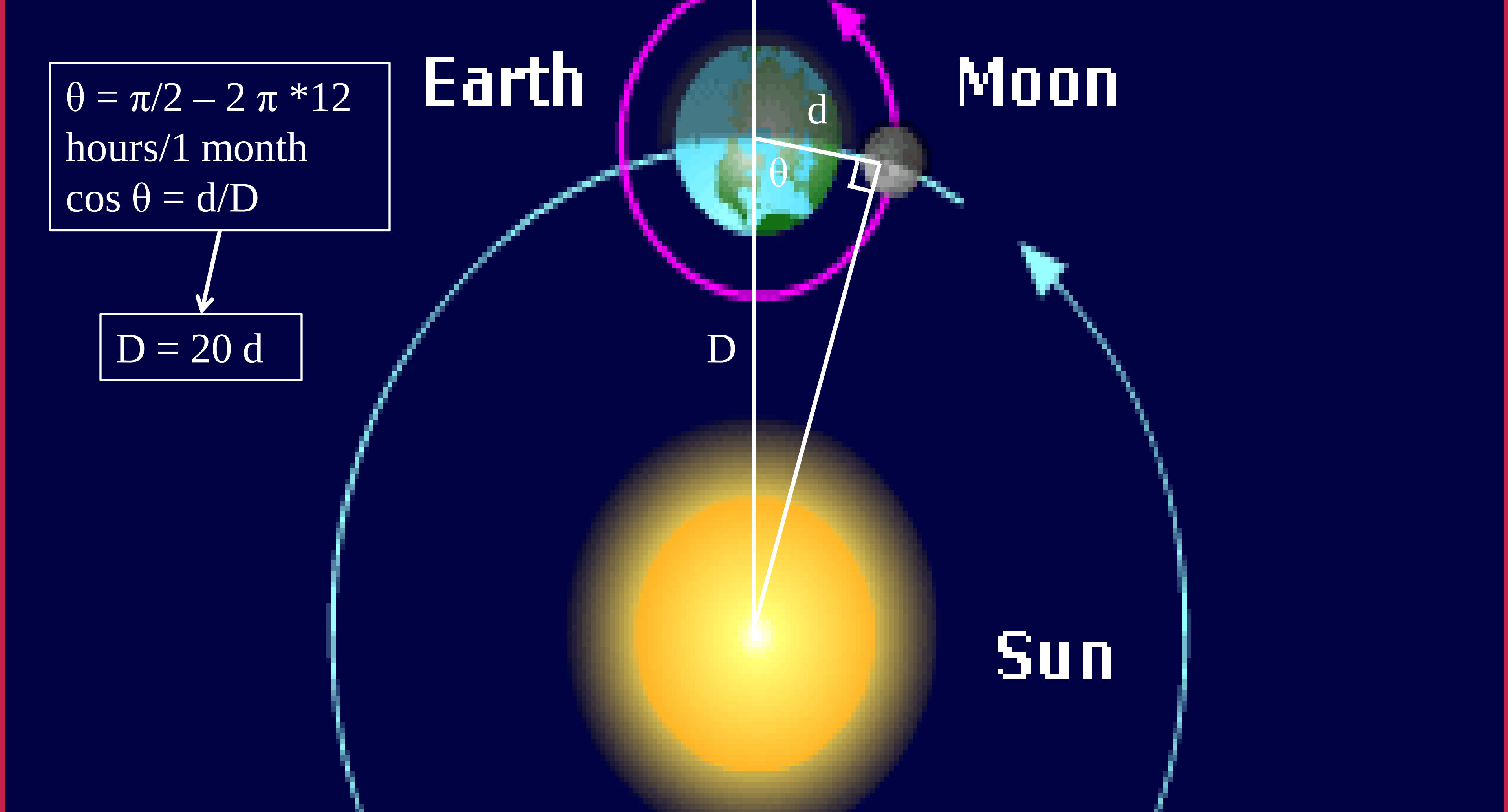
BBC

$\theta = \pi/2 - 2\pi * 12$ hours/1 month



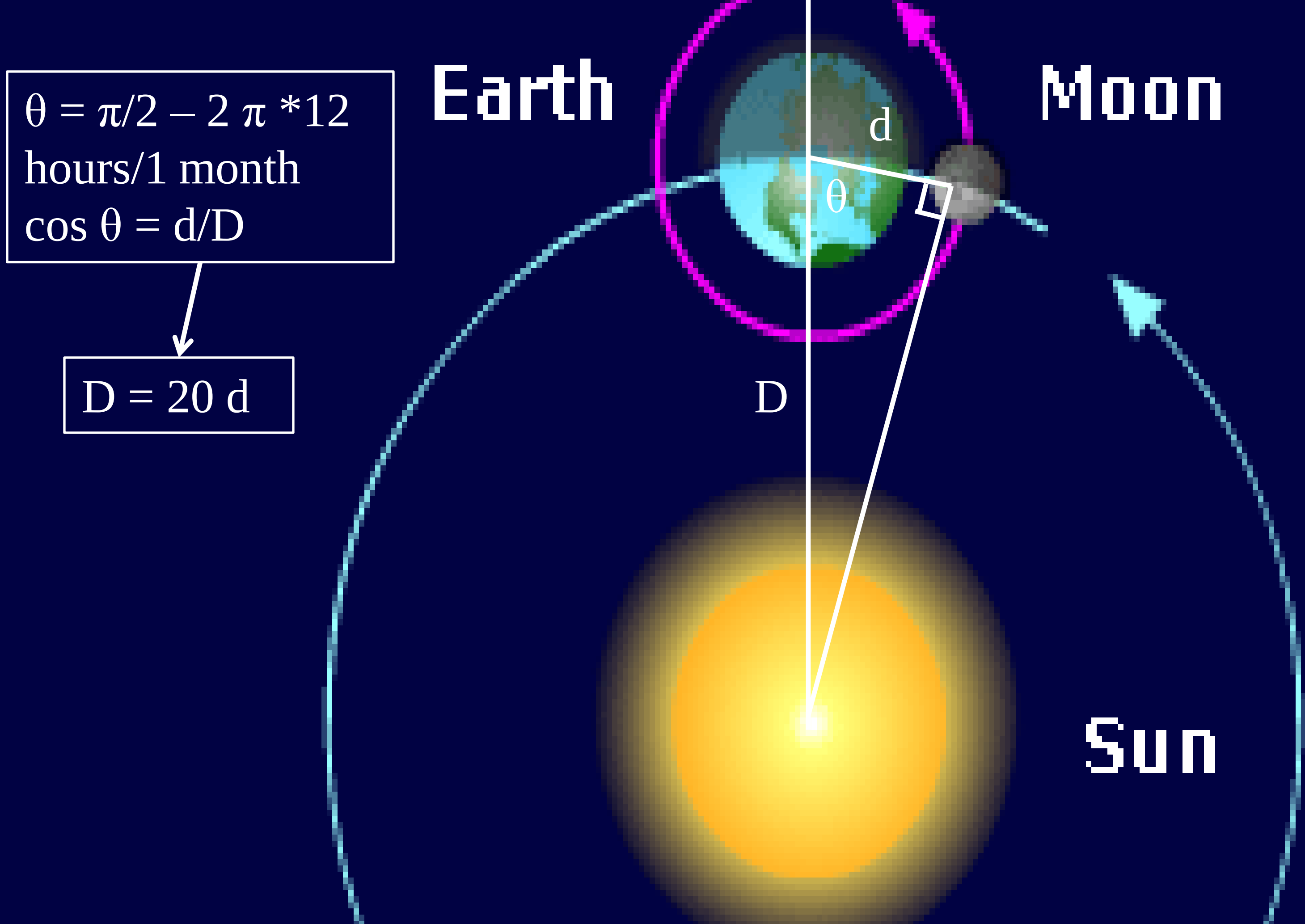
Aristarchus thought that half Moons occurred 12 hours before the midpoint of a new and full Moon.

BBC



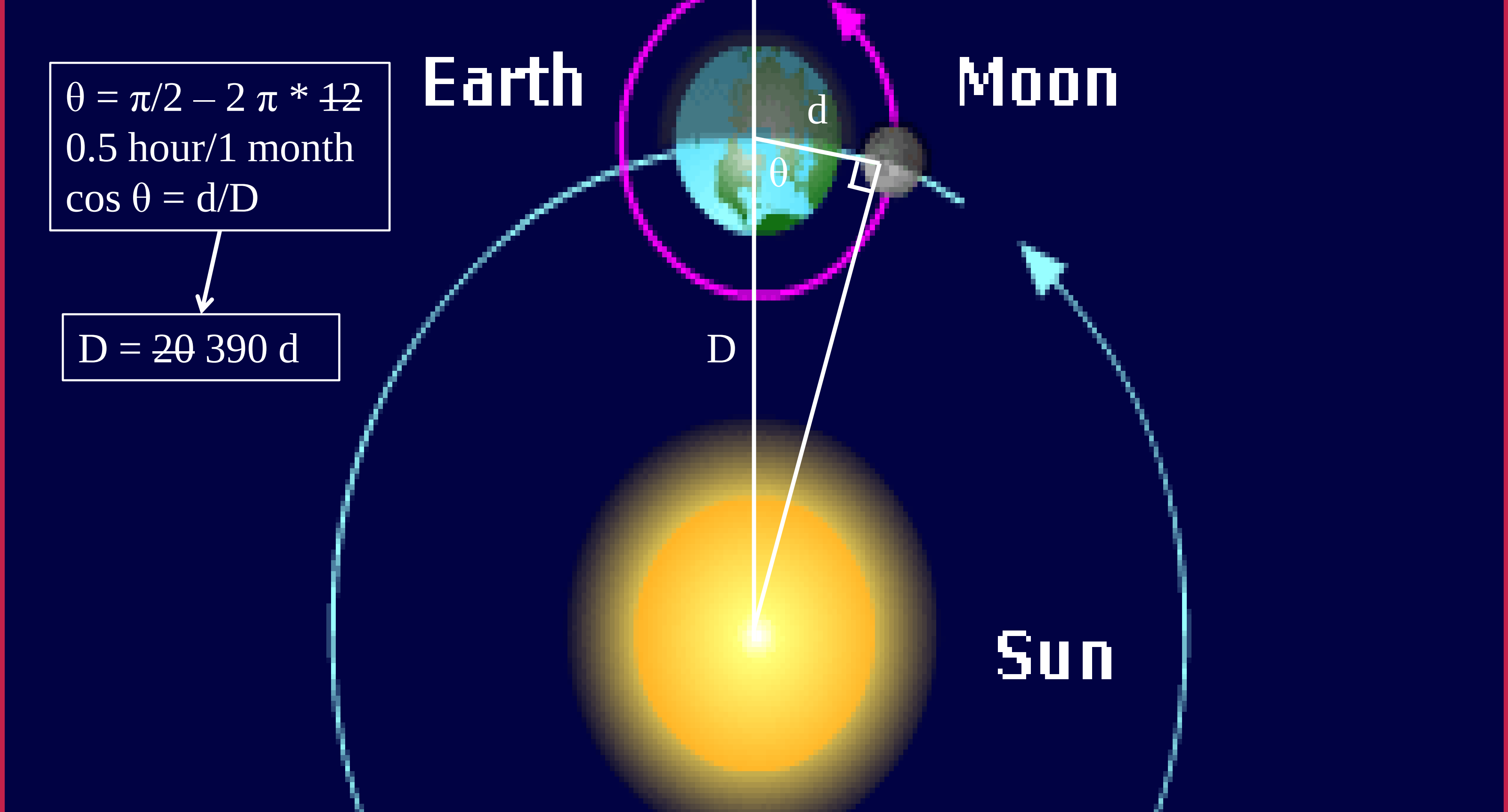
From this and trigonometry, he concluded that the Sun was 20 times further away than the Moon.

BBC



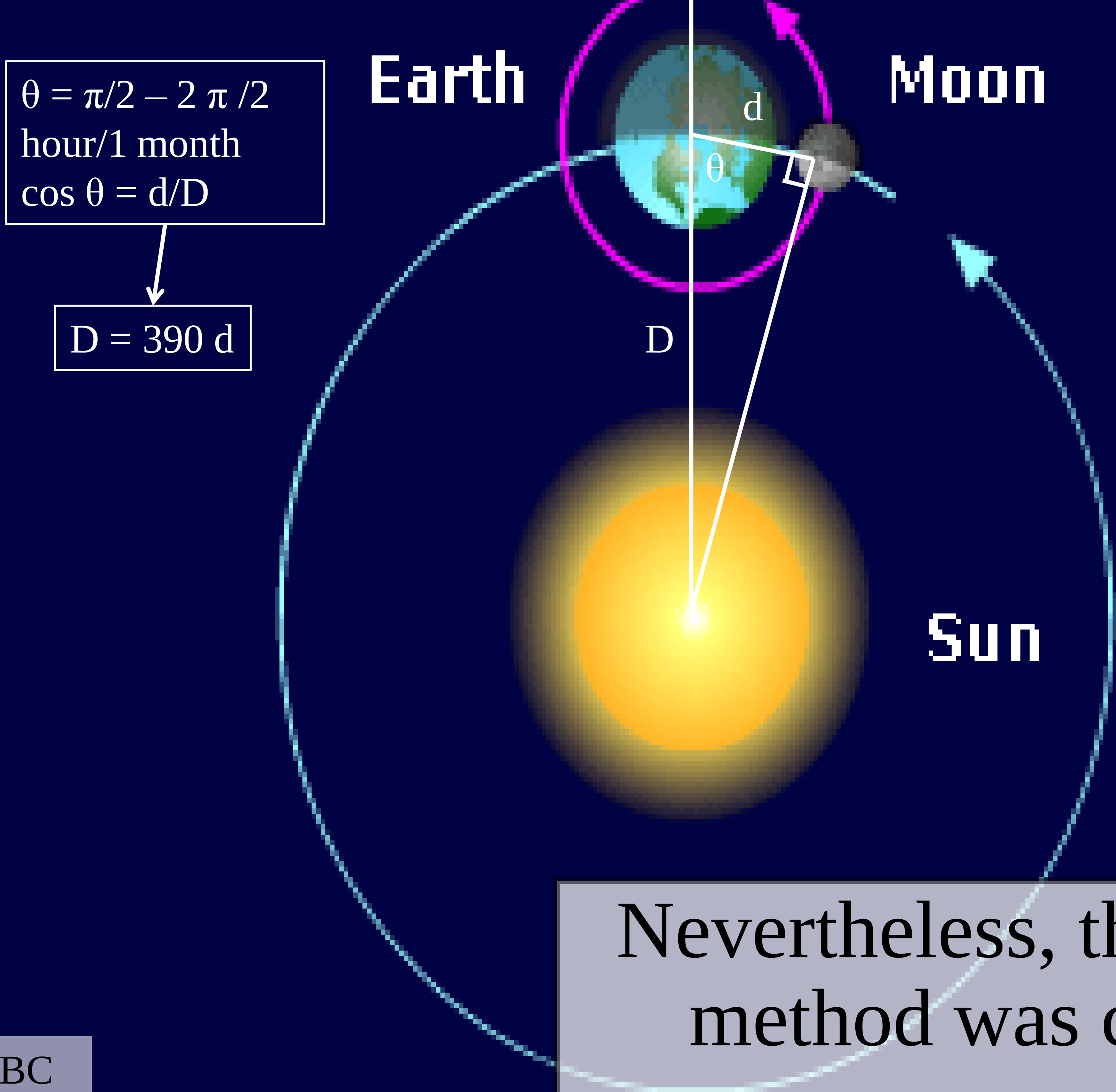
Unfortunately, with ancient Greek technology it was hard to time a new Moon perfectly.

BBC



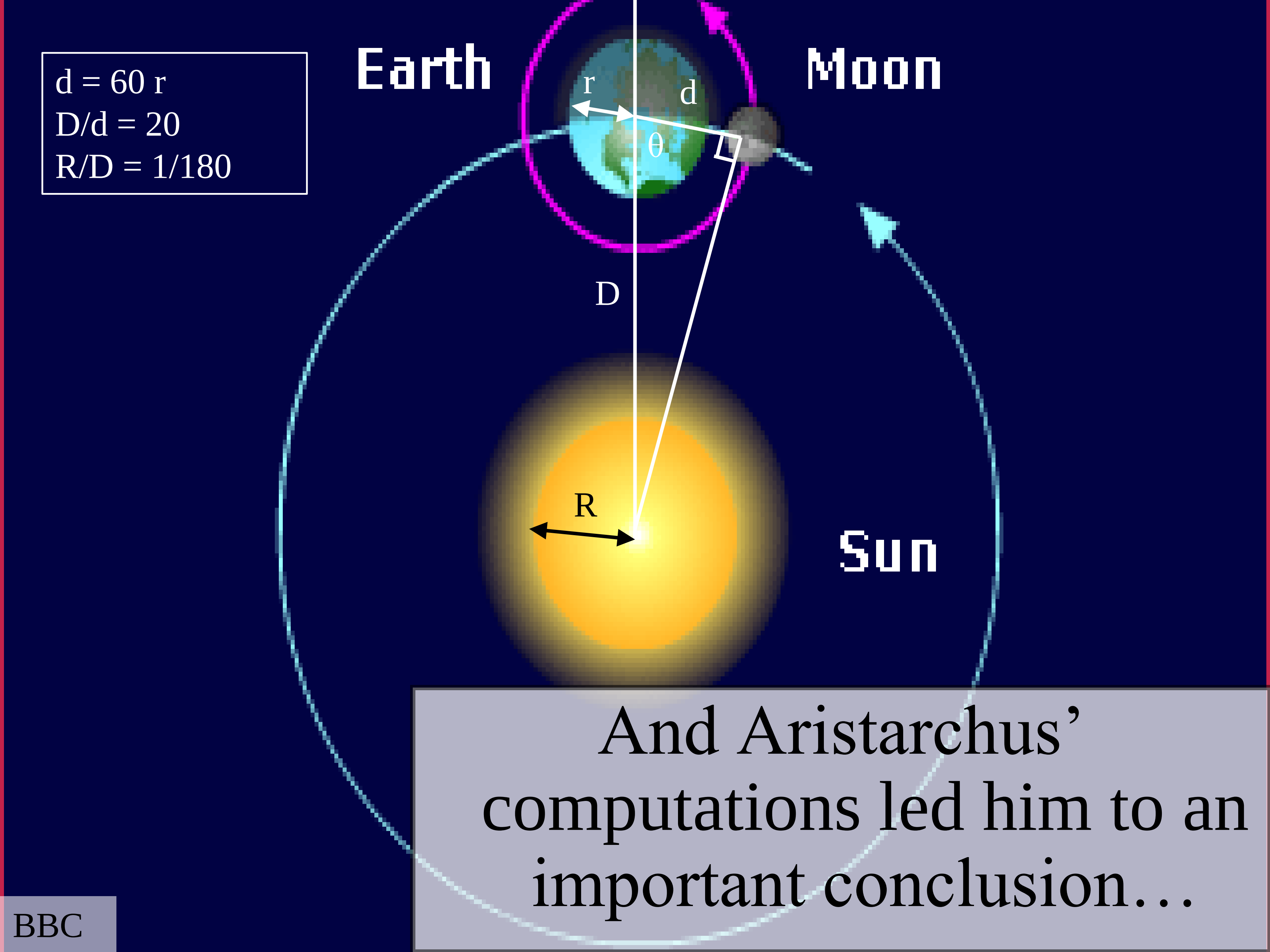
The true time discrepancy is ½ hour (not 12 hours), and the Sun is 390 times further away (not 20 times).

BBC



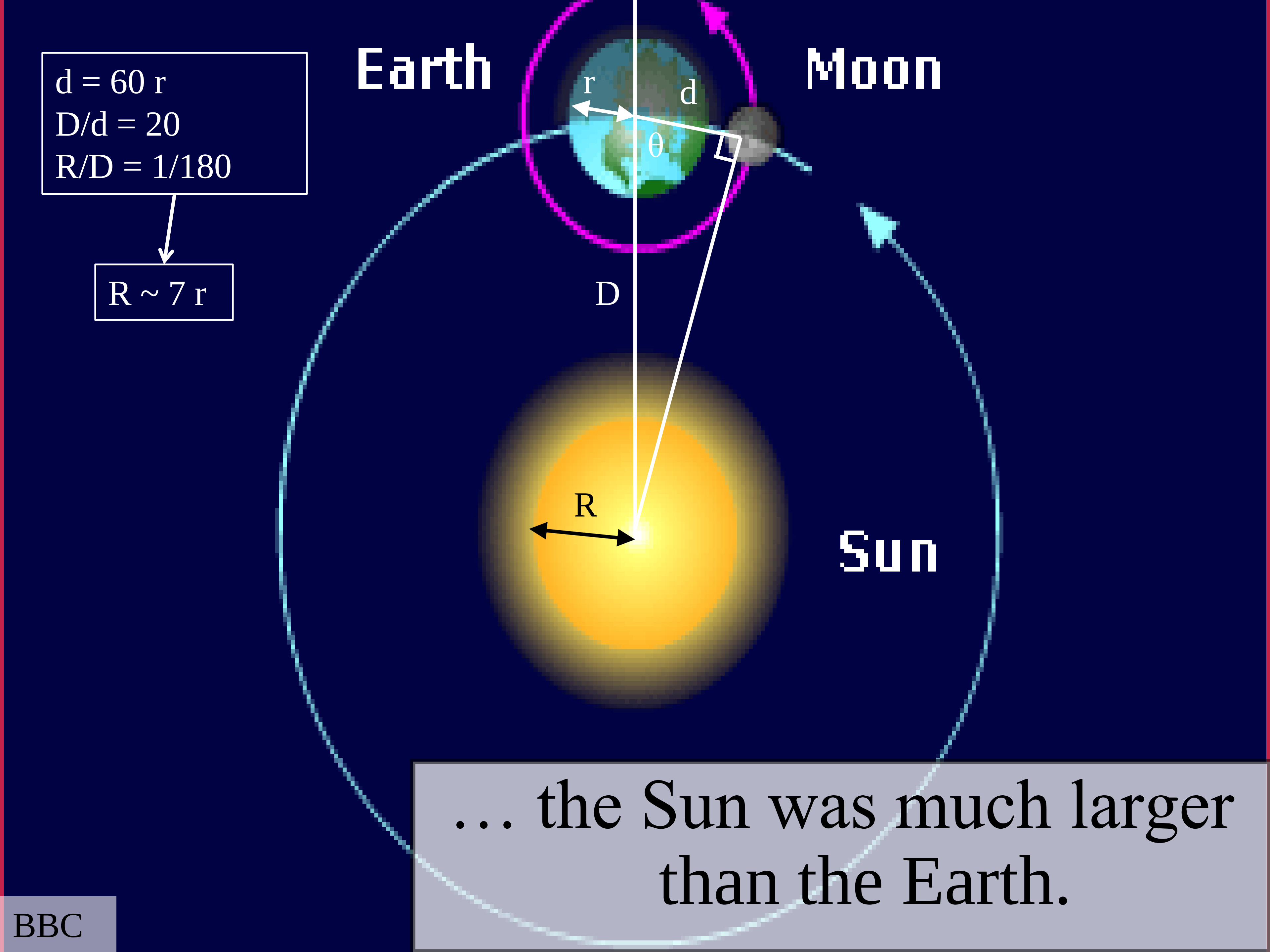
Nevertheless, the basic method was correct.

BBC

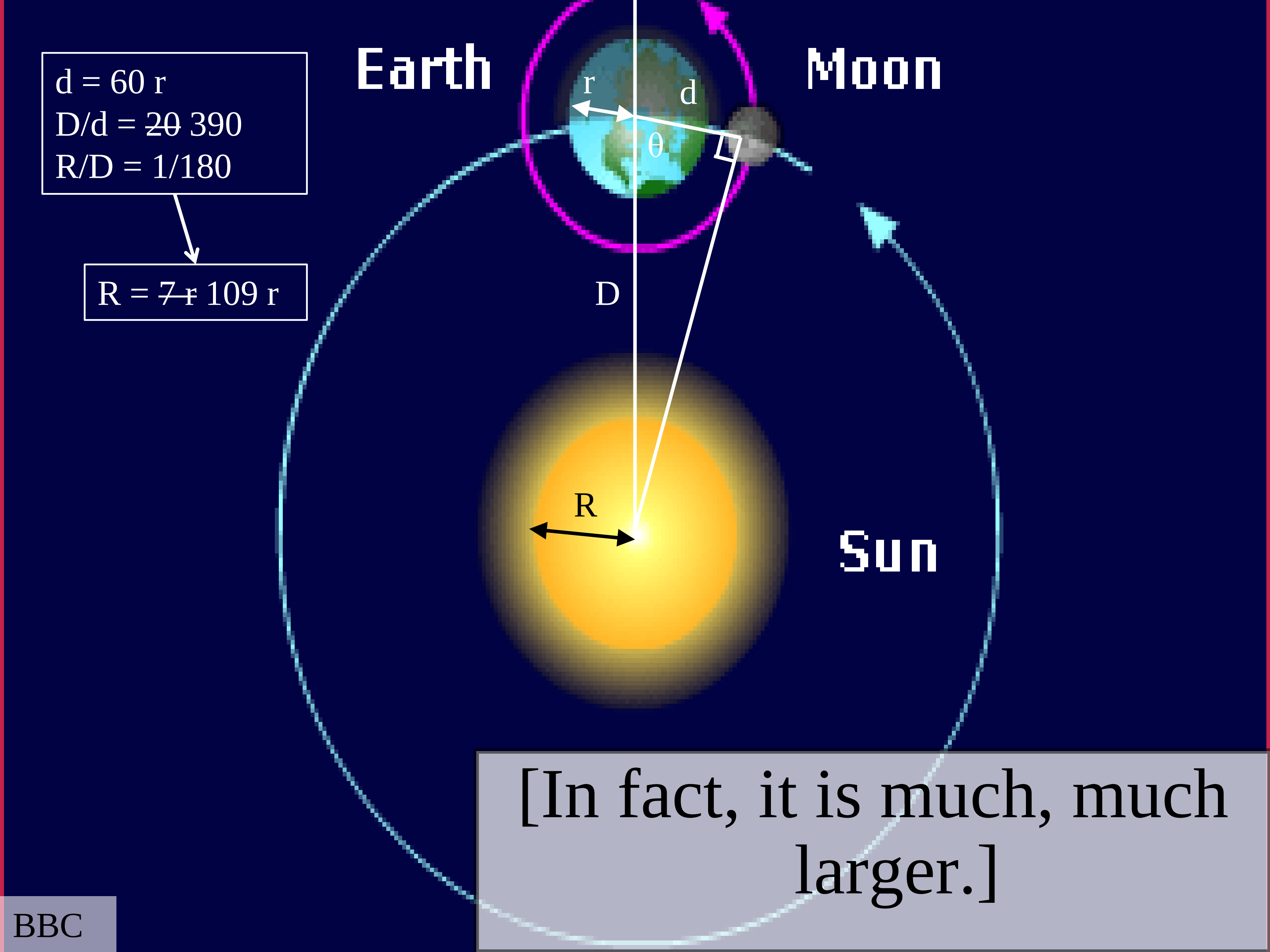


And Aristarchus' computations led him to an important conclusion…

BBC



… the Sun was much larger than the Earth.

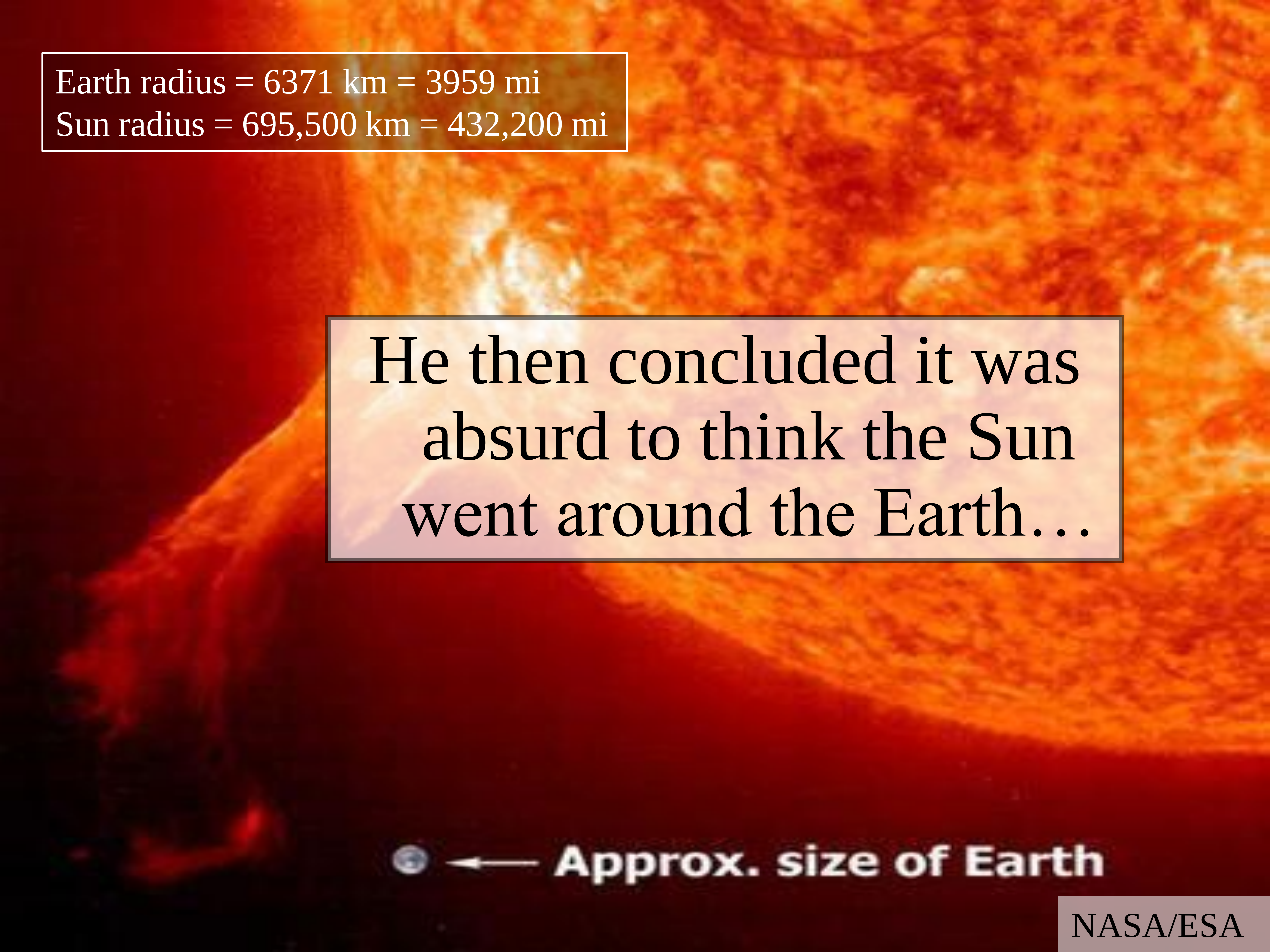


[In fact, it is much, much larger.]

BBC

Earth radius = 6371 km = 3959 mi
Sun radius = 695,500 km = 432,200 mi

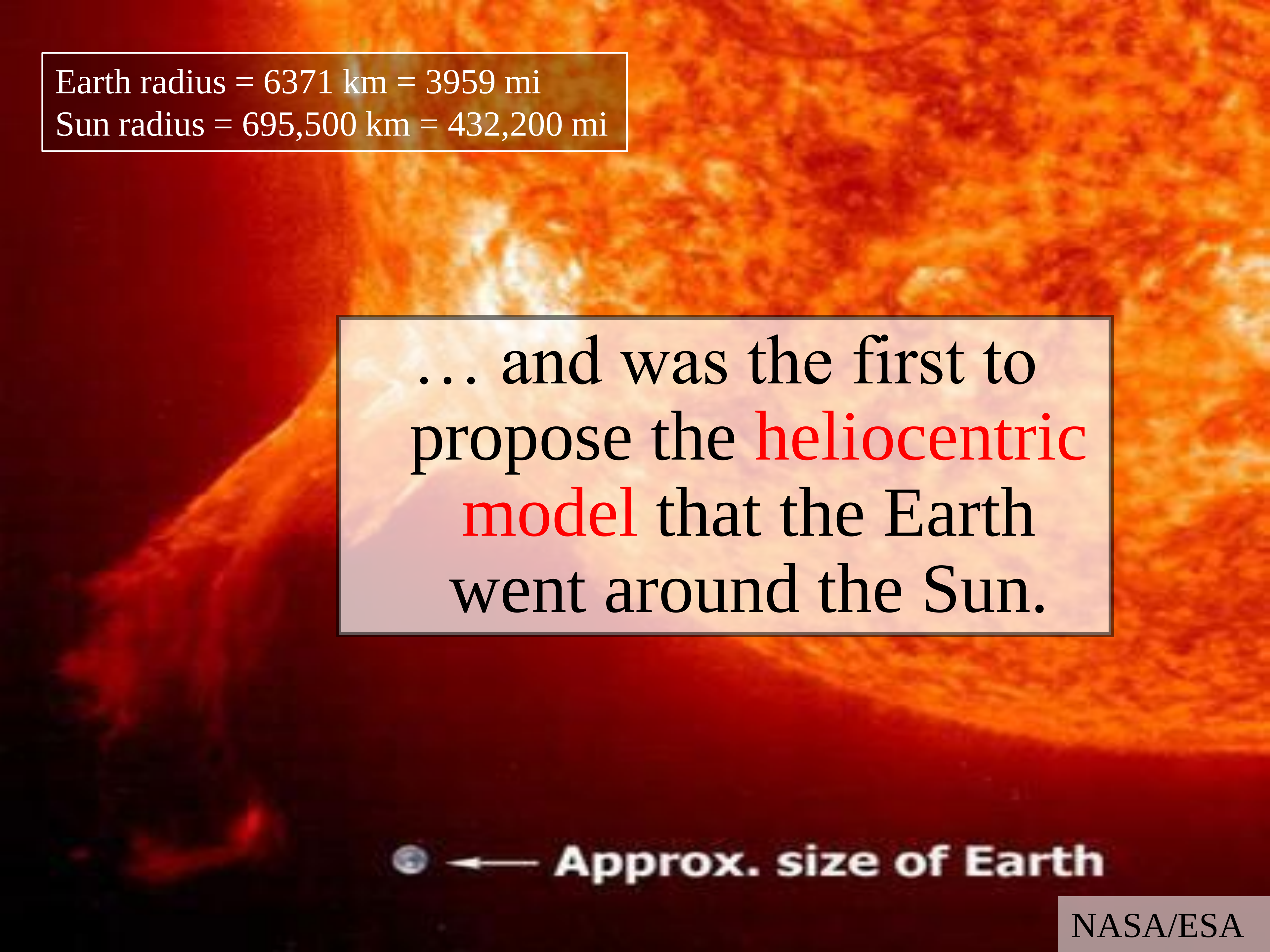
He then concluded it was absurd to think the Sun went around the Earth…



NASA/ESA

Earth radius = 6371 km = 3959 mi
Sun radius = 695,500 km = 432,200 mi

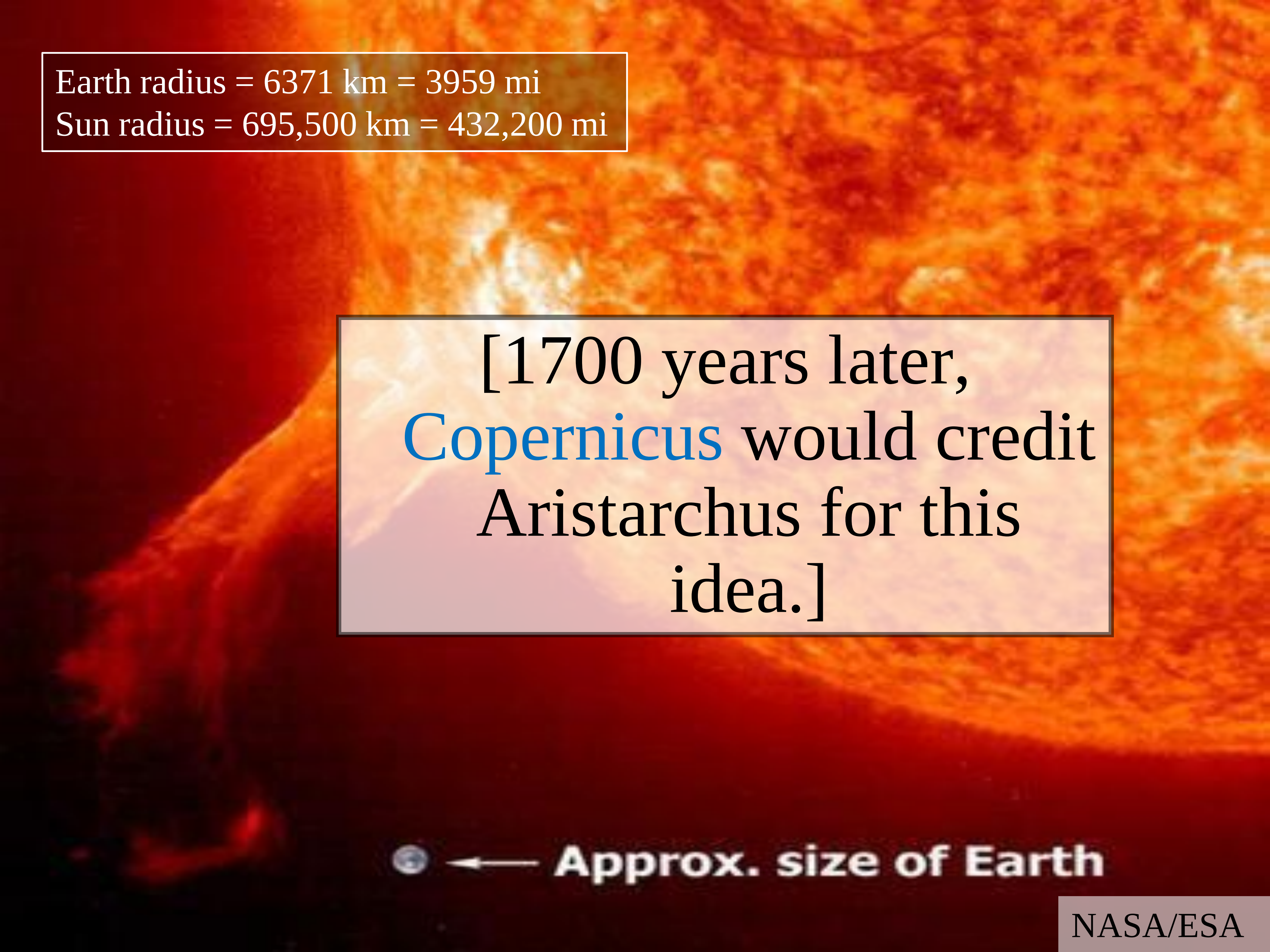
… and was the first to propose the heliocentric model that the Earth went around the Sun.



NASA/ESA

Earth radius = 6371 km = 3959 mi
Sun radius = 695,500 km = 432,200 mi

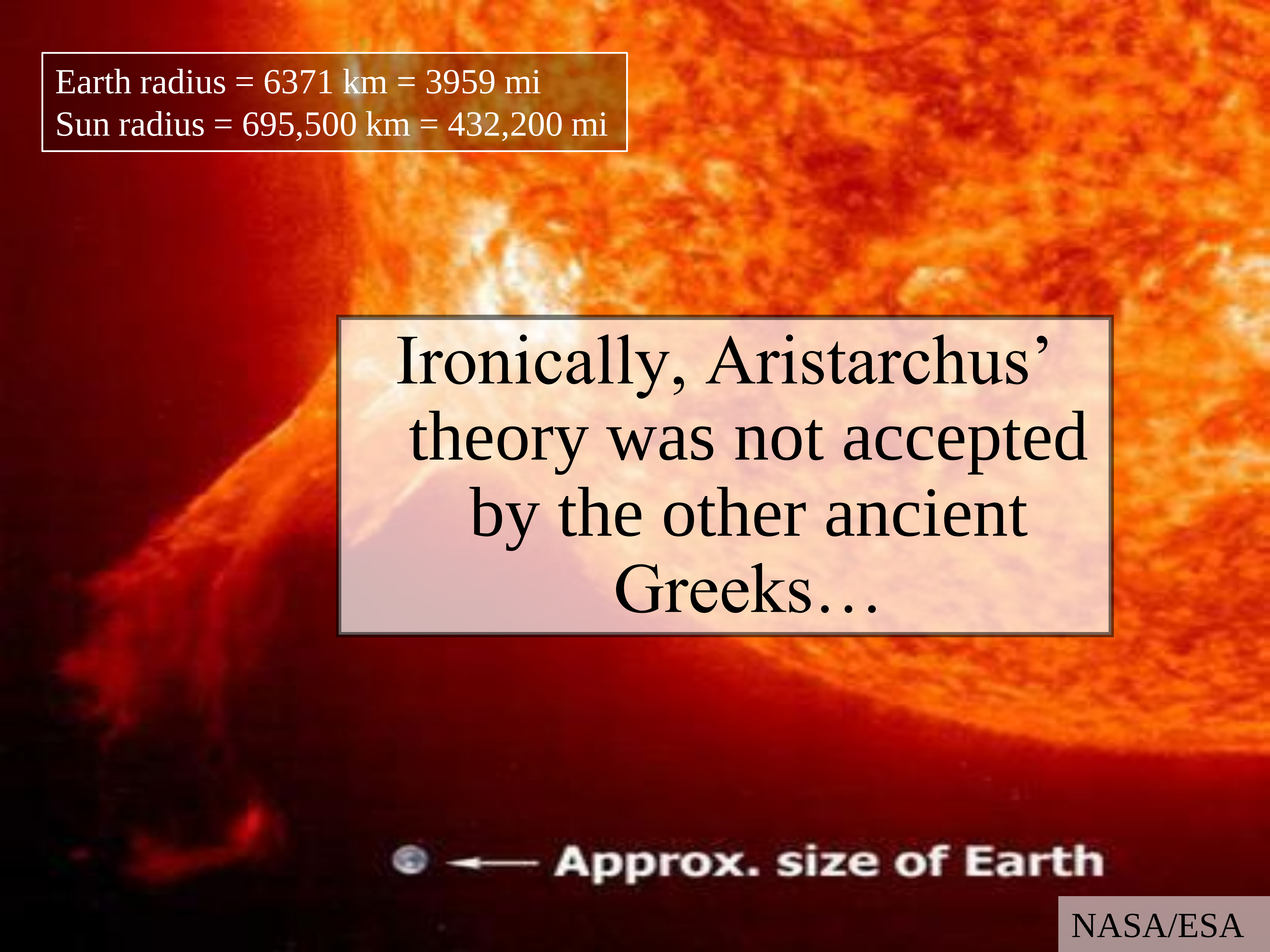
[1700 years later, Copernicus would credit Aristarchus for this idea.]



NASA/ESA

Earth radius = 6371 km = 3959 mi
Sun radius = 695,500 km = 432,200 mi

Ironically, Aristarchus' theory was not accepted by the other ancient Greeks…



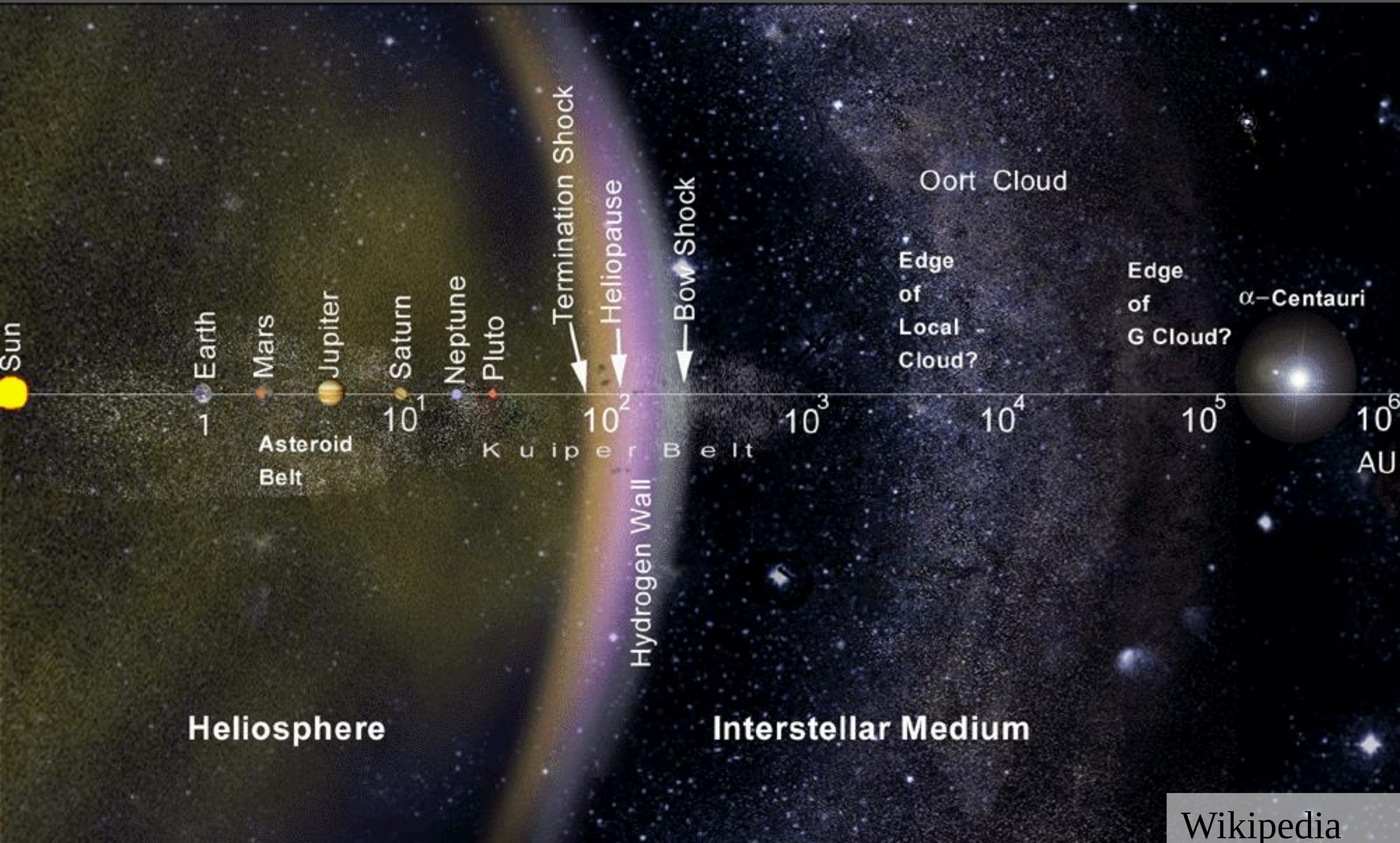
NASA/ESA

Earth radius = 6371 km = 3959 mi
Sun radius = 695,500 km = 432,200 mi

… but we’ll explain why later.

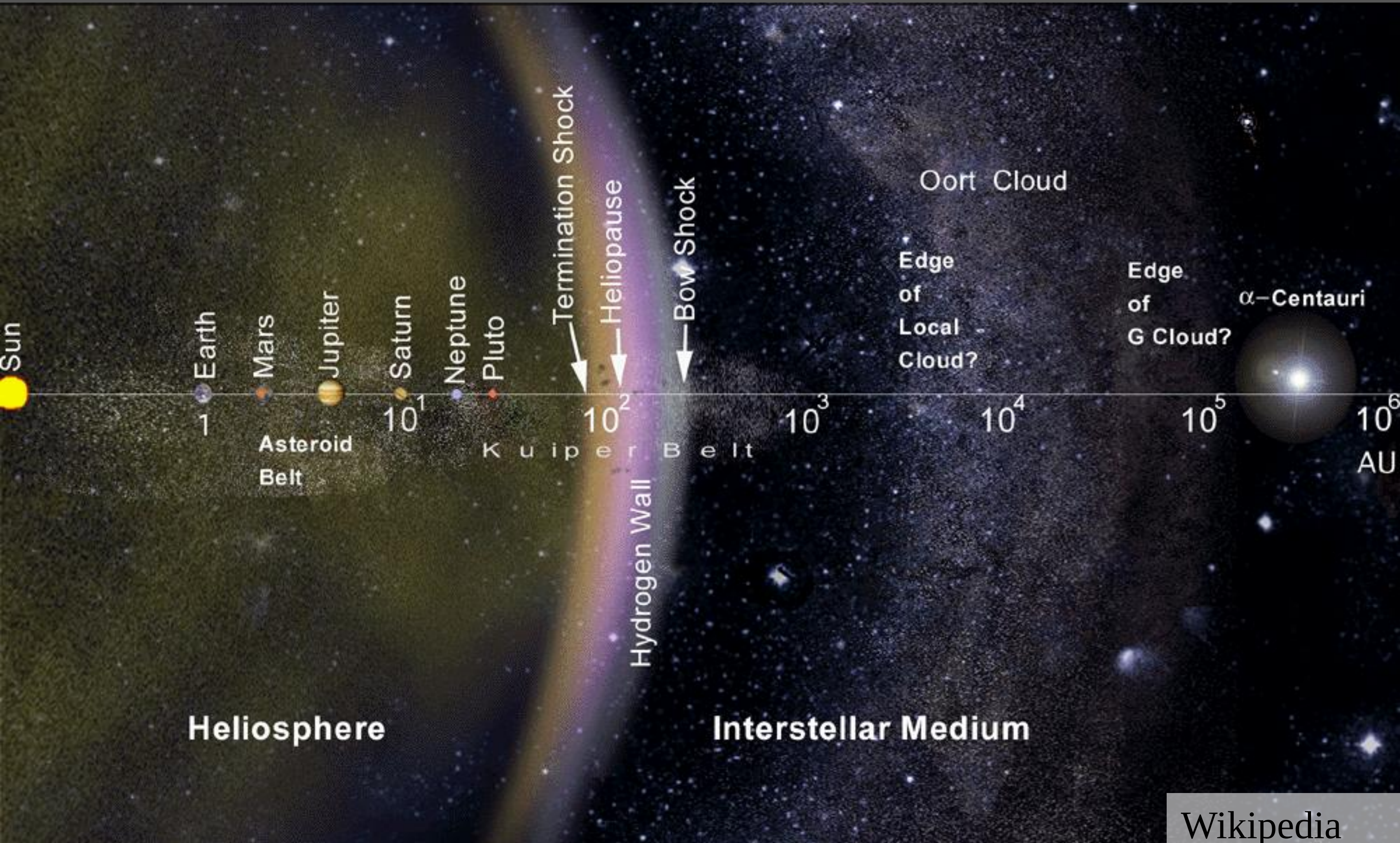
NASA/ESA

The distance from the Earth to the Sun is known as the Astronomical Unit (AU).



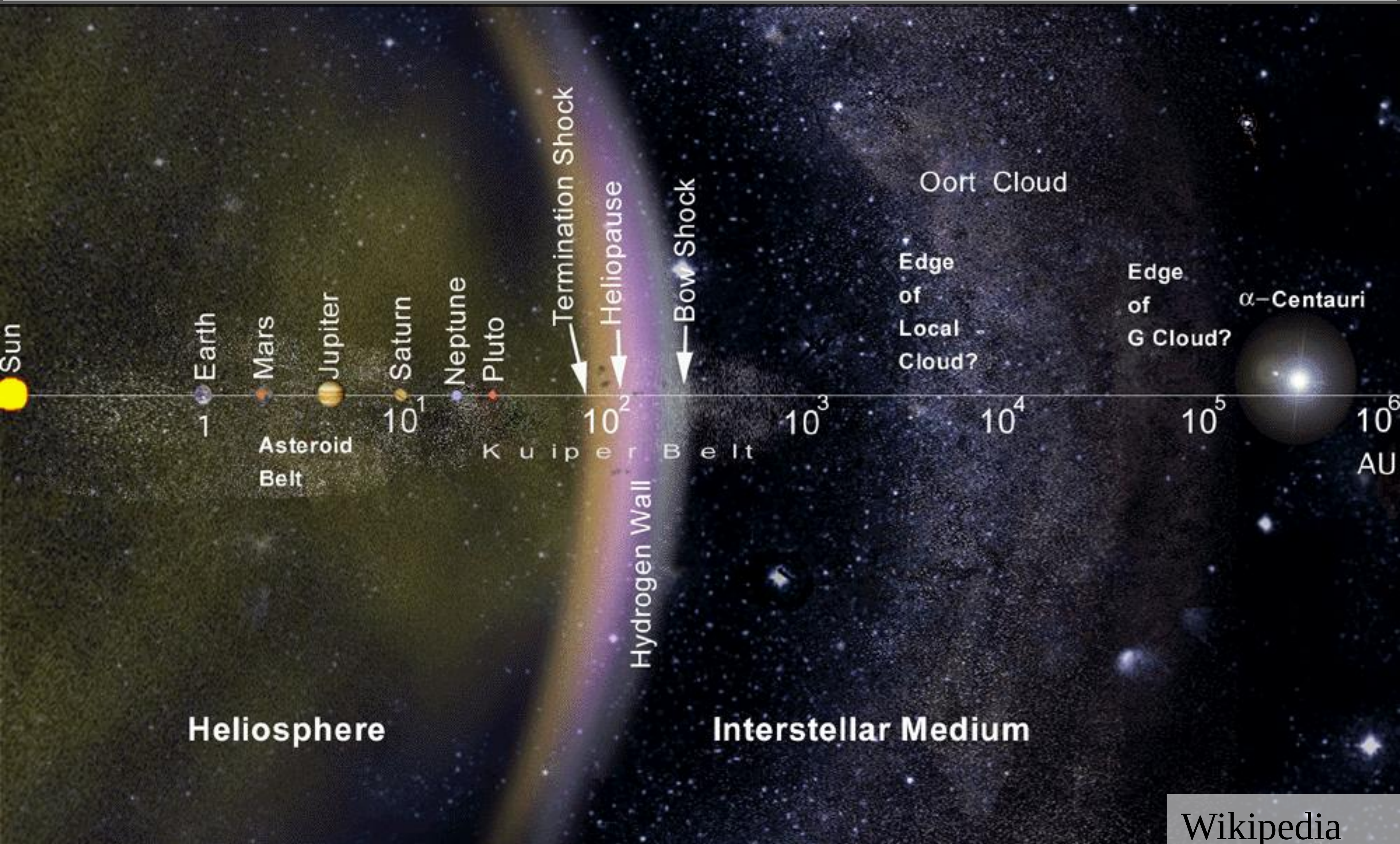
Wikipedia

It is an extremely important rung in the cosmic distance ladder.



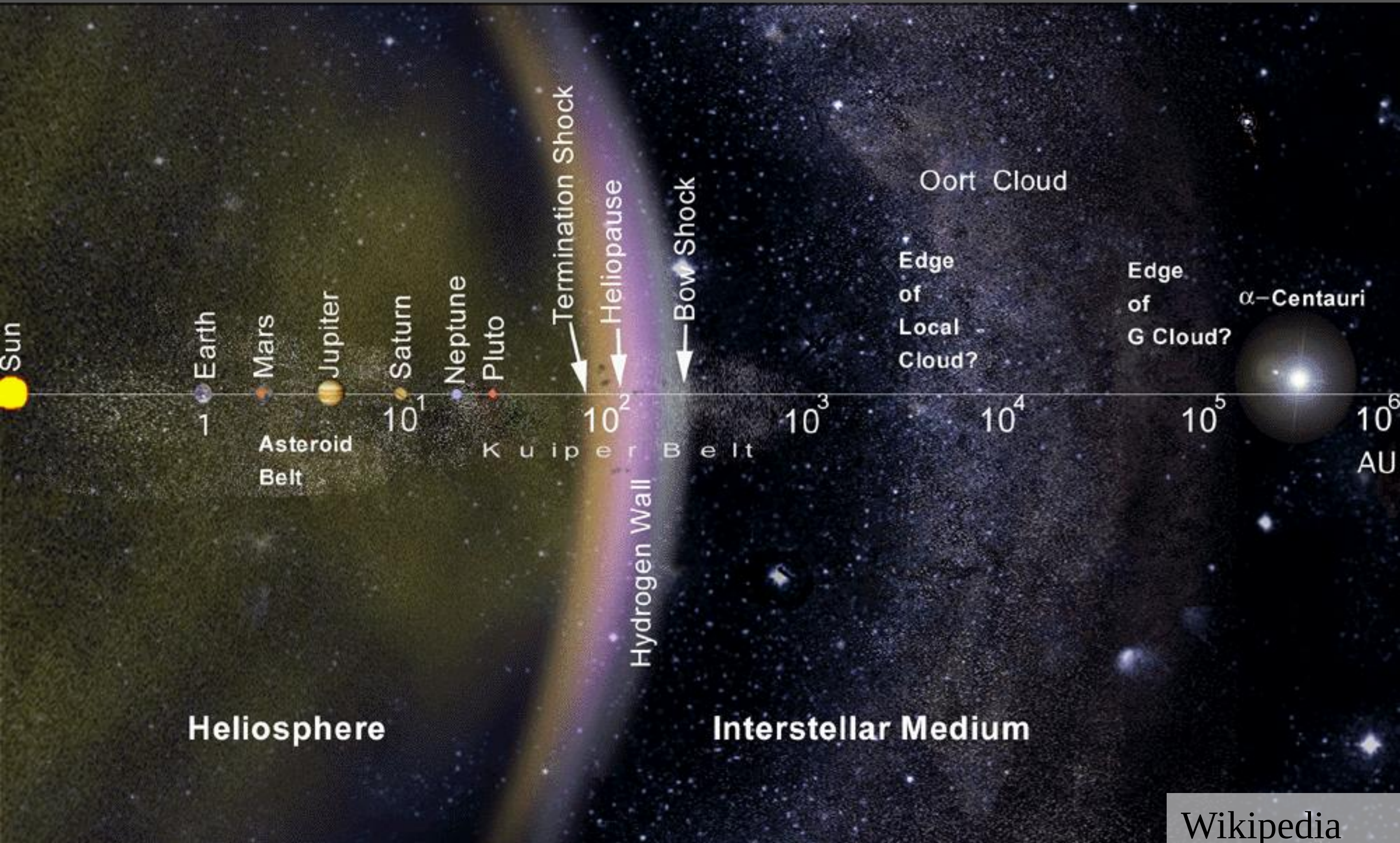
Wikipedia

Aristarchus' original estimate of the AU was inaccurate…



Wikipedia

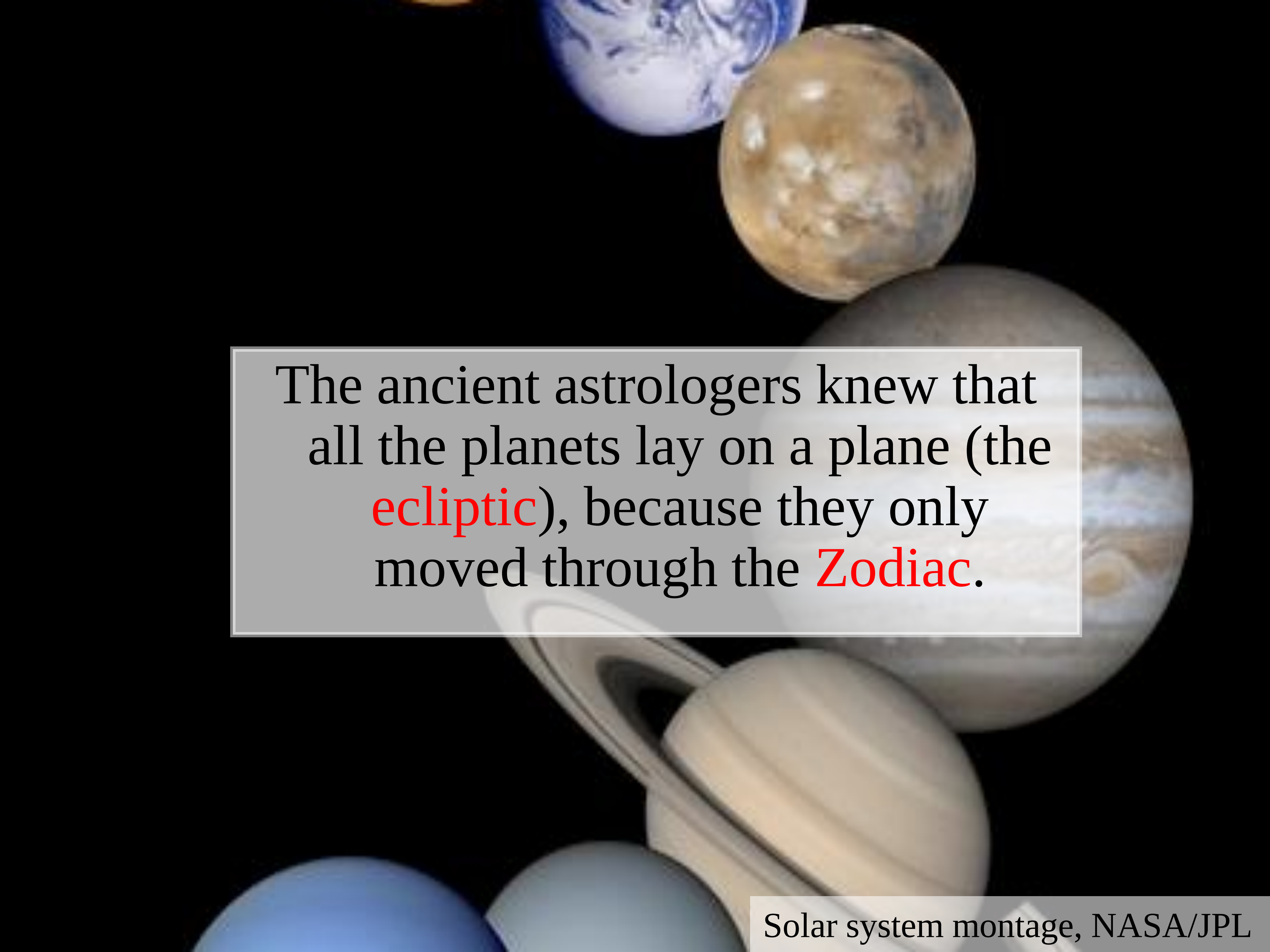
… but we'll see much more accurate ways to measure the AU later on.



Wikipedia

4th rung: the planets

Solar system montage, NASA/JPL

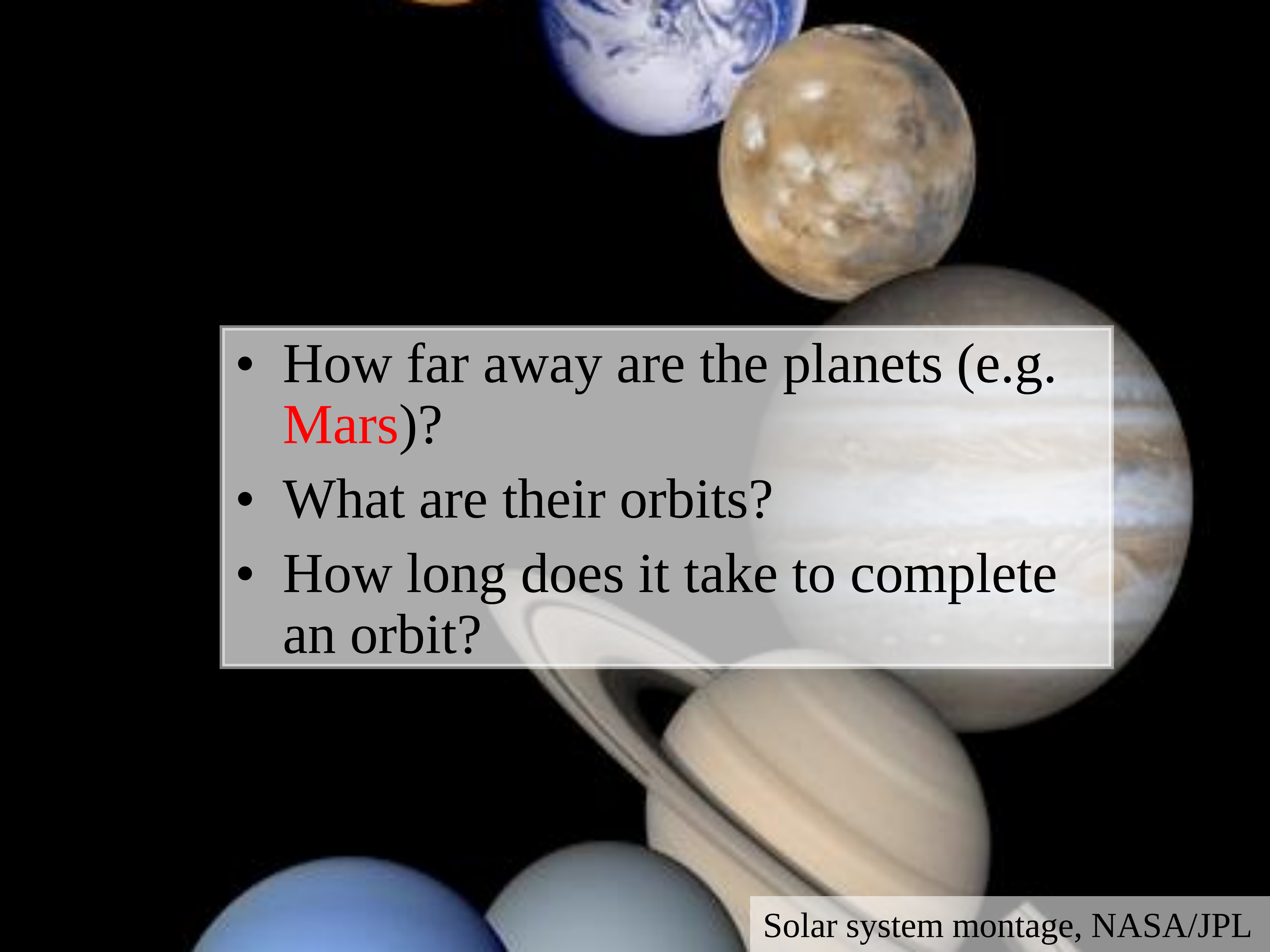


The ancient astrologers knew that all the planets lay on a plane (the ecliptic), because they only moved through the Zodiac.

Solar system montage, NASA/JPL

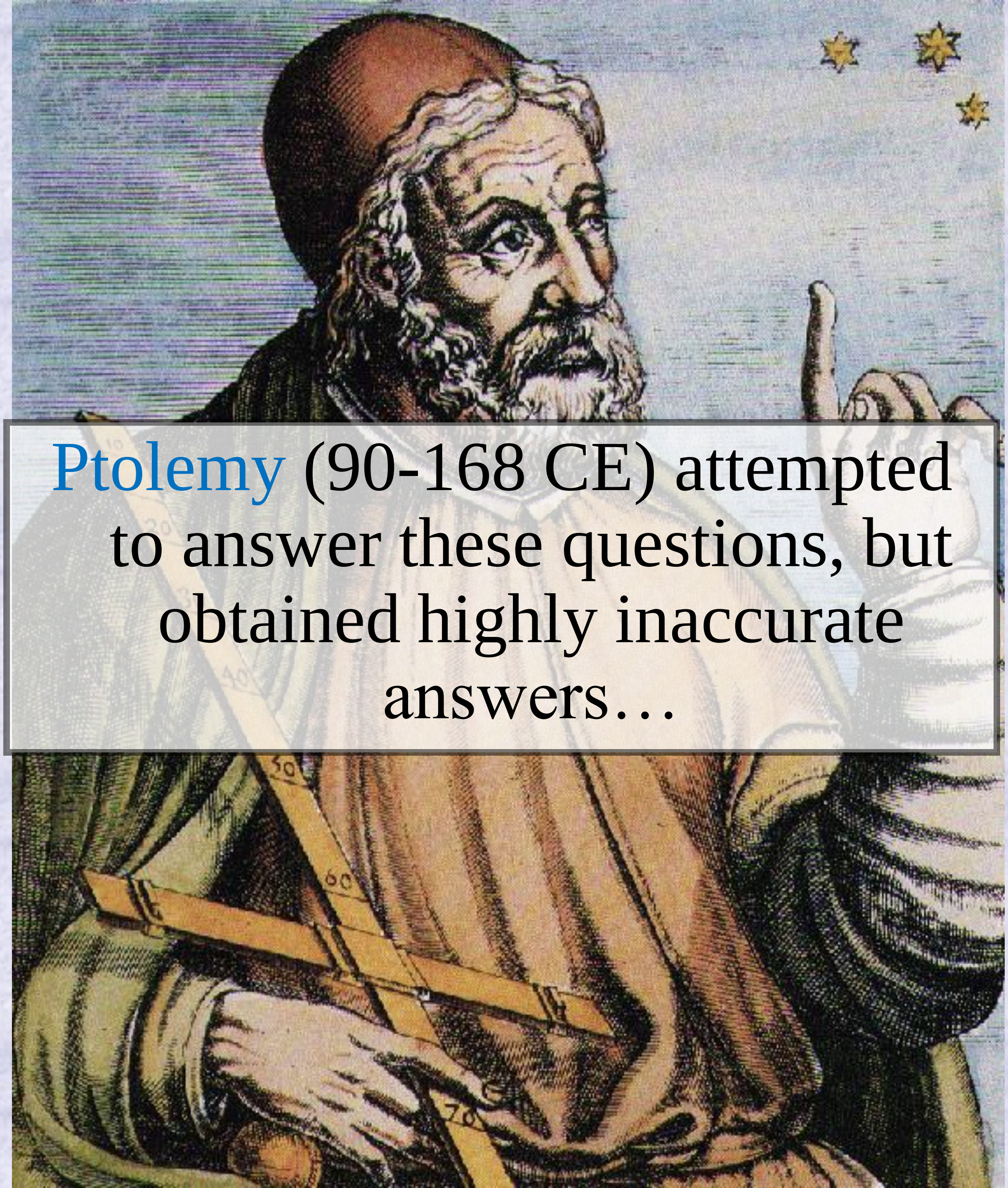
But this still left many questions unanswered:

Solar system montage, NASA/JPL



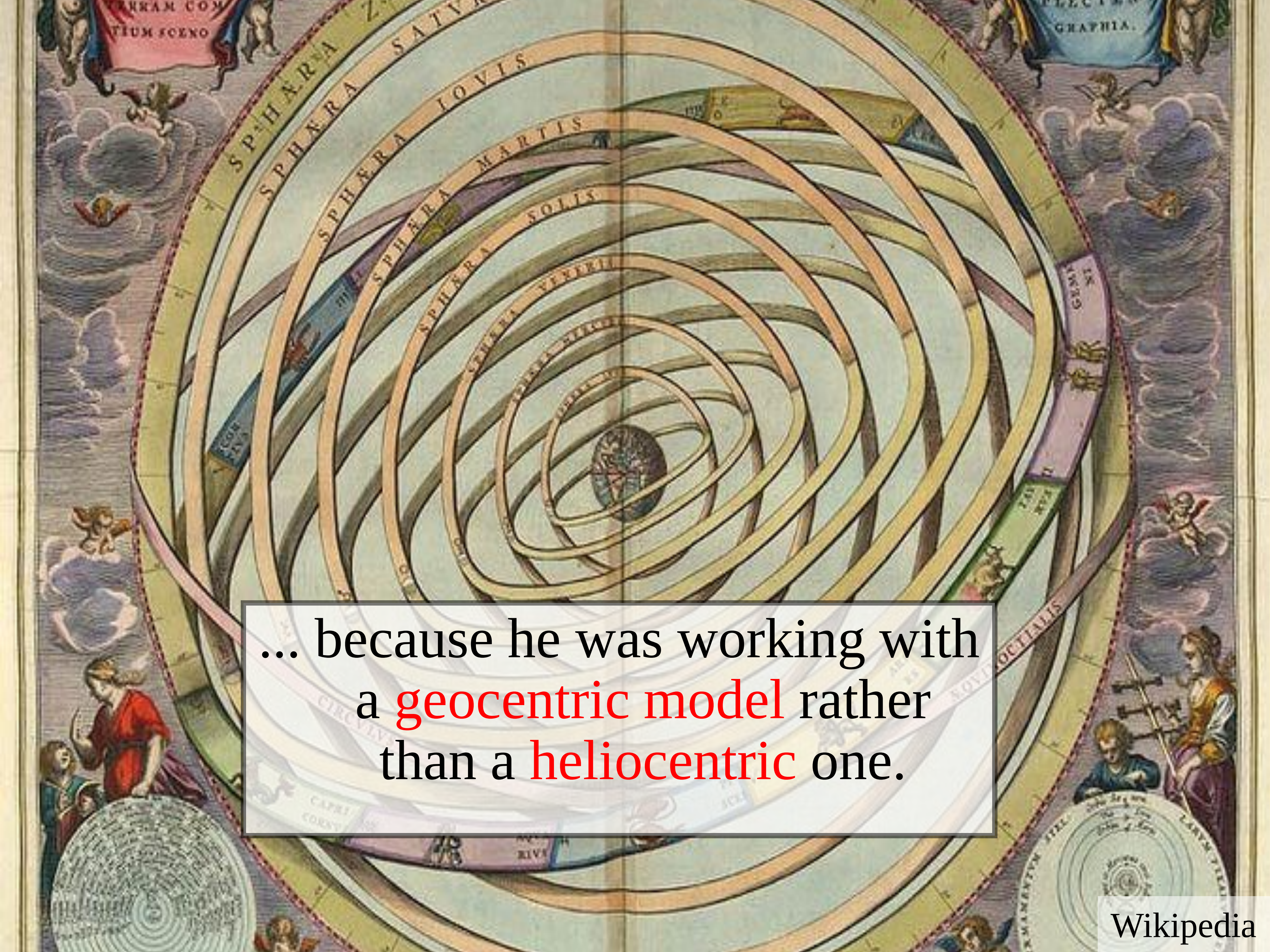
- How far away are the planets (e.g. Mars)?
- What are their orbits?
- How long does it take to complete an orbit?

Solar system montage, NASA/JPL



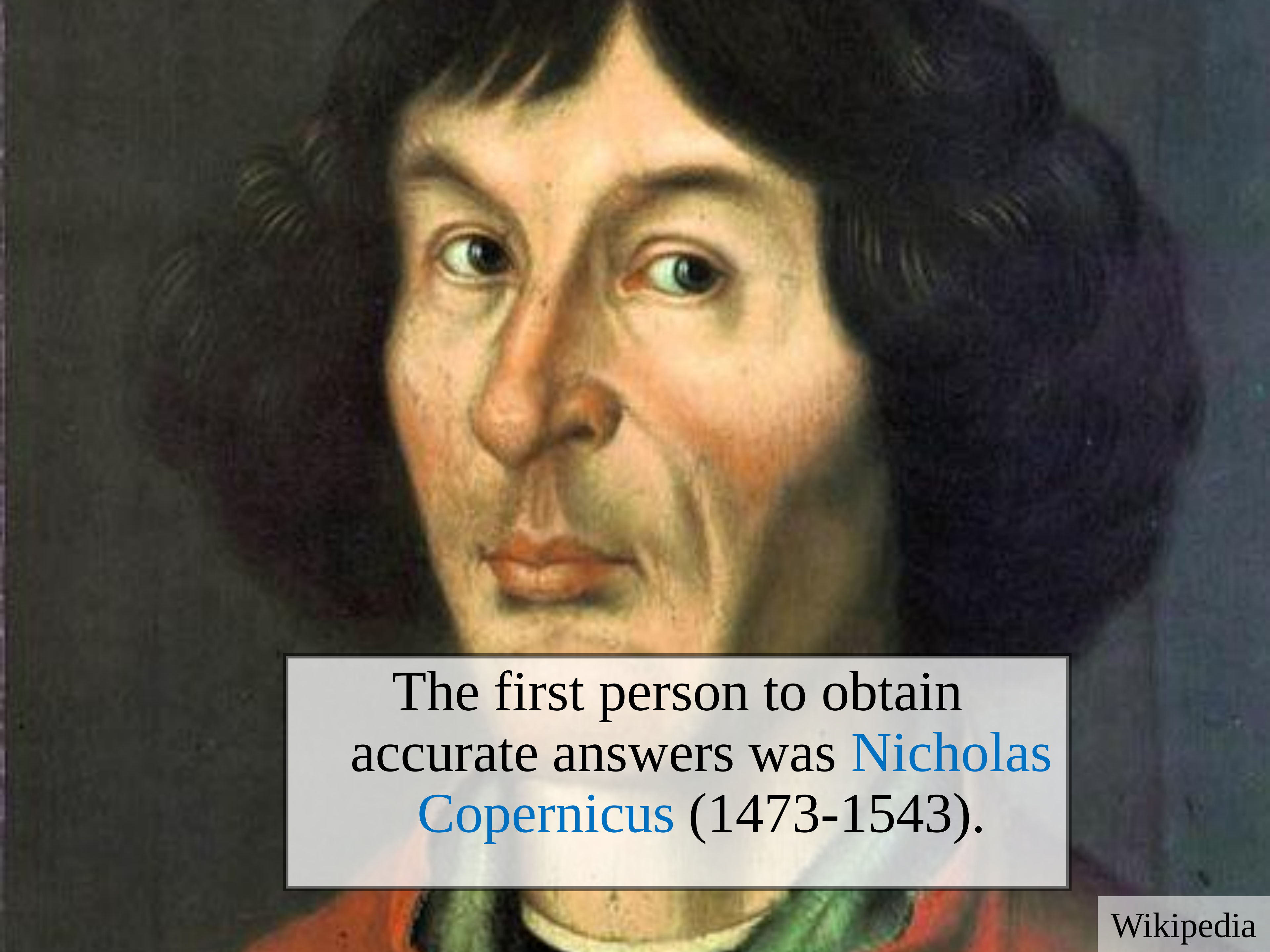
Ptolemy (90-168 CE) attempted to answer these questions, but obtained highly inaccurate answers…

Wikipedia



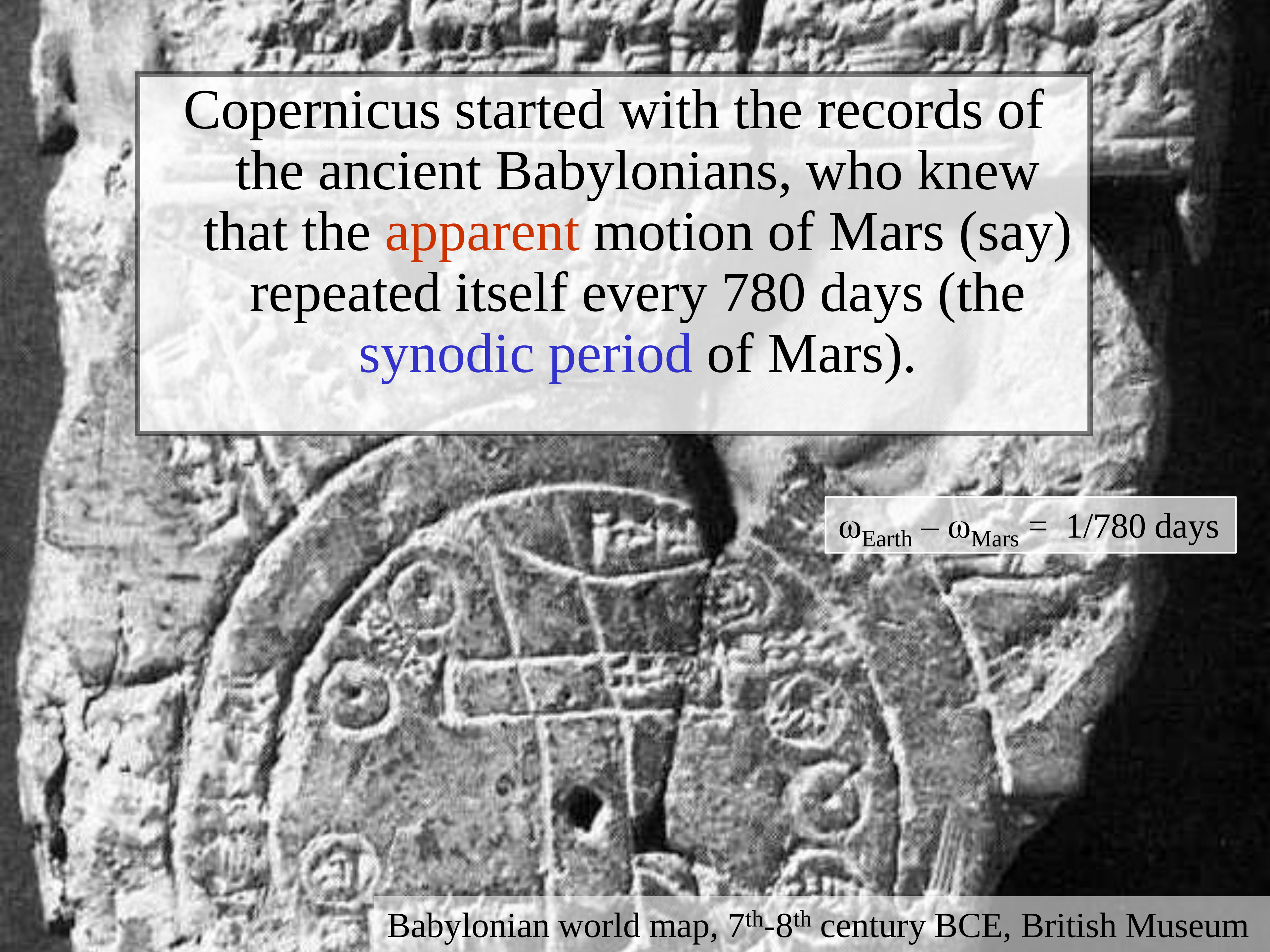
... because he was working with a geocentric model rather than a heliocentric one.

Wikipedia



The first person to obtain accurate answers was Nicholas Copernicus (1473-1543).

Wikipedia

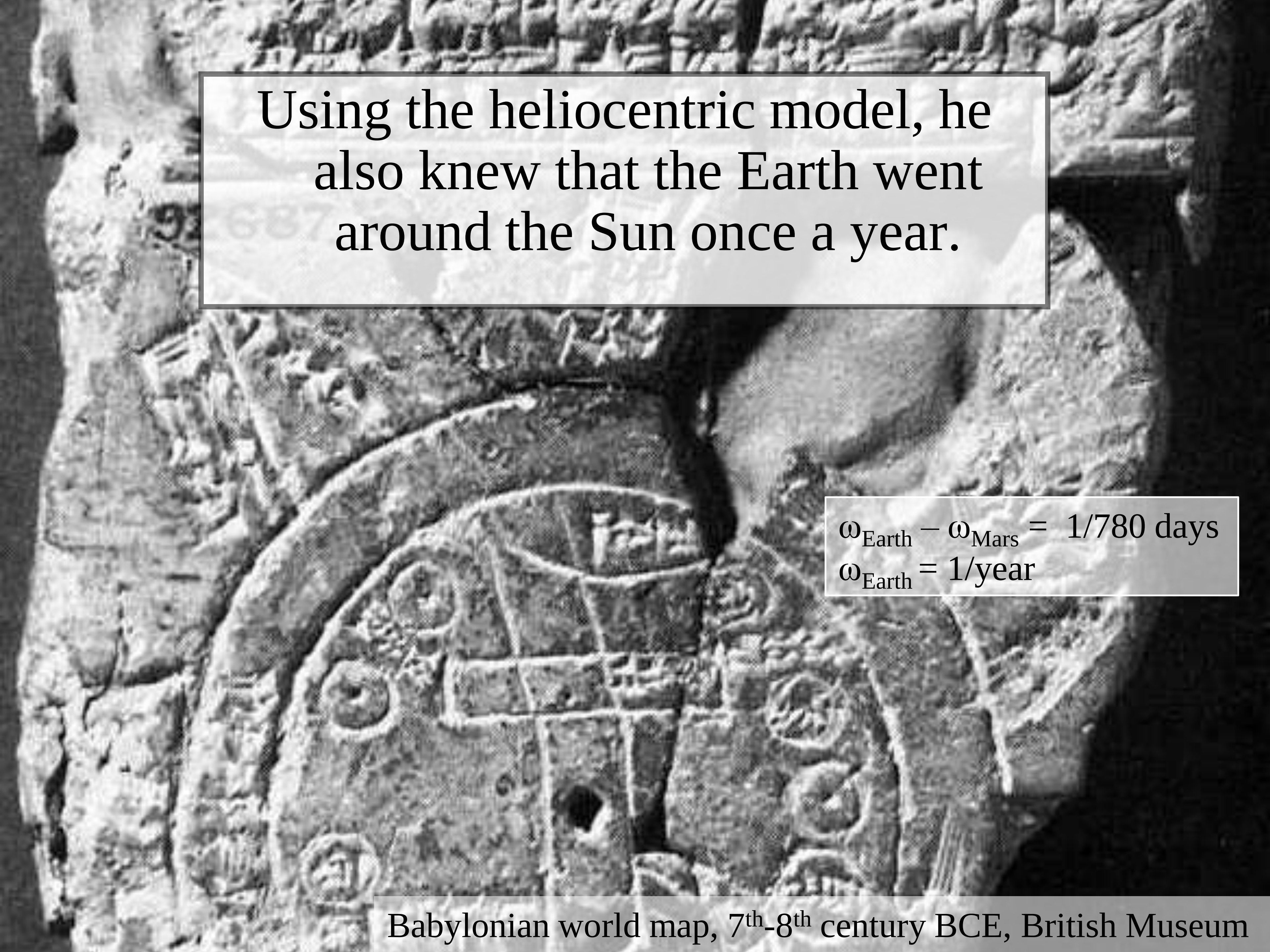


Copernicus started with the records of the ancient Babylonians, who knew that the apparent motion of Mars (say) repeated itself every 780 days (the synodic period of Mars).

$\omega_{Earth} - \omega_{Mars} = 1/780$ days

Babylonian world map, 7th-8th century BCE, British Museum

Using the heliocentric model, he also knew that the Earth went around the Sun once a year.



$\omega_{Earth} - \omega_{Mars} = 1/780$ days

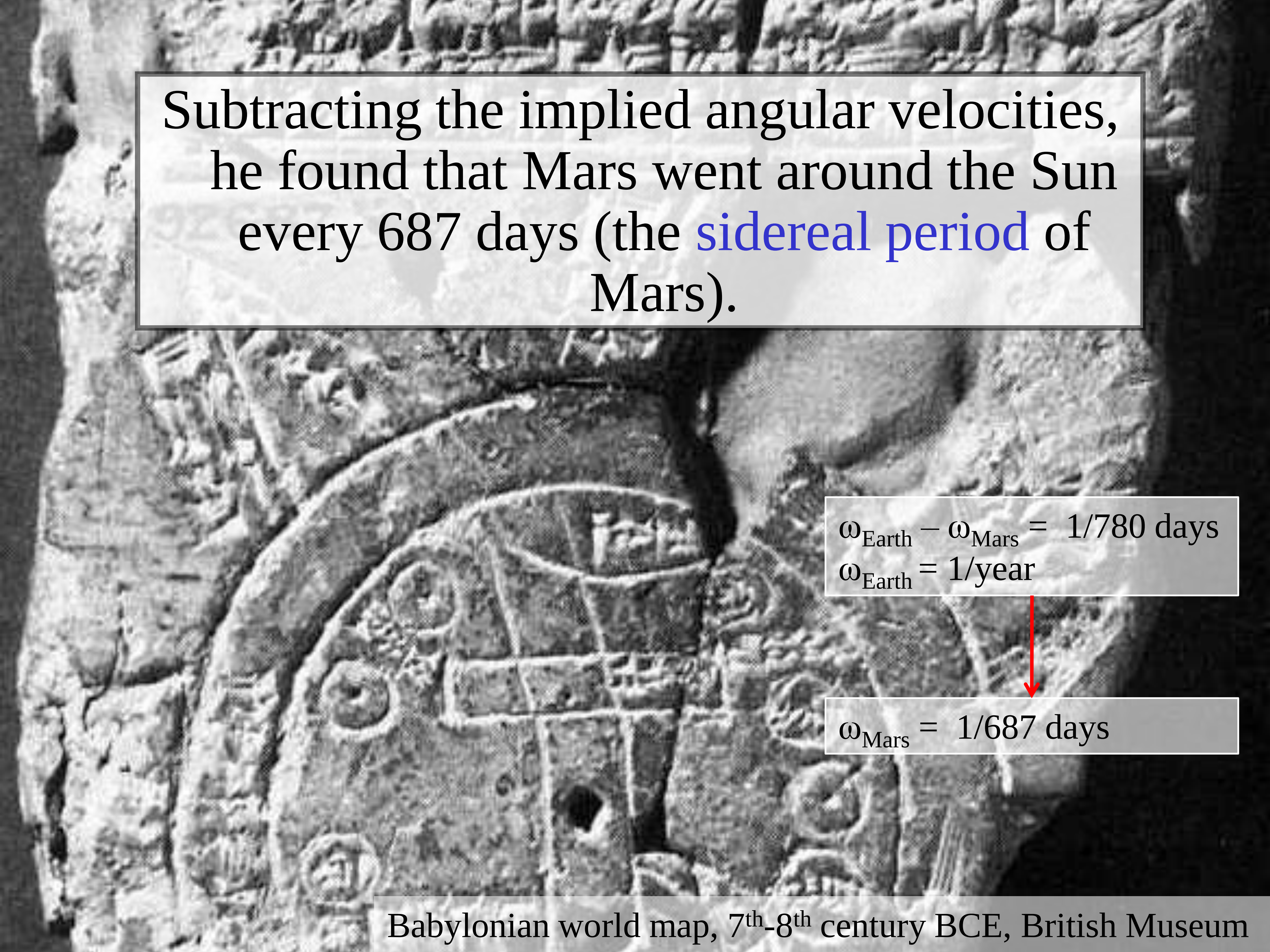
$\omega_{Earth} = 1/year$

Babylonian world map, 7th-8th century BCE, British Museum

Subtracting the implied angular velocities, he found that Mars went around the Sun every 687 days (the sidereal period of Mars).

$\omega_{Earth} - \omega_{Mars}$ = 1/780 days
ω_{Earth} = 1/year

ω_{Mars} = 1/687 days

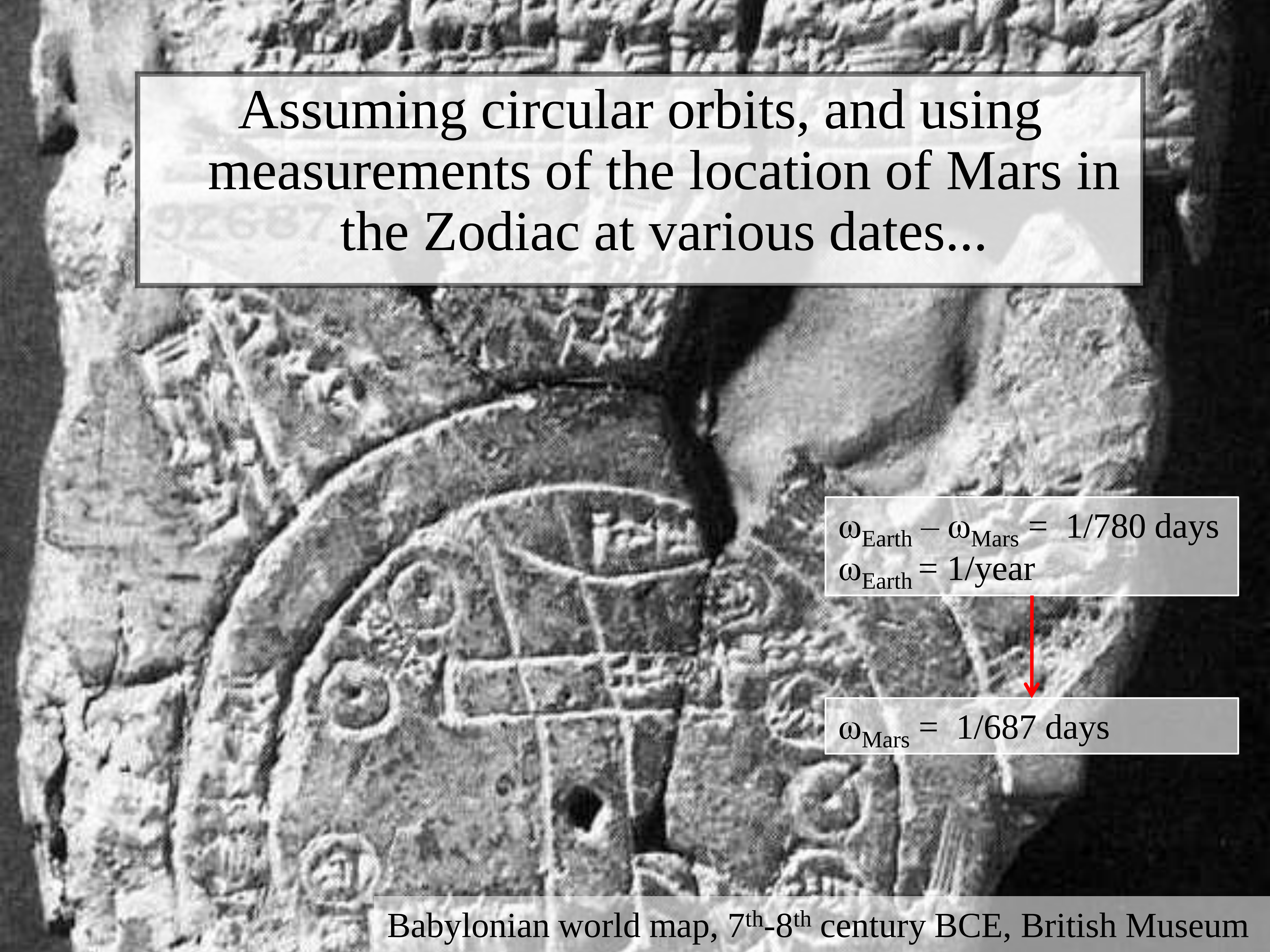


Babylonian world map, 7th-8th century BCE, British Museum

Assuming circular orbits, and using measurements of the location of Mars in the Zodiac at various dates...

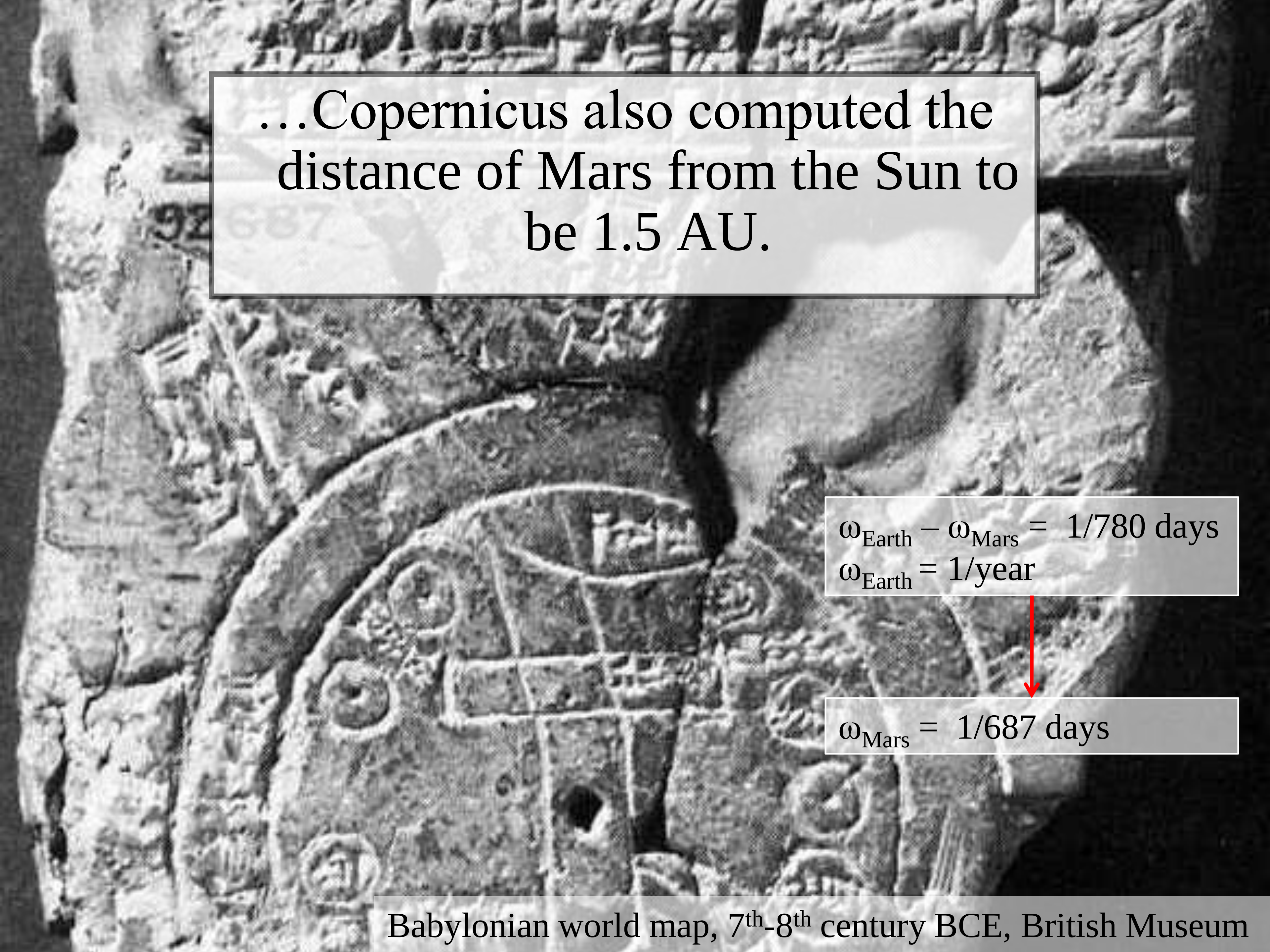
$\omega_{Earth} - \omega_{Mars} = 1/780$ days
$\omega_{Earth} = 1/year$

$\omega_{Mars} = 1/687$ days



Babylonian world map, 7th-8th century BCE, British Museum

…Copernicus also computed the distance of Mars from the Sun to be 1.5 AU.

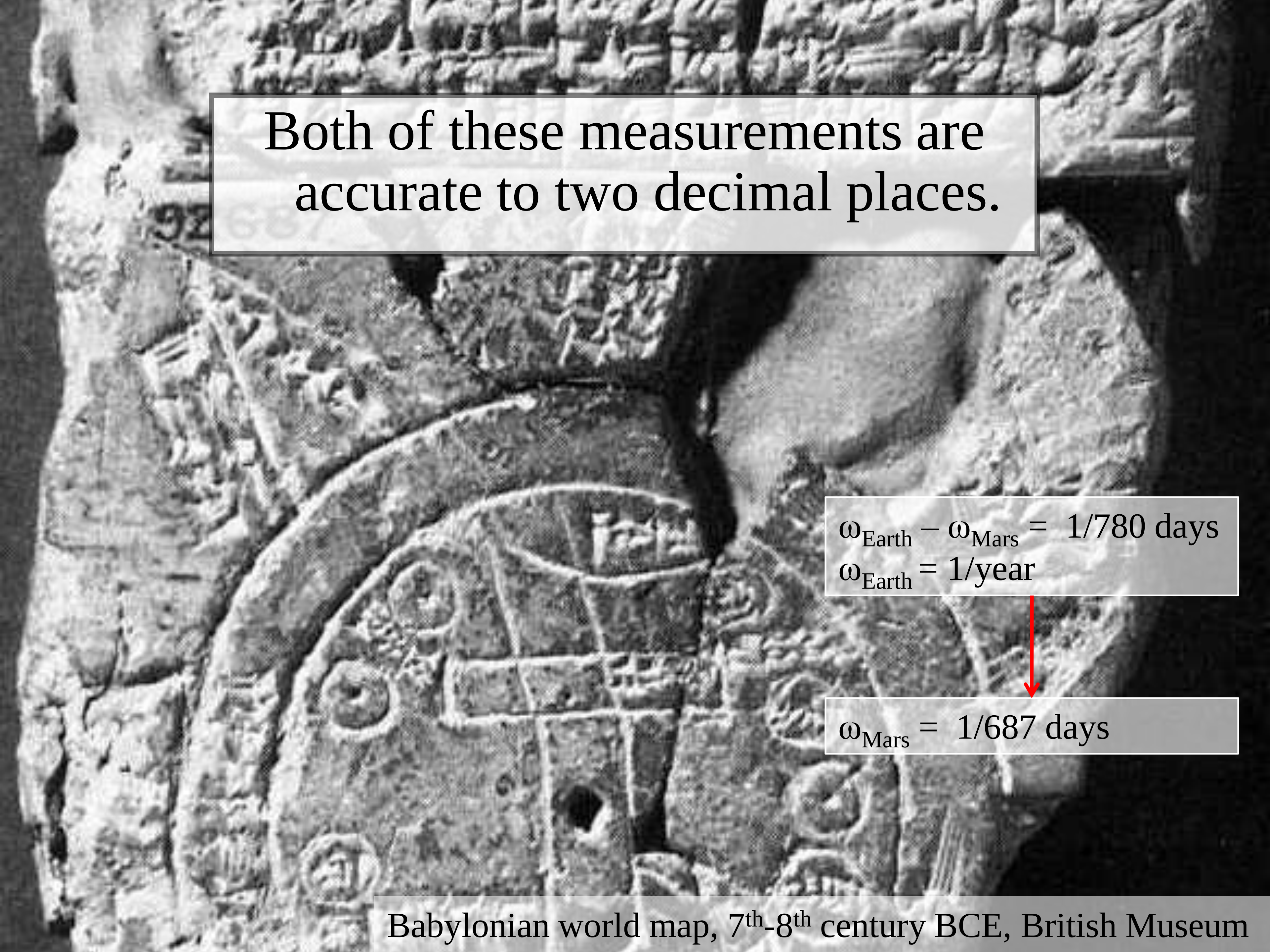


$\omega_{Earth} - \omega_{Mars} = 1/780$ days

$\omega_{Earth} = 1/year$

$\omega_{Mars} = 1/687$ days

Babylonian world map, 7th-8th century BCE, British Museum

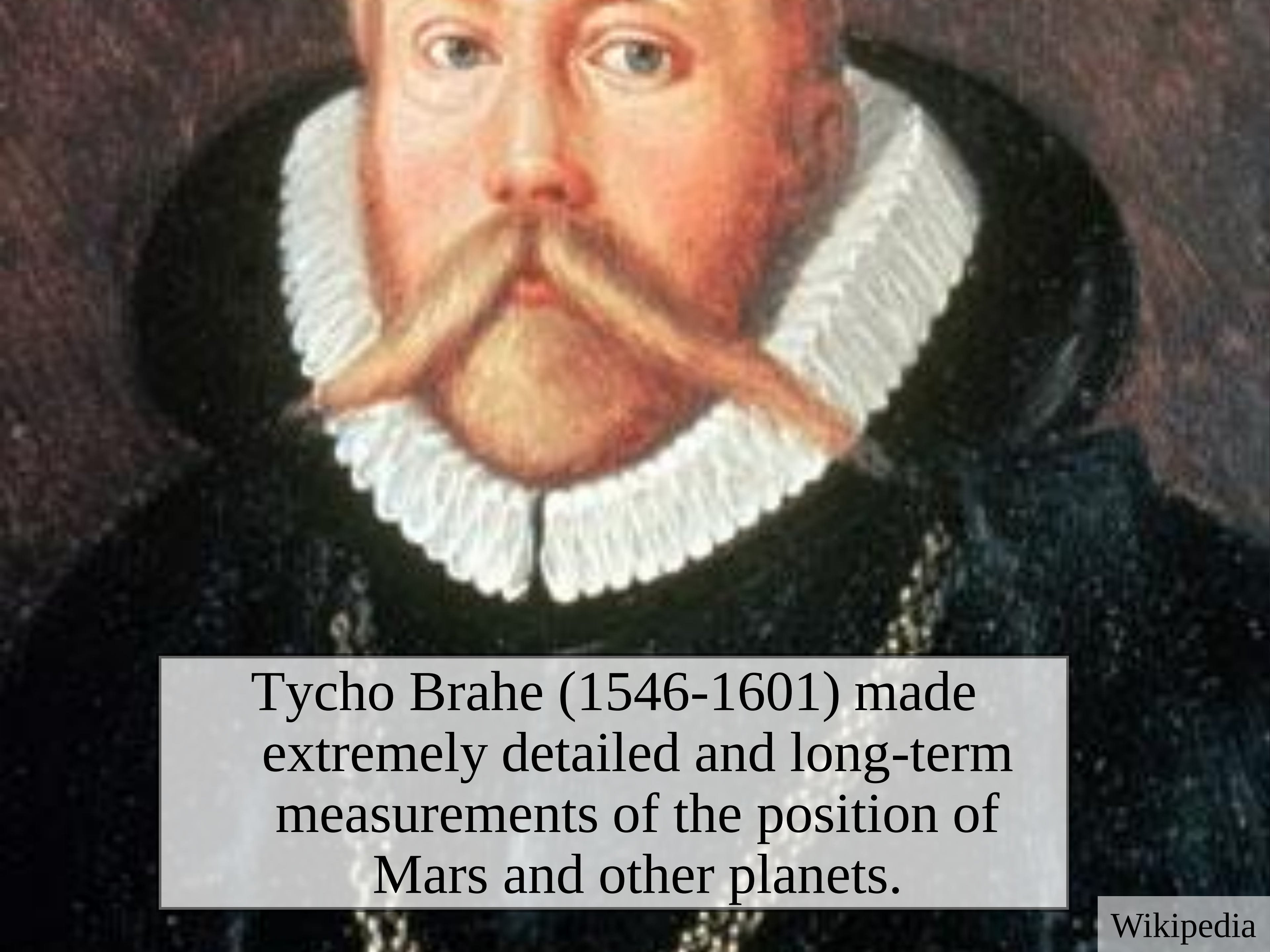


Both of these measurements are accurate to two decimal places.

$\omega_{Earth} - \omega_{Mars} = 1/780$ days
$\omega_{Earth} = 1/year$

$\omega_{Mars} = 1/687$ days

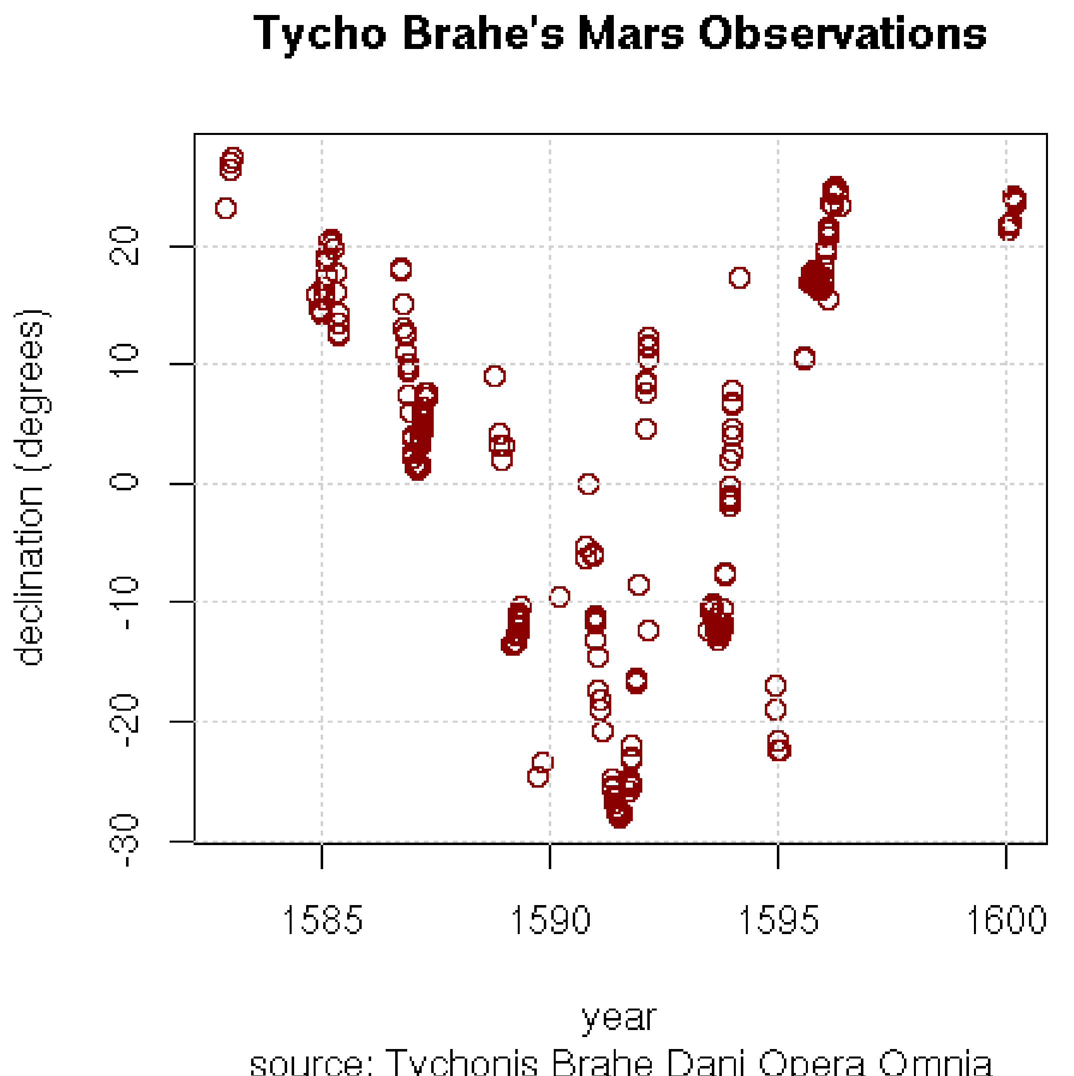
Babylonian world map, 7th-8th century BCE, British Museum

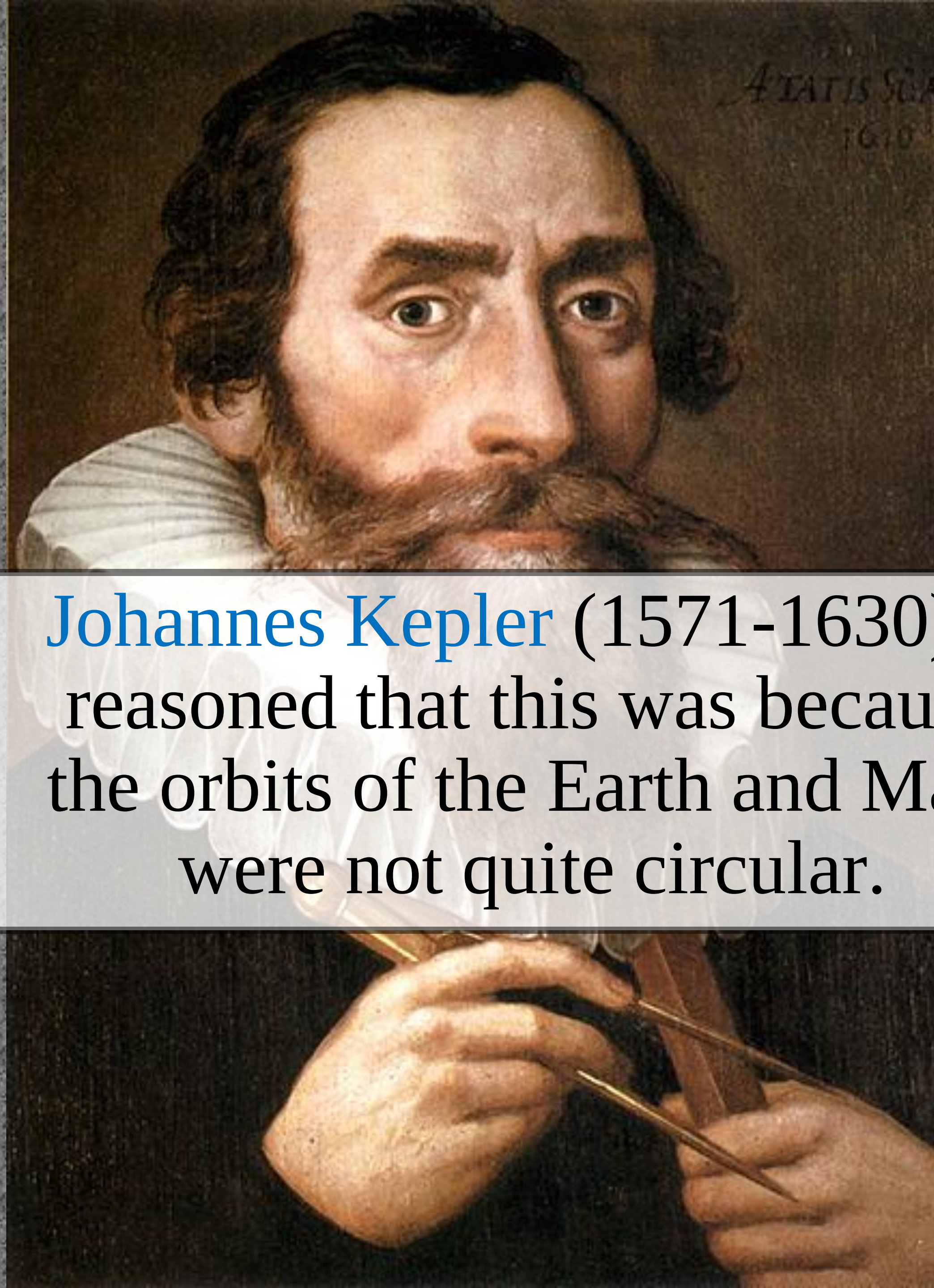


Tycho Brahe (1546-1601) made extremely detailed and long-term measurements of the position of Mars and other planets.

Wikipedia

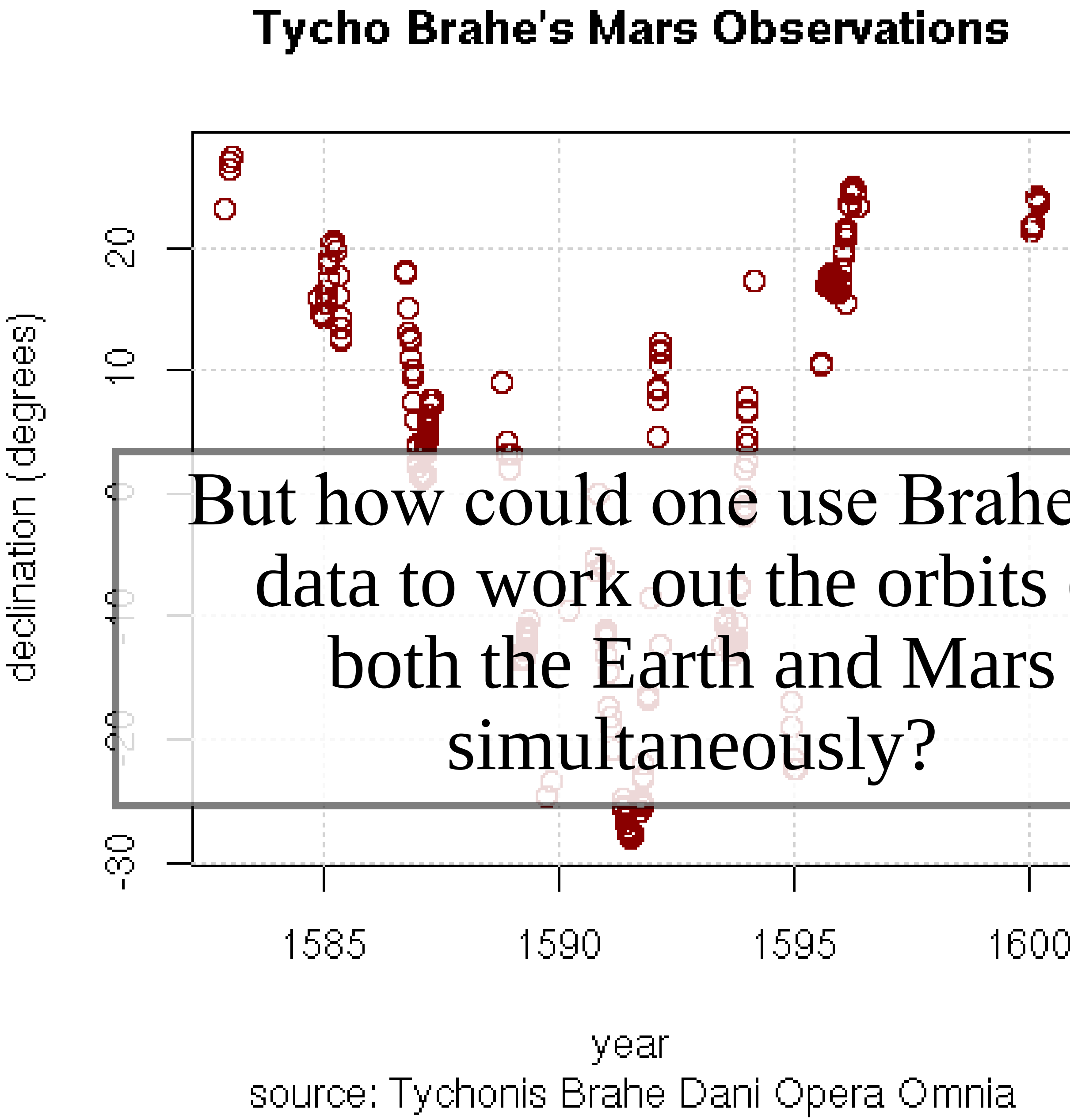
Unfortunately, his data deviated slightly from the predictions of the Copernican model.



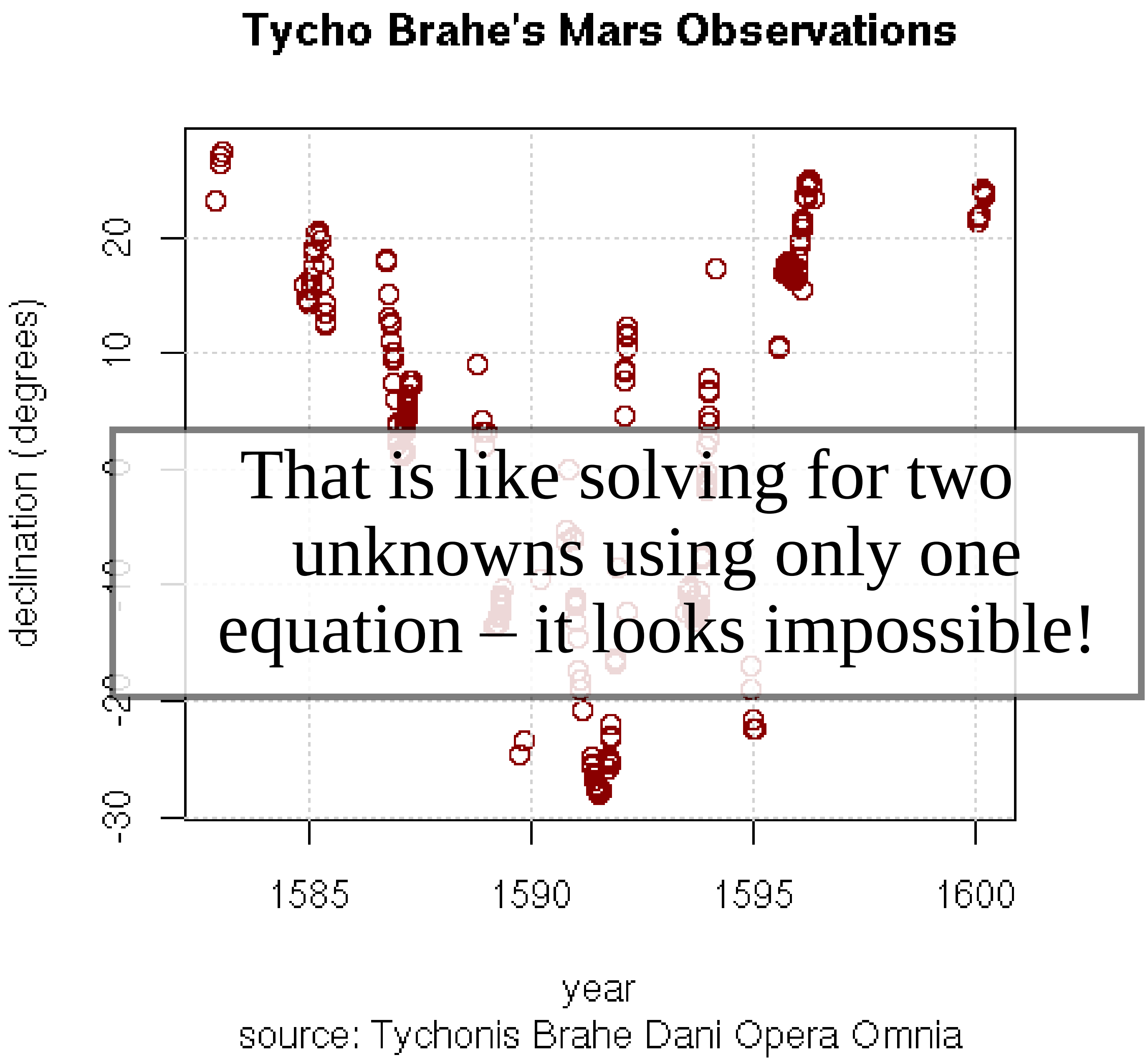


Johannes Kepler (1571-1630) reasoned that this was because the orbits of the Earth and Mars were not quite circular.

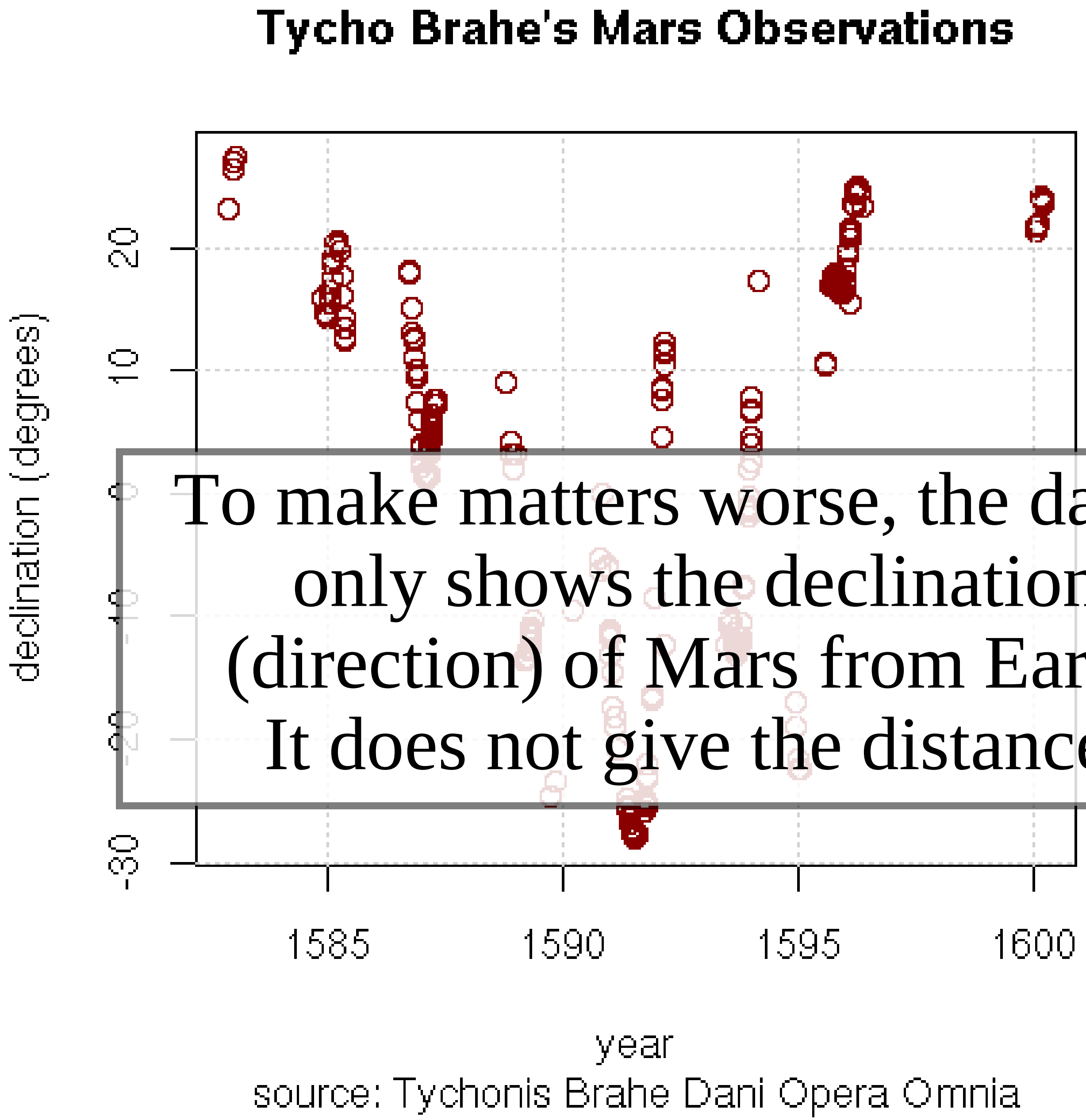
Wikipedia



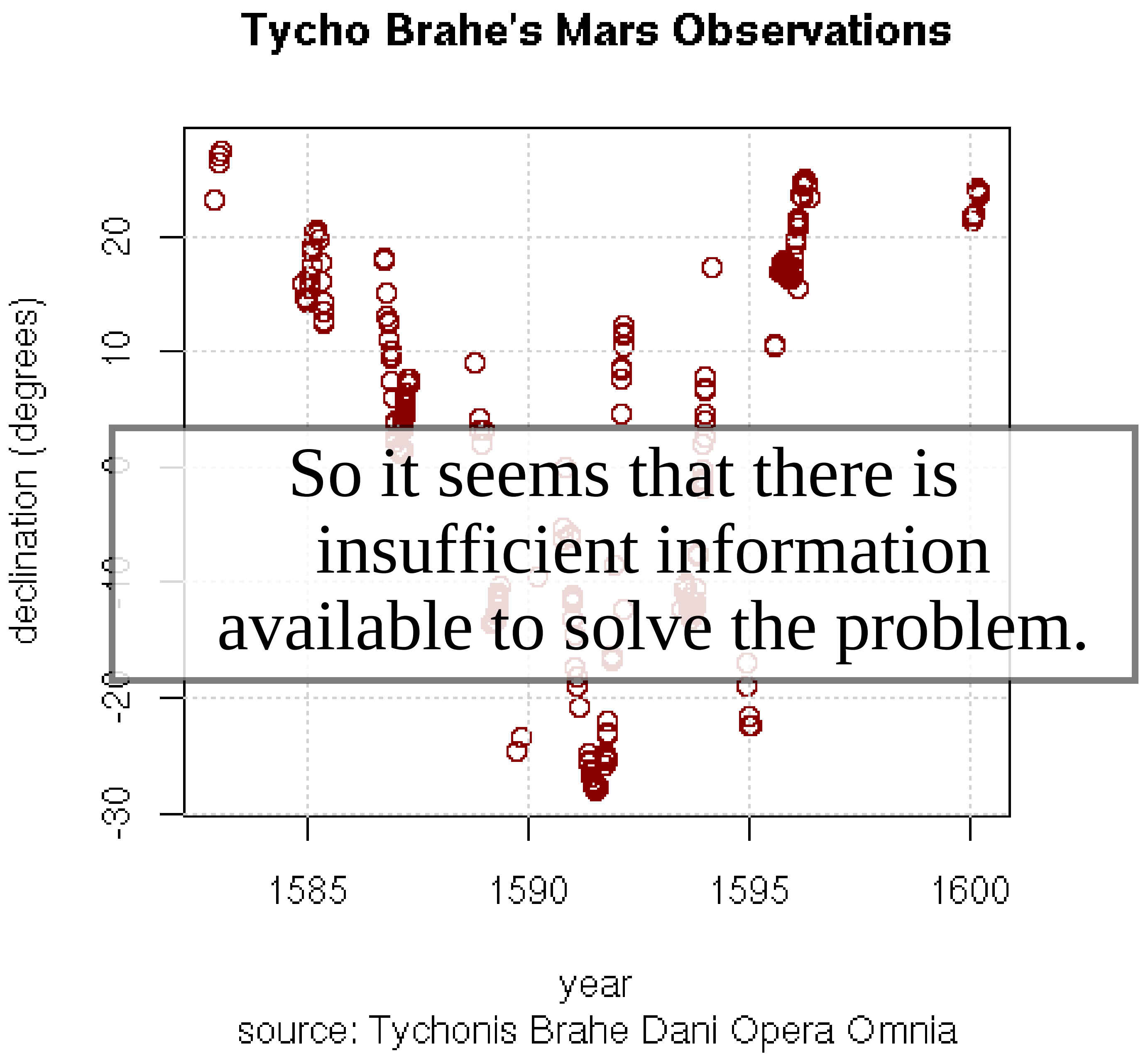
But how could one use Brahe's data to work out the orbits of both the Earth and Mars simultaneously?



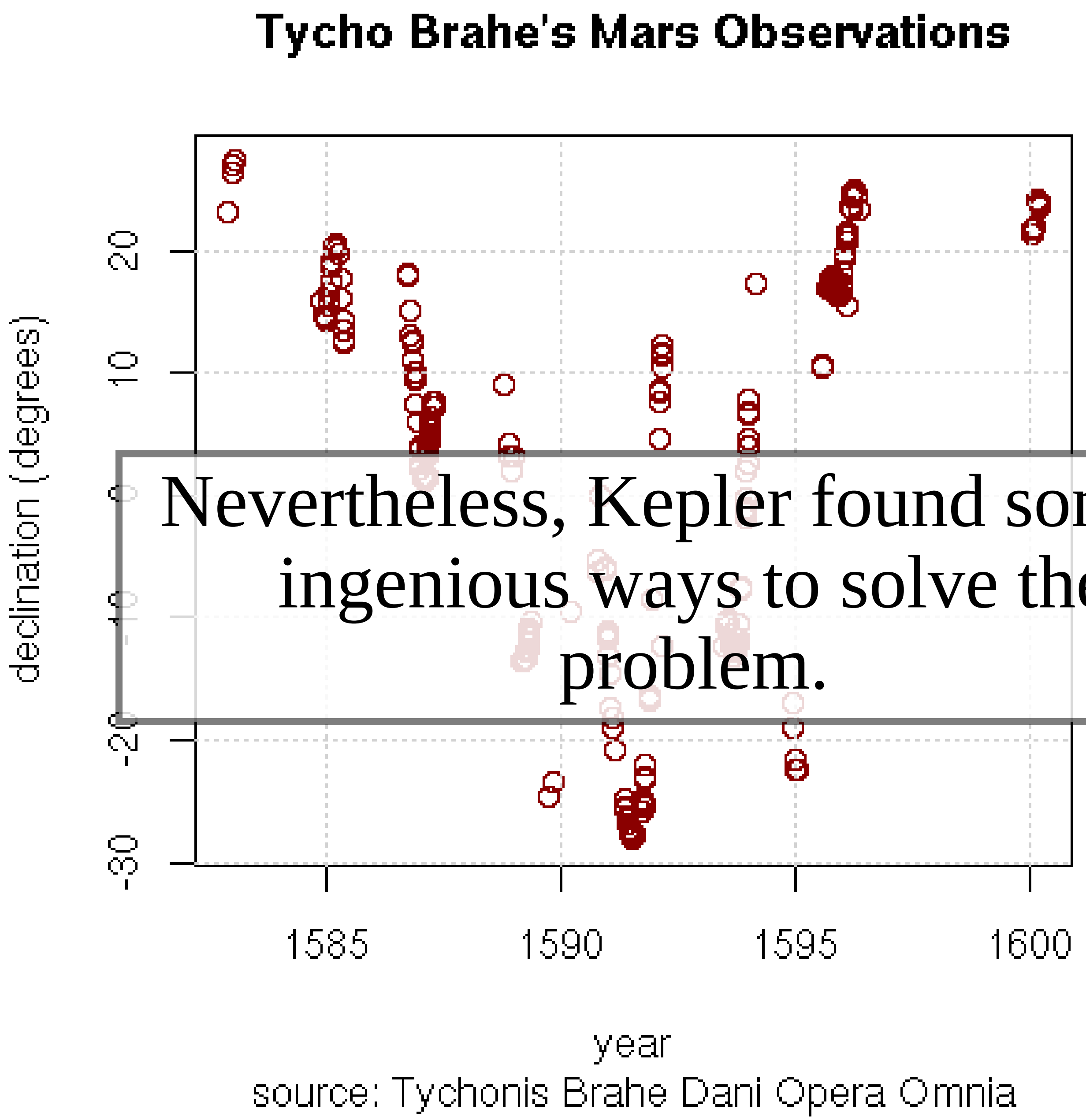
That is like solving for two unknowns using only one equation – it looks impossible!



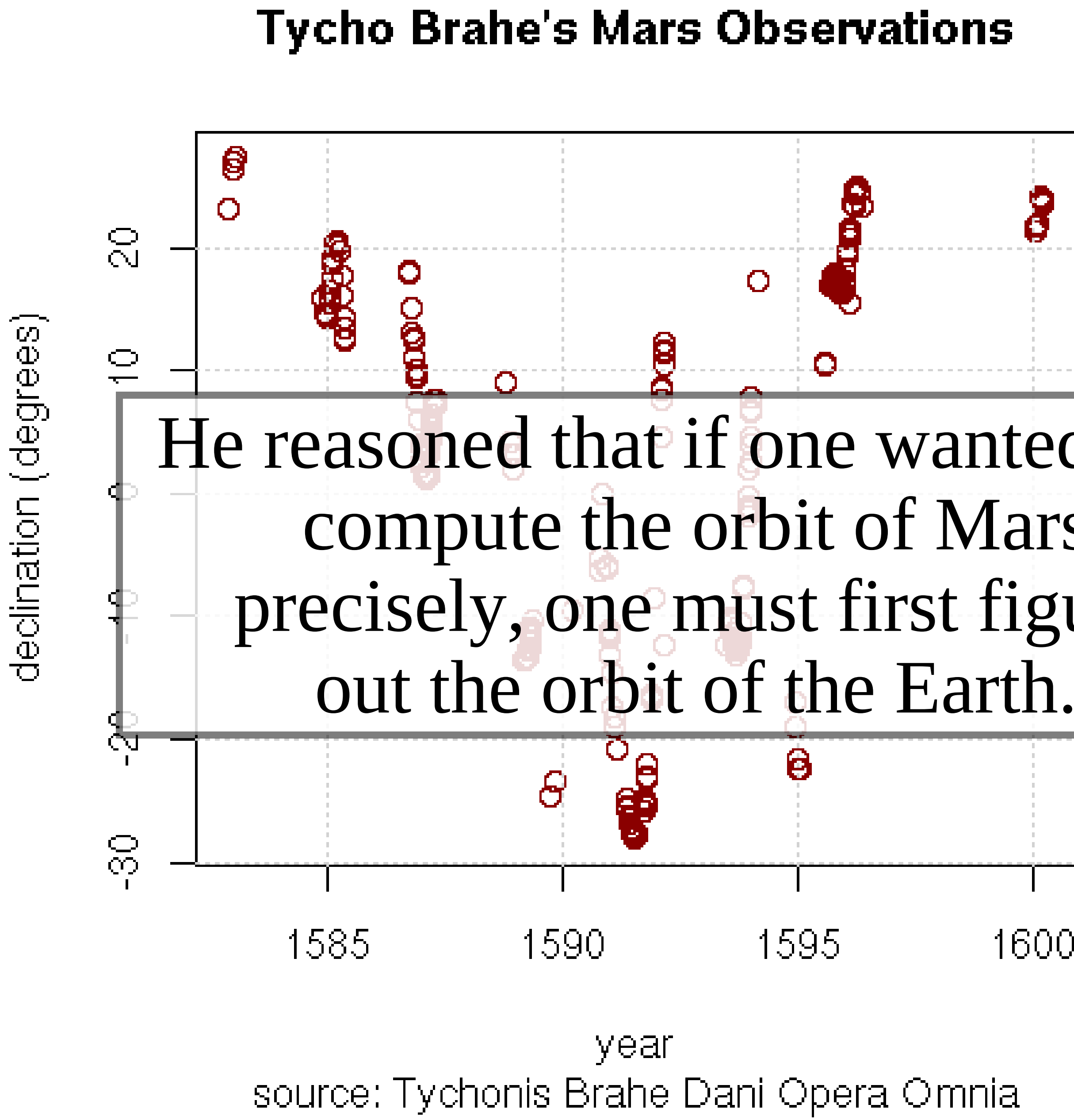
To make matters worse, the data only shows the declination (direction) of Mars from Earth. It does not give the distance.



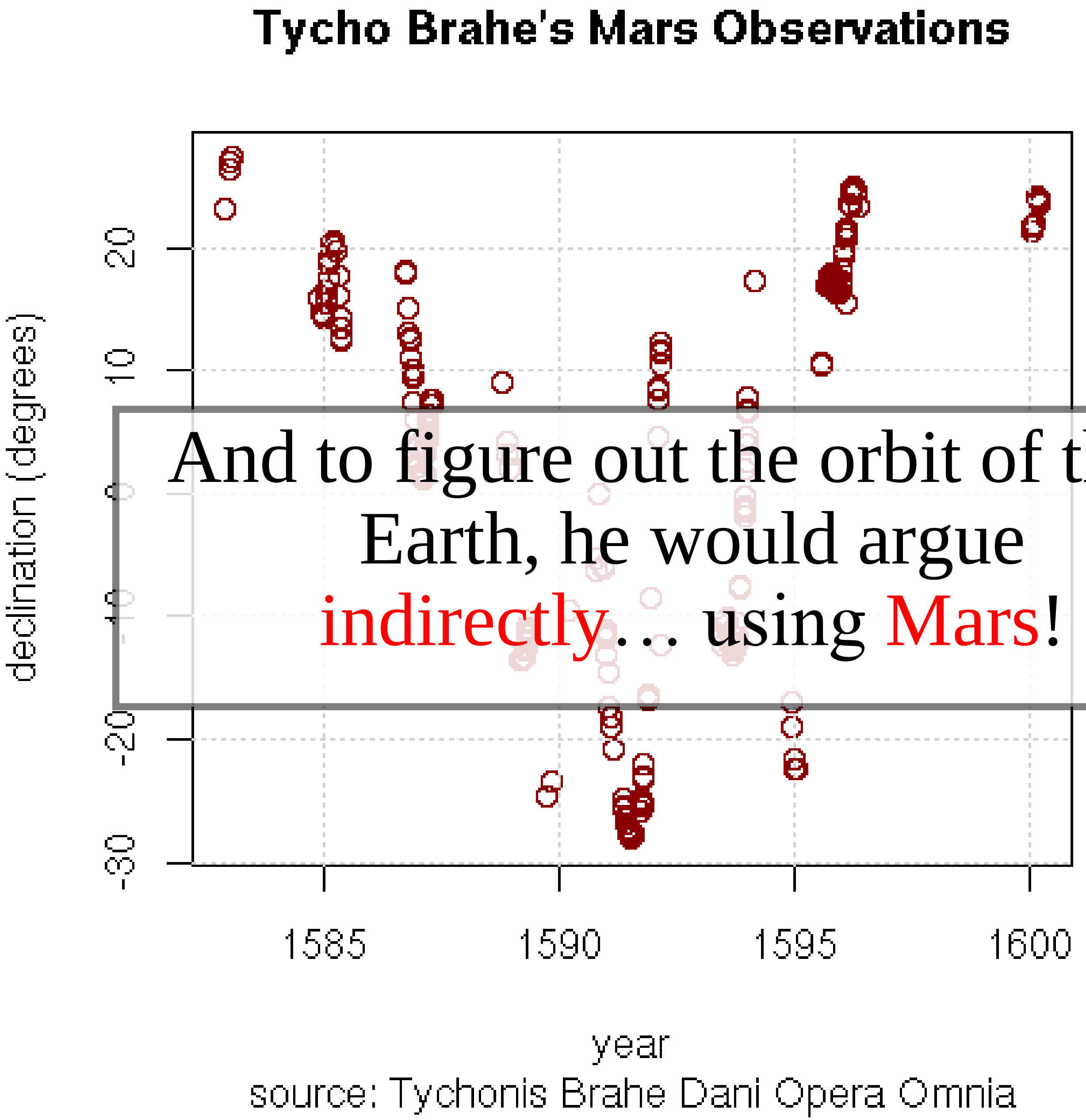
So it seems that there is insufficient information available to solve the problem.



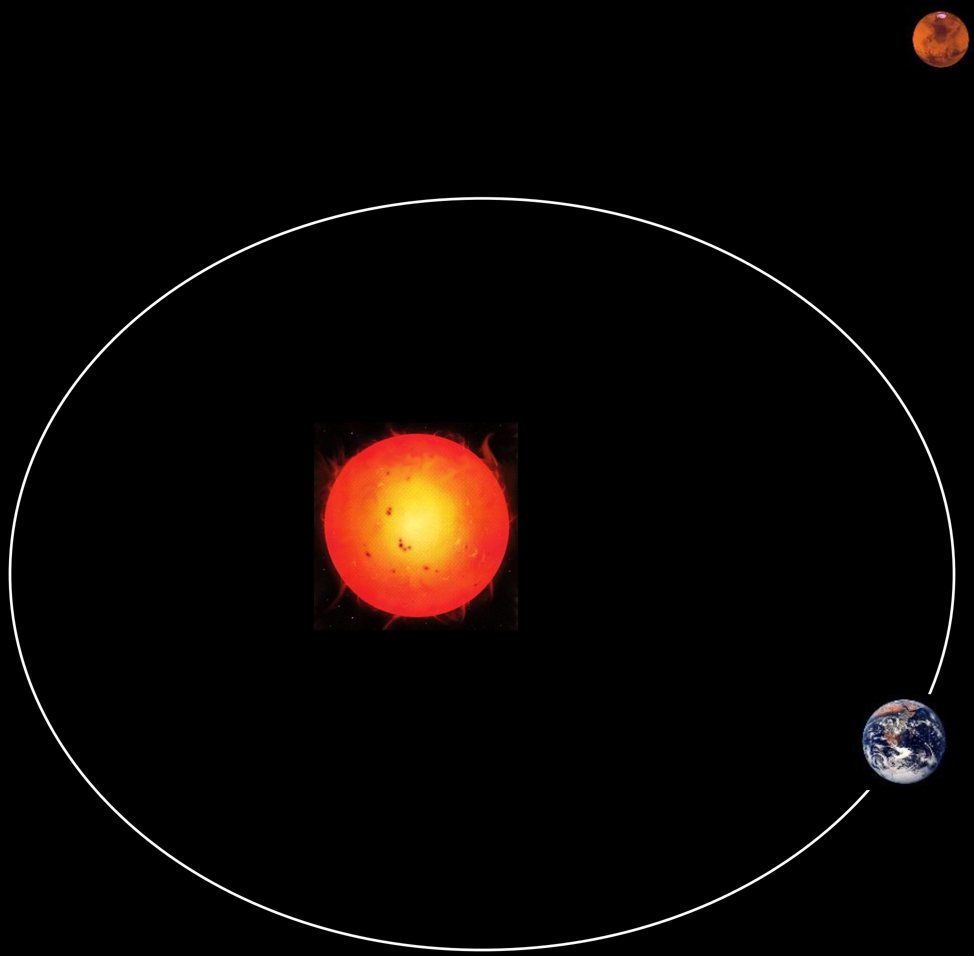
Nevertheless, Kepler found some ingenious ways to solve the problem.



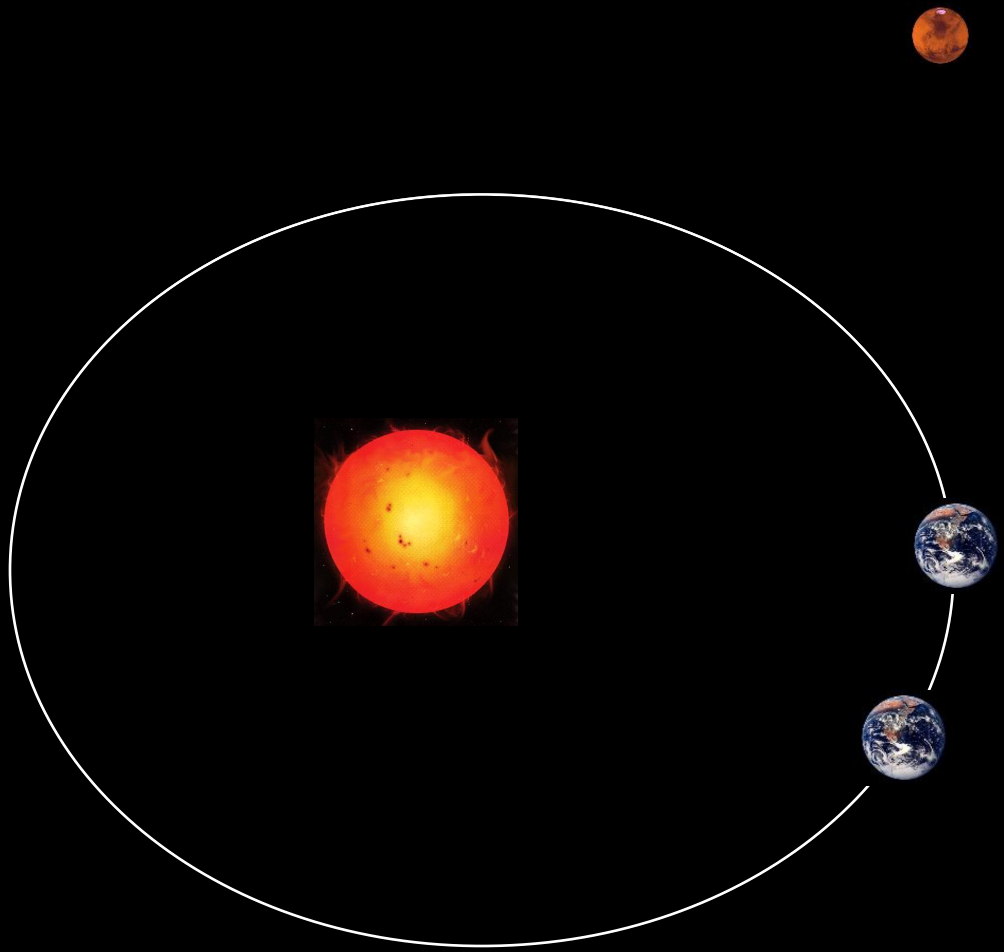
He reasoned that if one wanted to compute the orbit of Mars precisely, one must first figure out the orbit of the Earth.



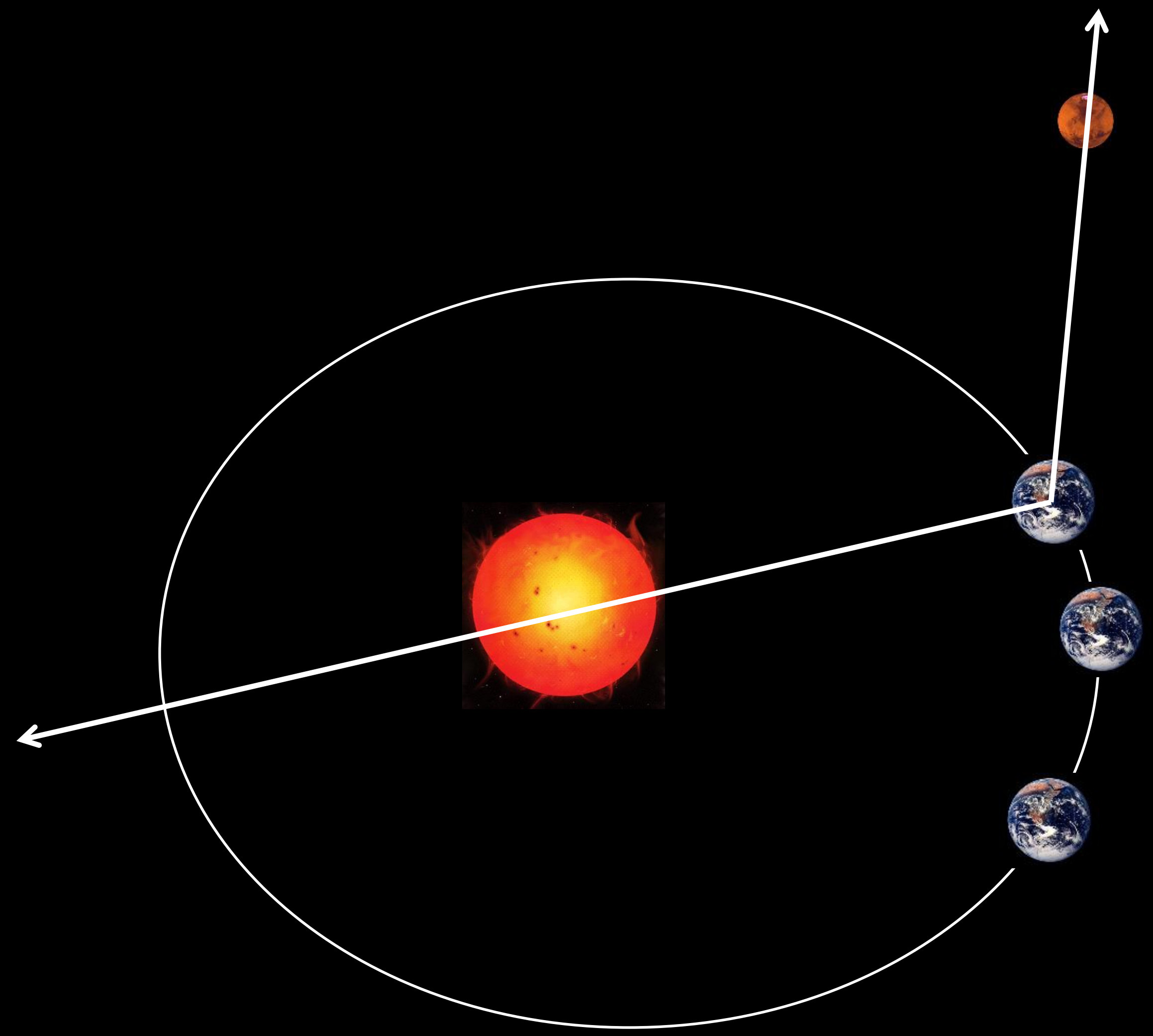
And to figure out the orbit of the Earth, he would argue indirectly… using Mars!



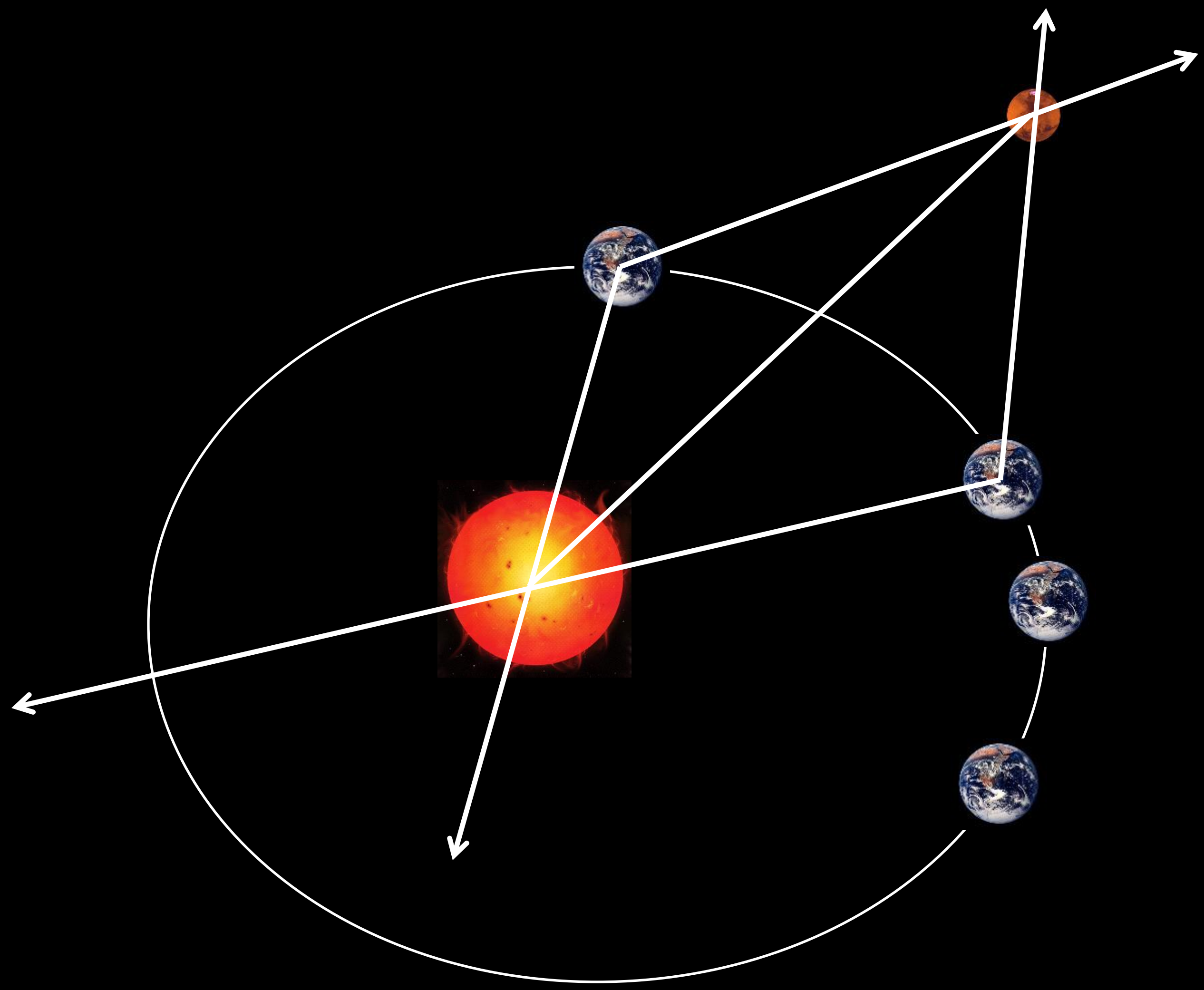
To explain how this works, let’s first suppose that Mars is fixed, rather than orbiting the Sun.



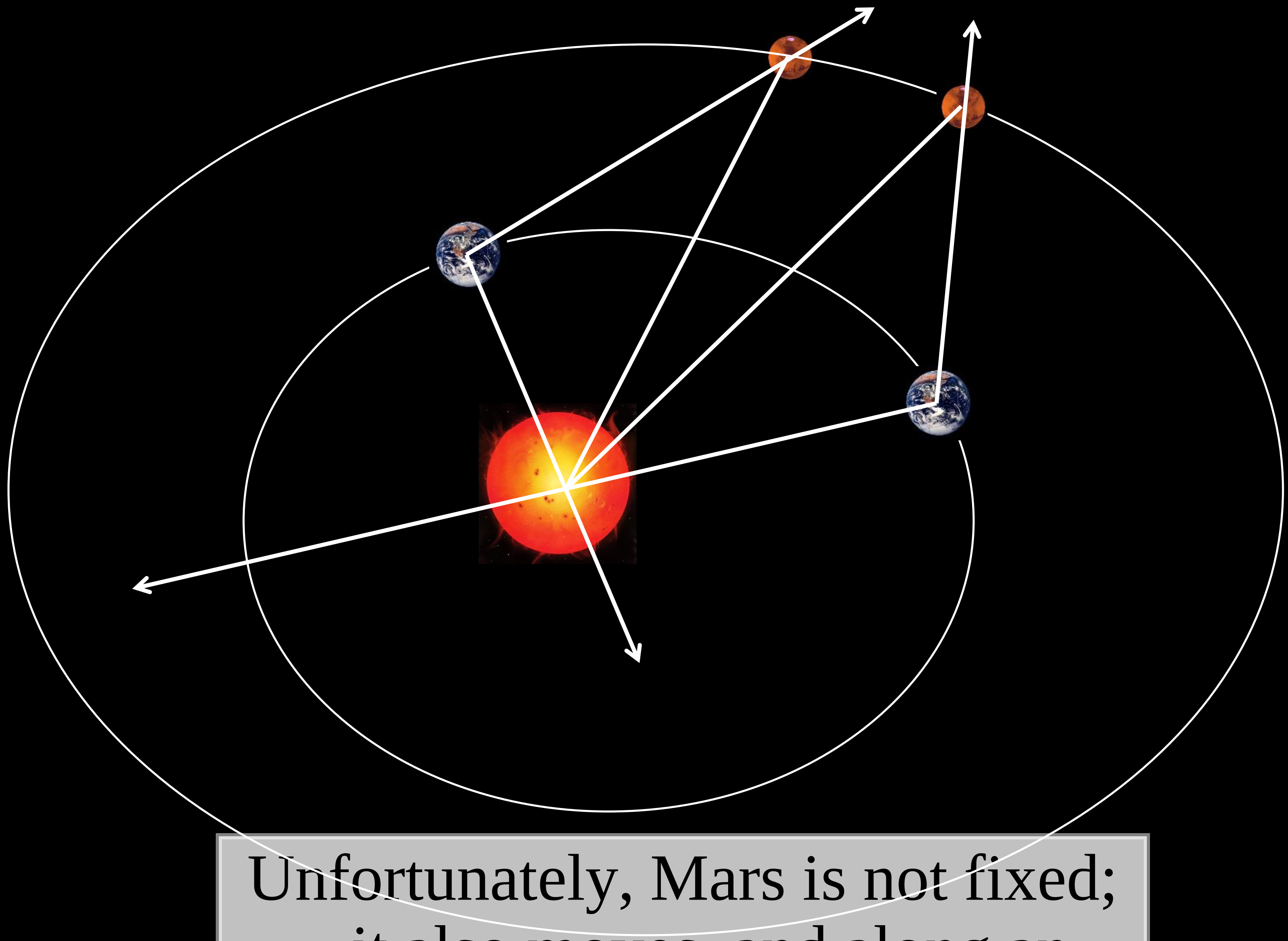
But the Earth is moving in an unknown orbit.



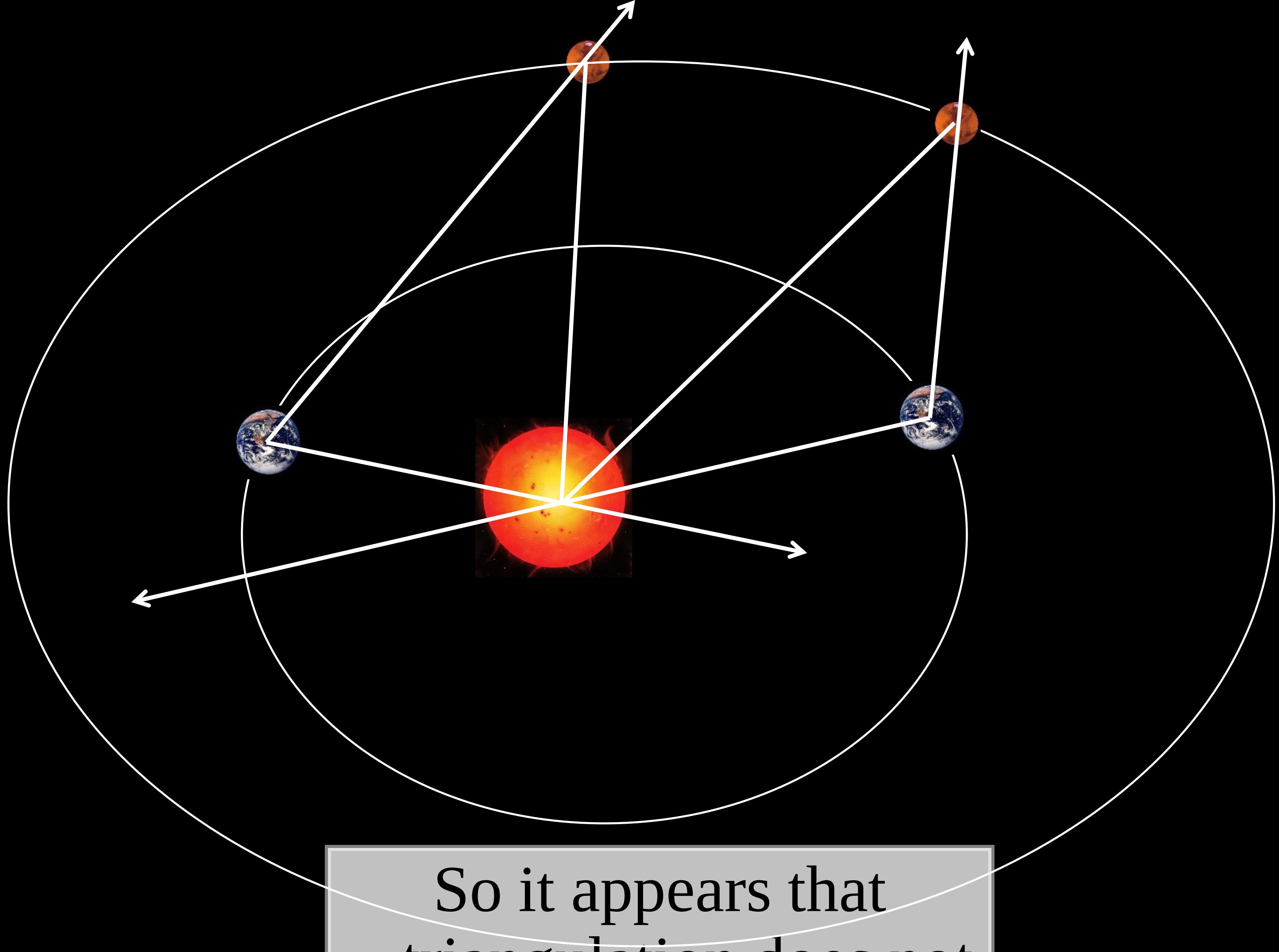
At any given time, one can measure the position of the Sun and Mars from Earth, with respect to the fixed stars (the Zodiac).



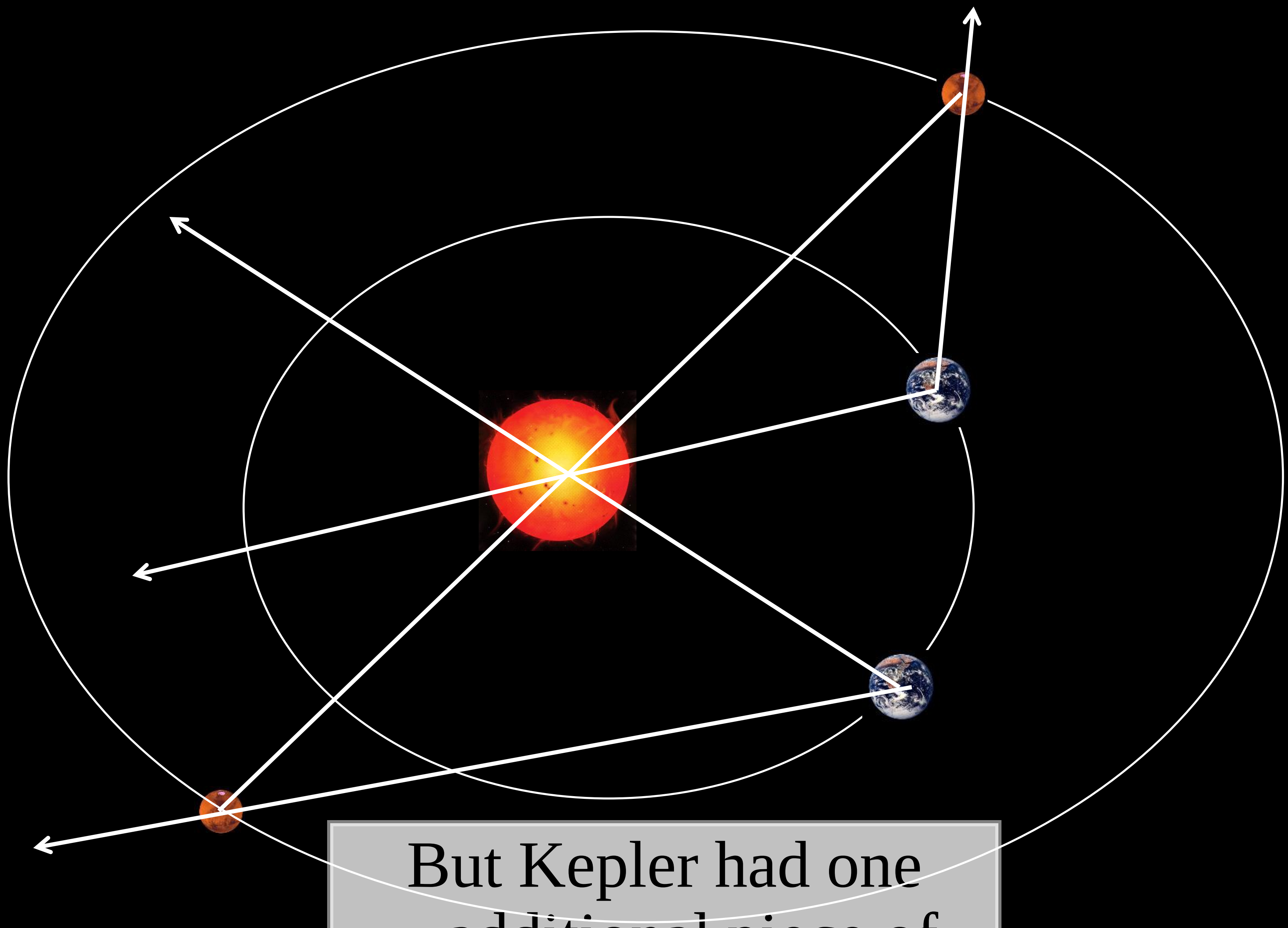
Assuming that the Sun and Mars are fixed, one can then triangulate to determine the position of the Earth relative to the Sun and Mars.



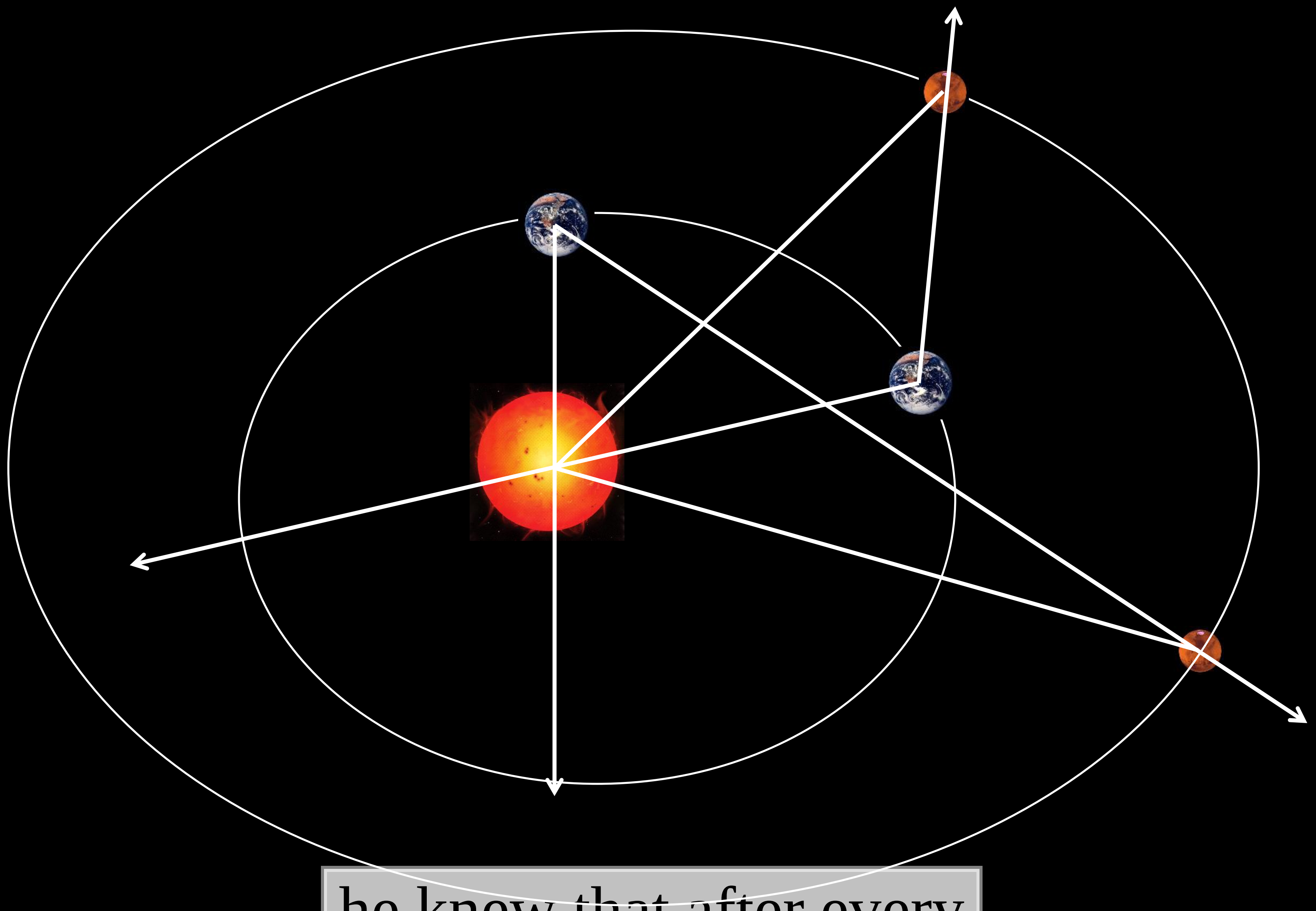
Unfortunately, Mars is not fixed; it also moves, and along an unknown orbit.



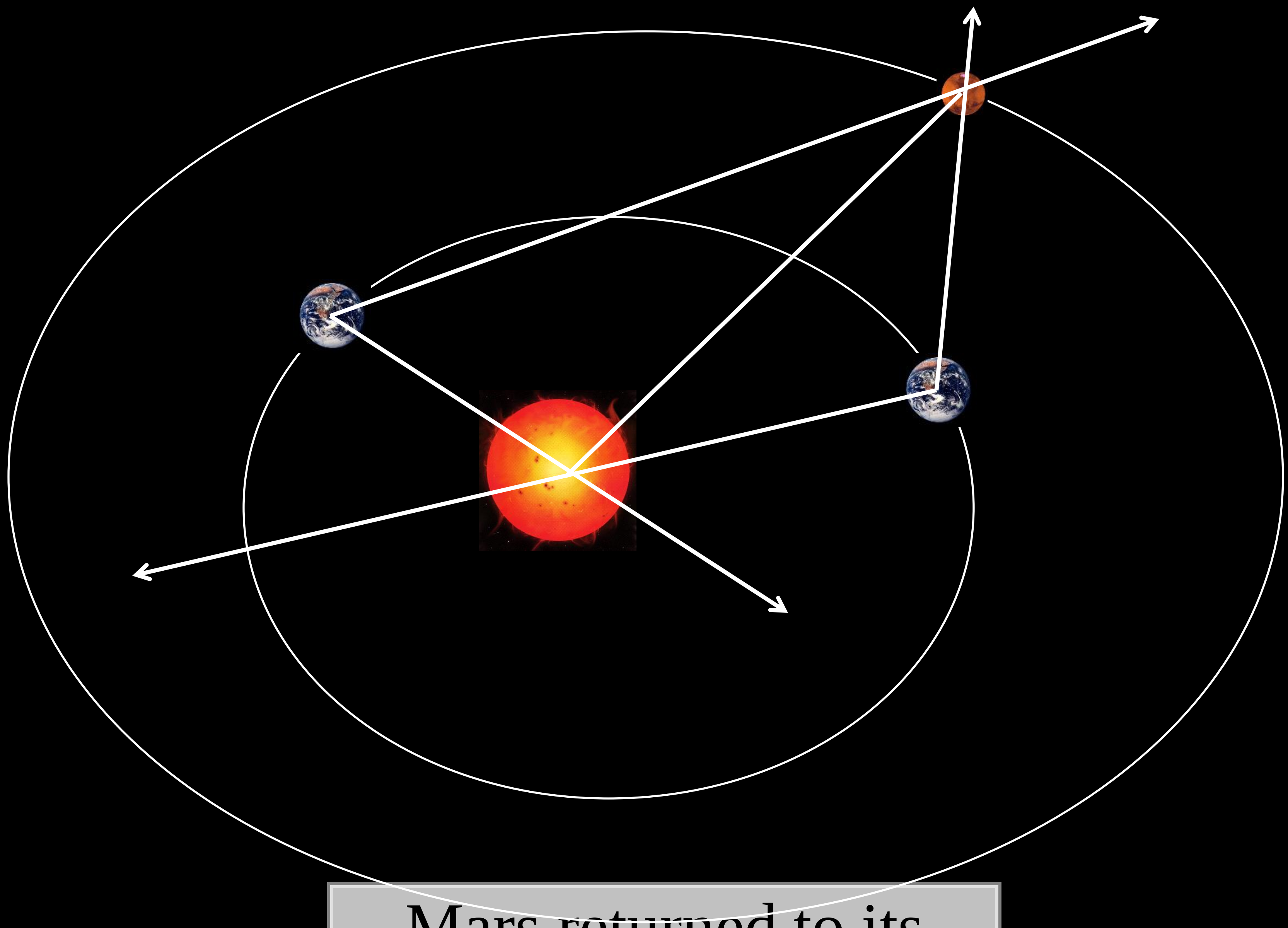
So it appears that triangulation does not work.



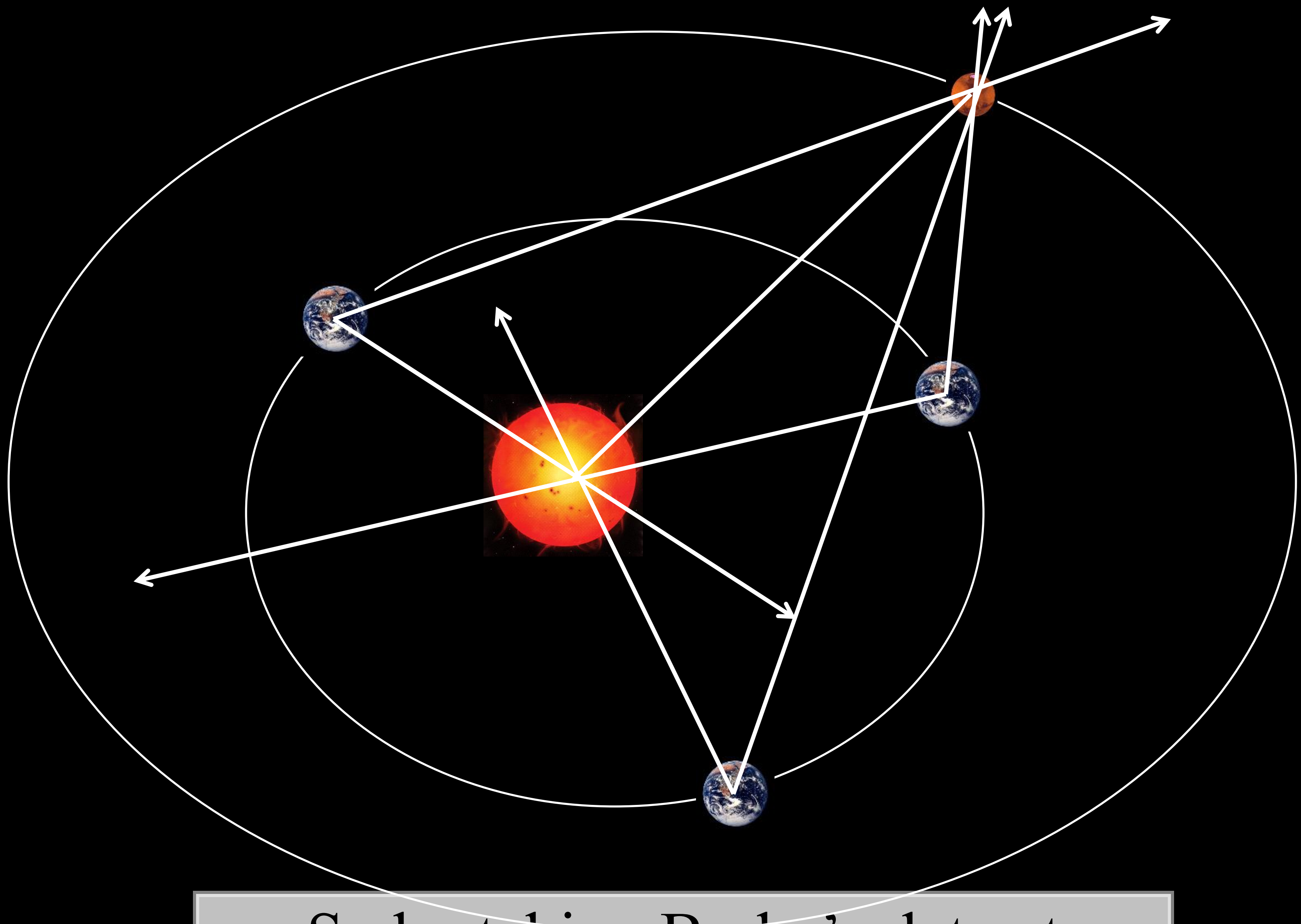
But Kepler had one additional piece of information:



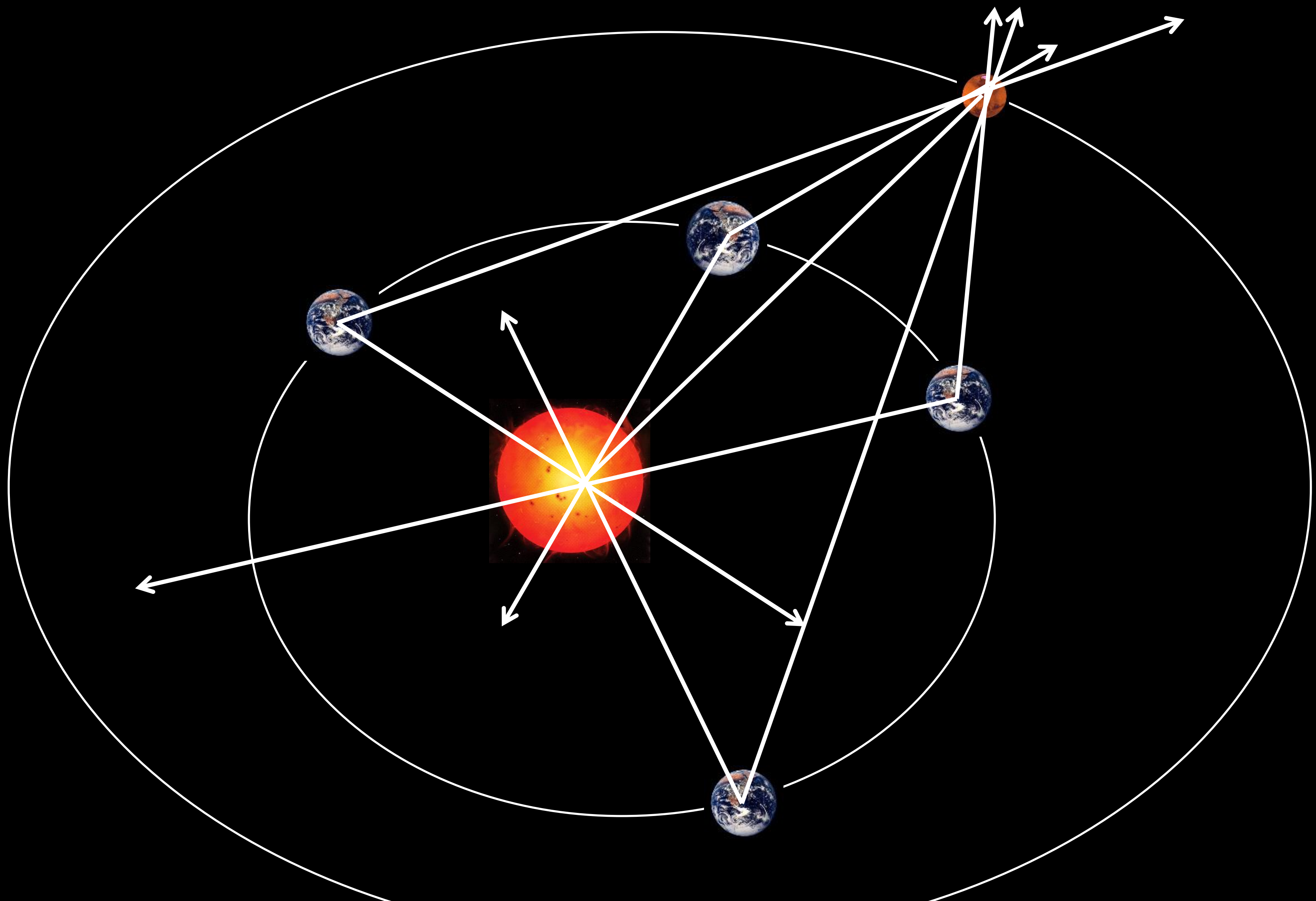
he knew that after every
687 days…



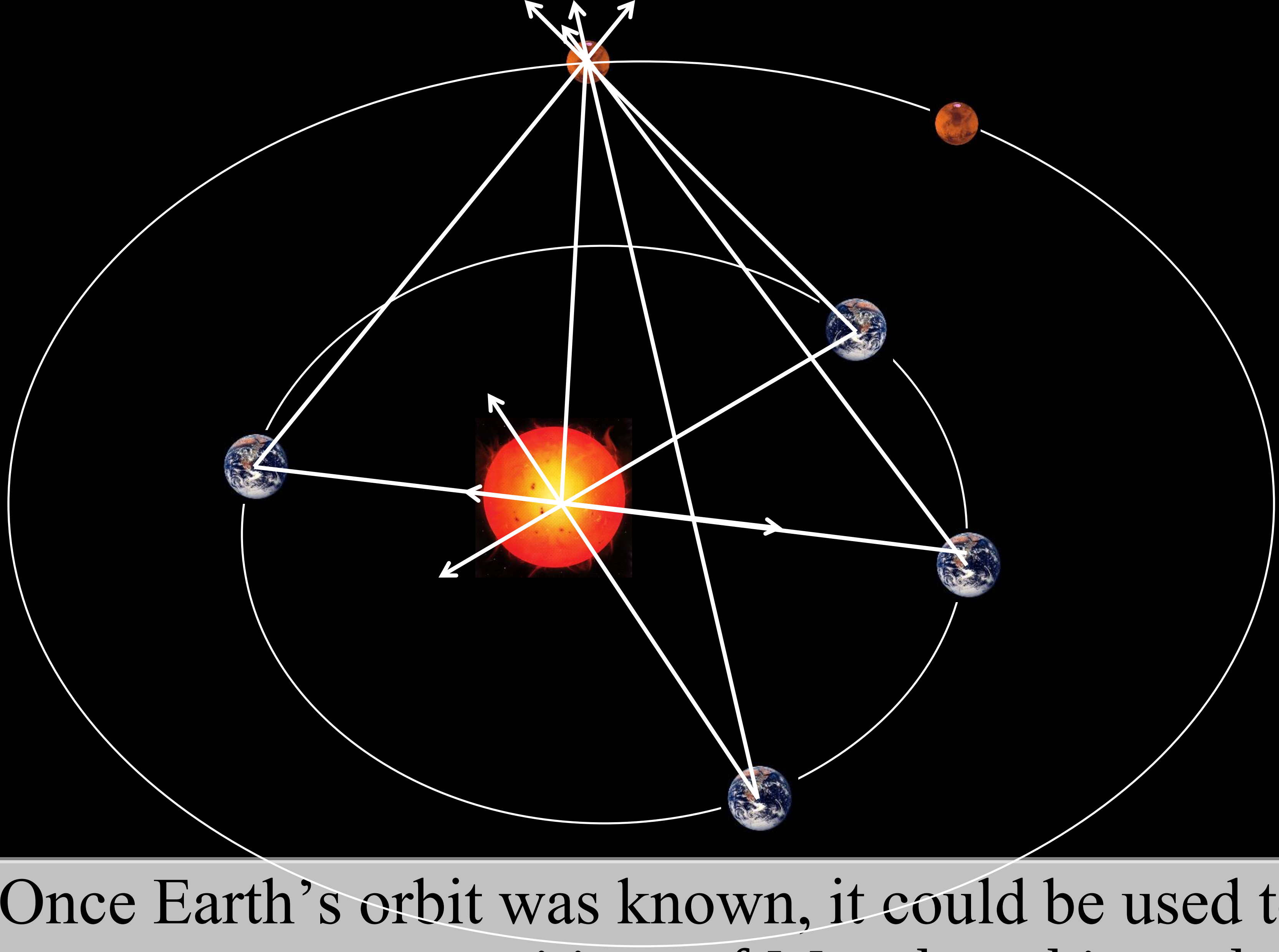
Mars returned to its original position.



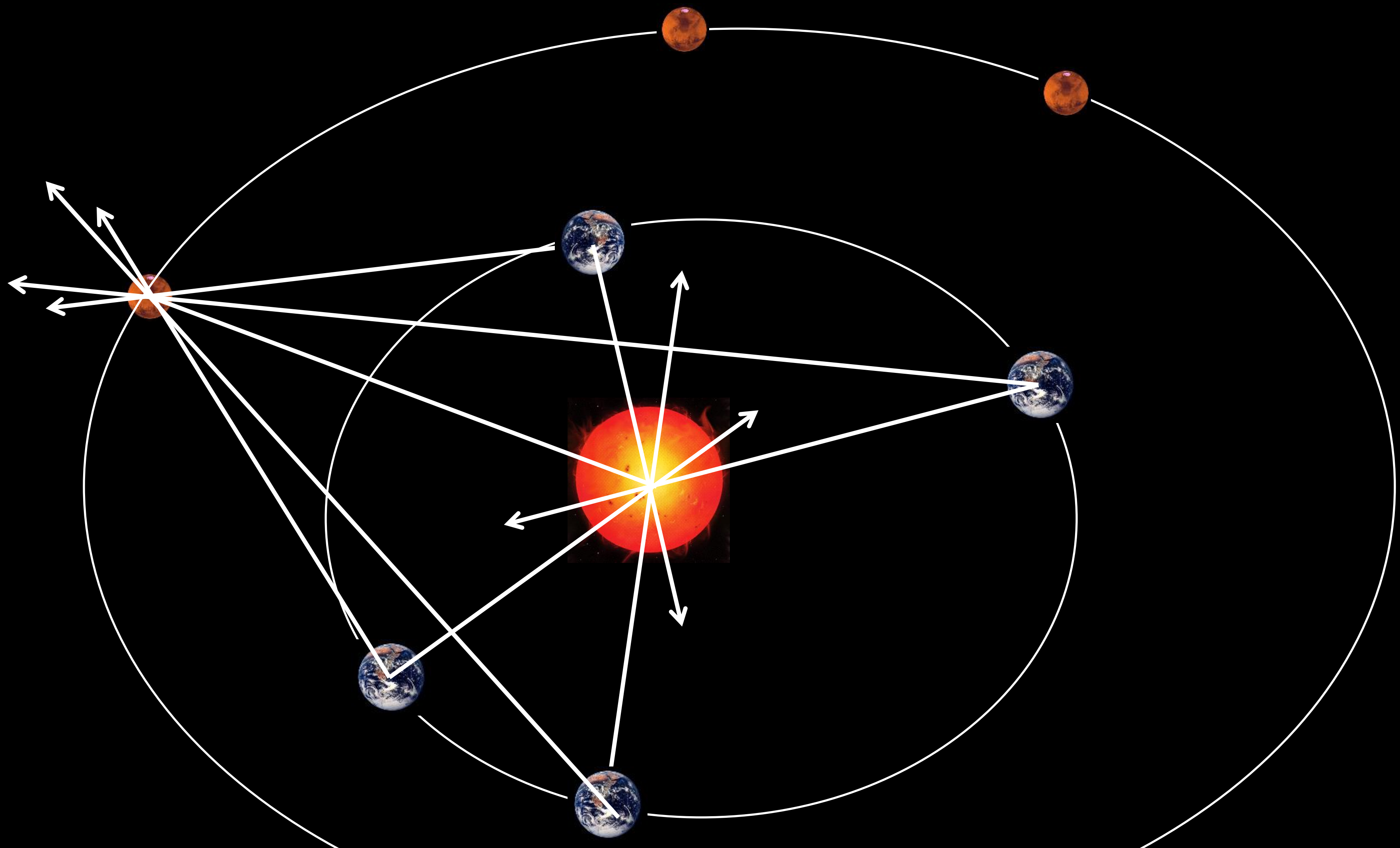
So by taking Brahe's data at intervals of 687 days…



… Kepler could triangulate and compute Earth's orbit relative to any position of Mars.



Once Earth's orbit was known, it could be used to compute more positions of Mars by taking other sequences of data separated by 687 days…



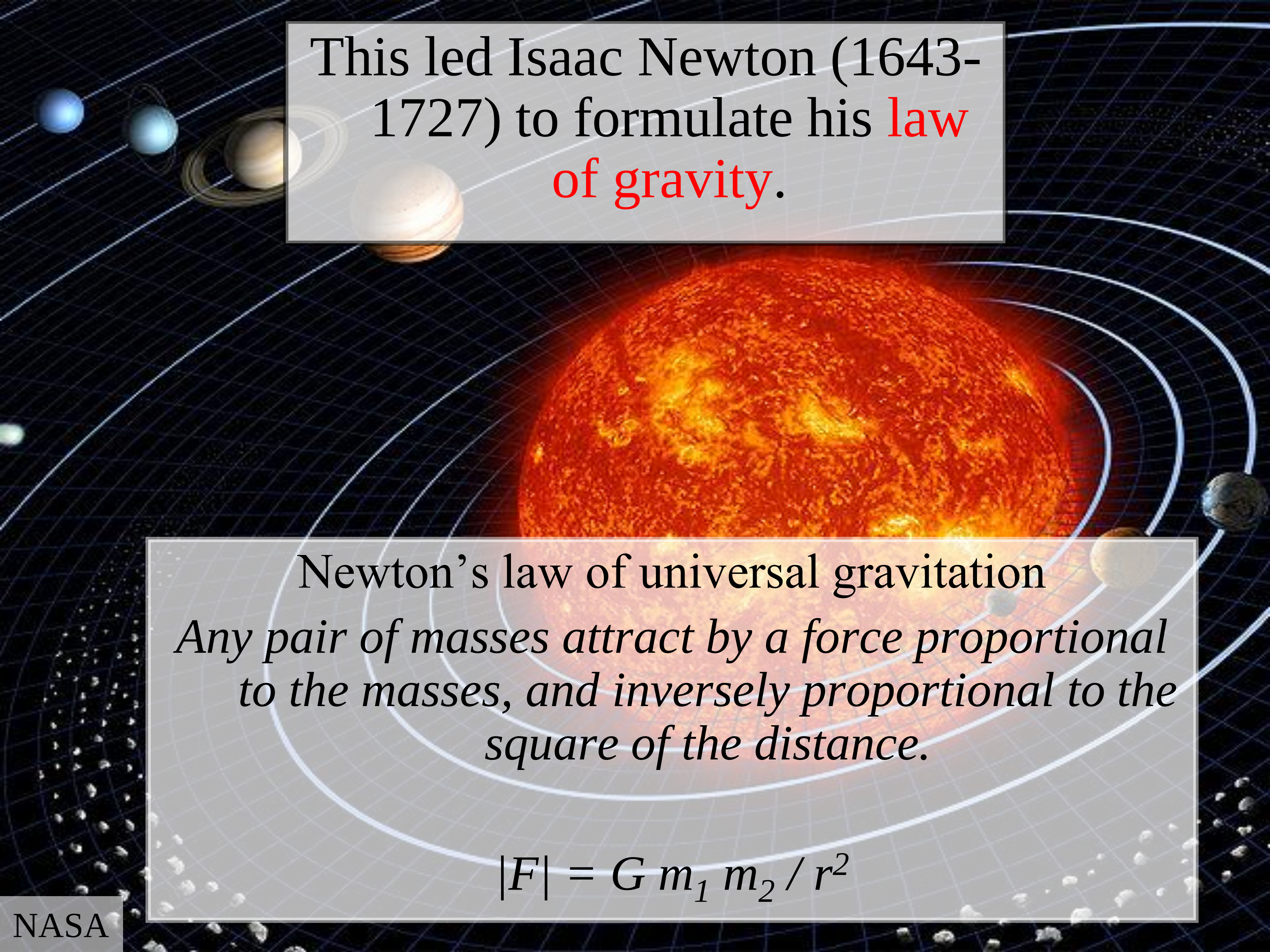
… which allows one to compute the orbit of Mars.

Using the data for Mars and other planets, Kepler formulated his three laws of planetary motion.

Kepler’s laws of planetary motion

1. *Planets orbit in ellipses, with the Sun as one of the foci.*
2. *A planet sweeps out equal areas in equal times.*
3. *The square of the period of an orbit is proportional to the cube of its semi-major axis.*

NASA



This led Isaac Newton (1643-1727) to formulate his law of gravity.

Newton’s law of universal gravitation

Any pair of masses attract by a force proportional to the masses, and inversely proportional to the square of the distance.

$$|F| = G\, m_1\, m_2 / r^2$$

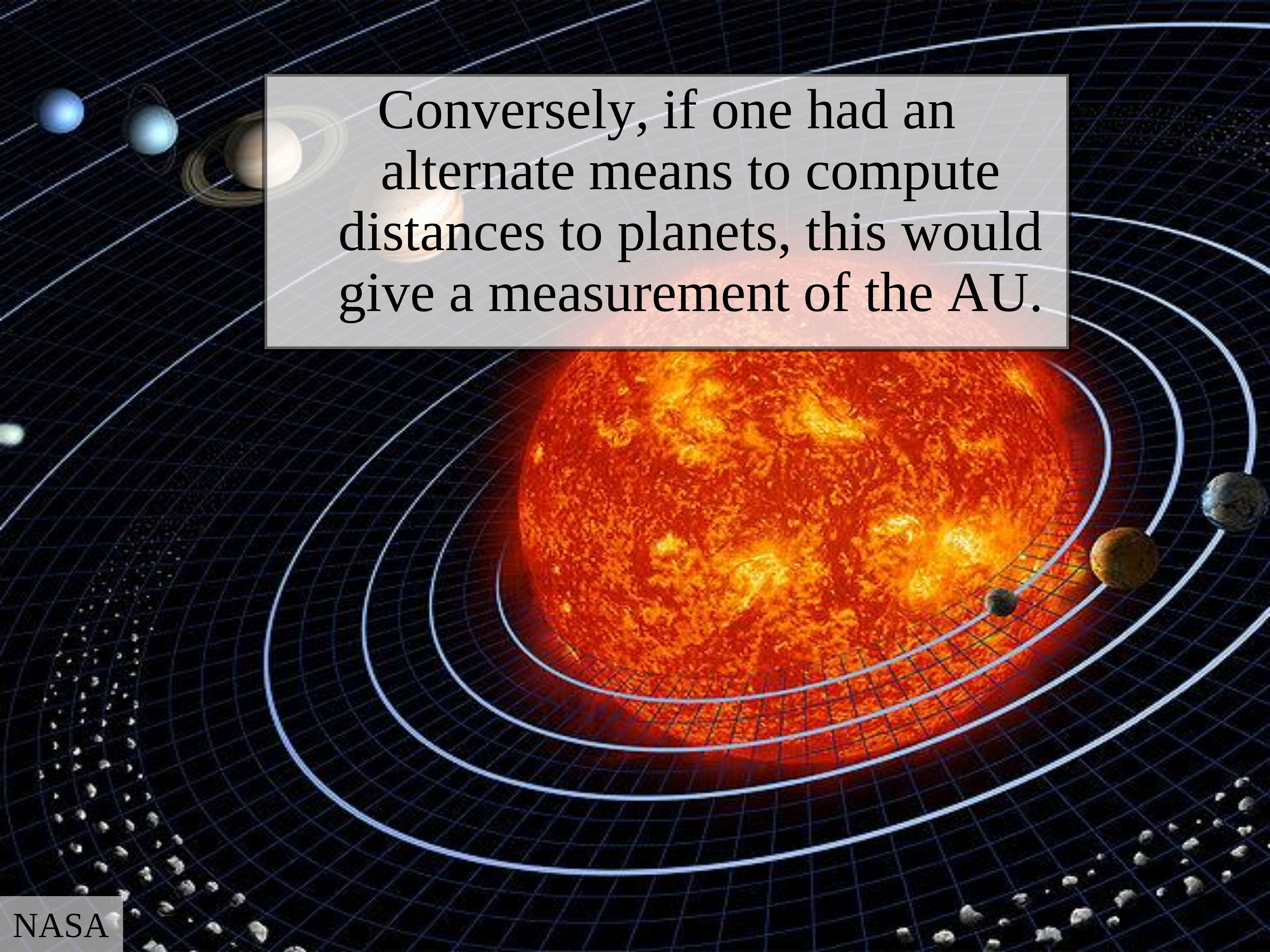
NASA

Kepler's methods allowed for very precise measurements of planetary distances in terms of the AU.

Mercury: 0.307-0.466 AU
Venus: 0.718-0.728 AU
Earth: 0.98-1.1 AU
Mars: 1.36-1.66 AU
Jupiter: 4.95-5.46 AU
Saturn: 9.05-10.12 AU
Uranus: 18.4-20.1 AU
Neptune: 29.8-30.4 AU

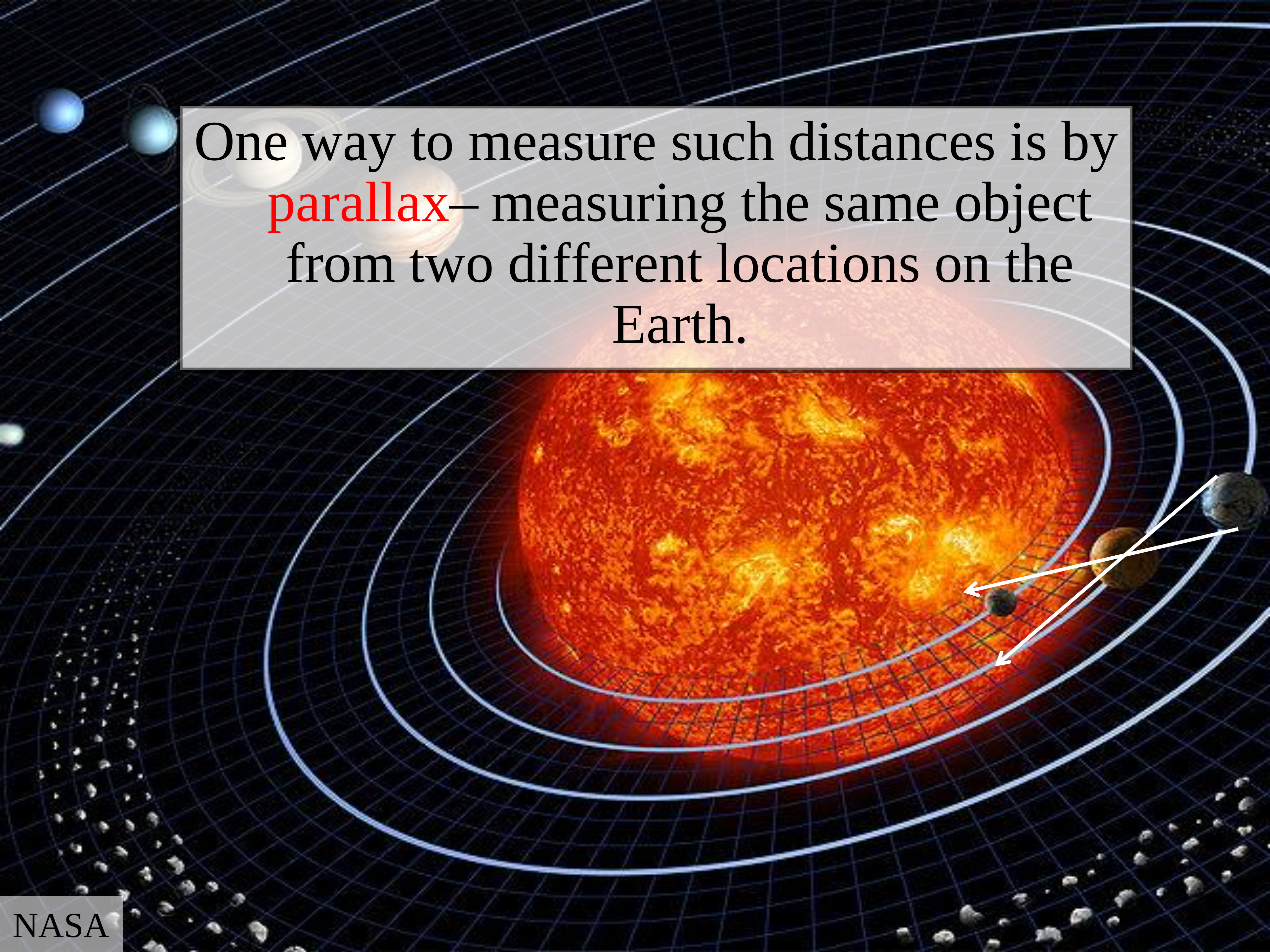
NASA

Conversely, if one had an alternate means to compute distances to planets, this would give a measurement of the AU.



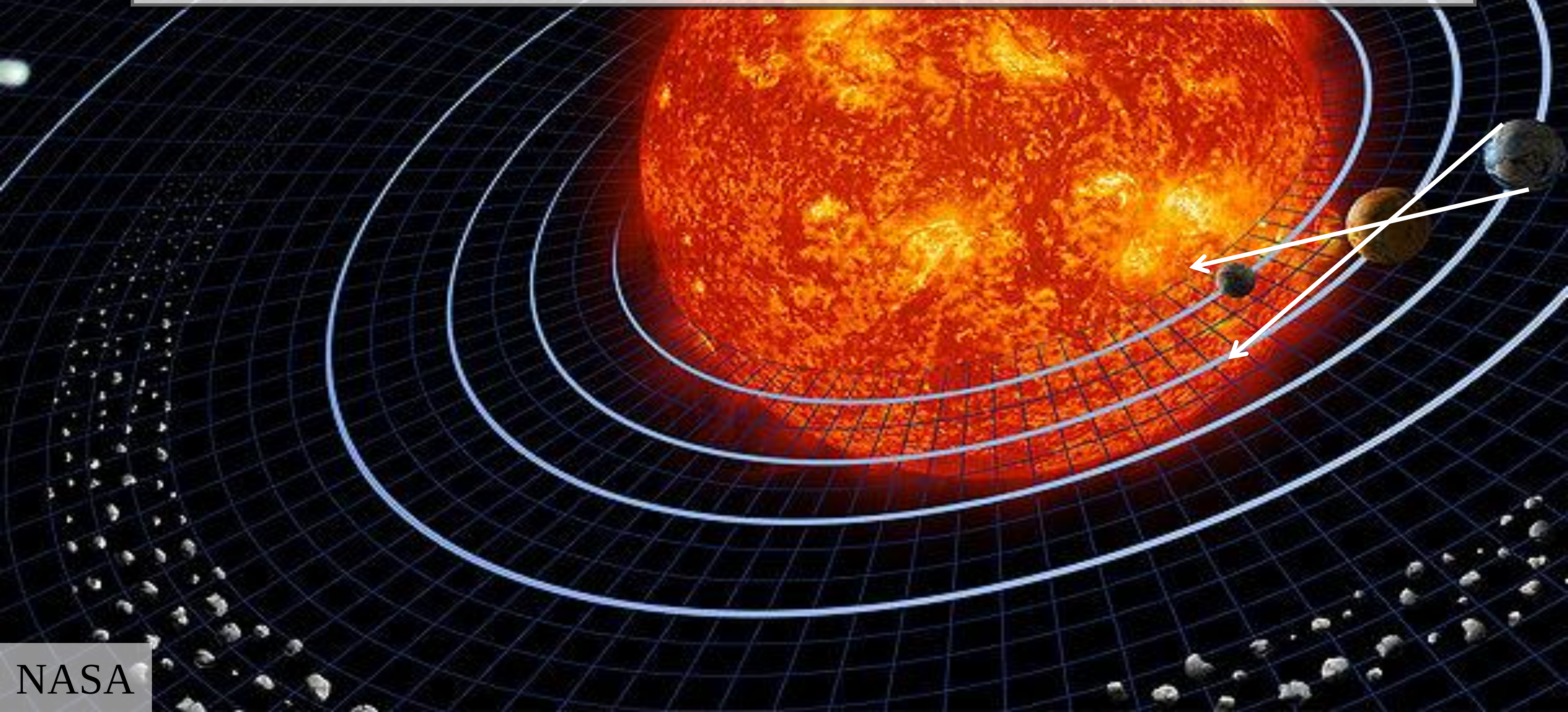
NASA

One way to measure such distances is by parallax– measuring the same object from two different locations on the Earth.



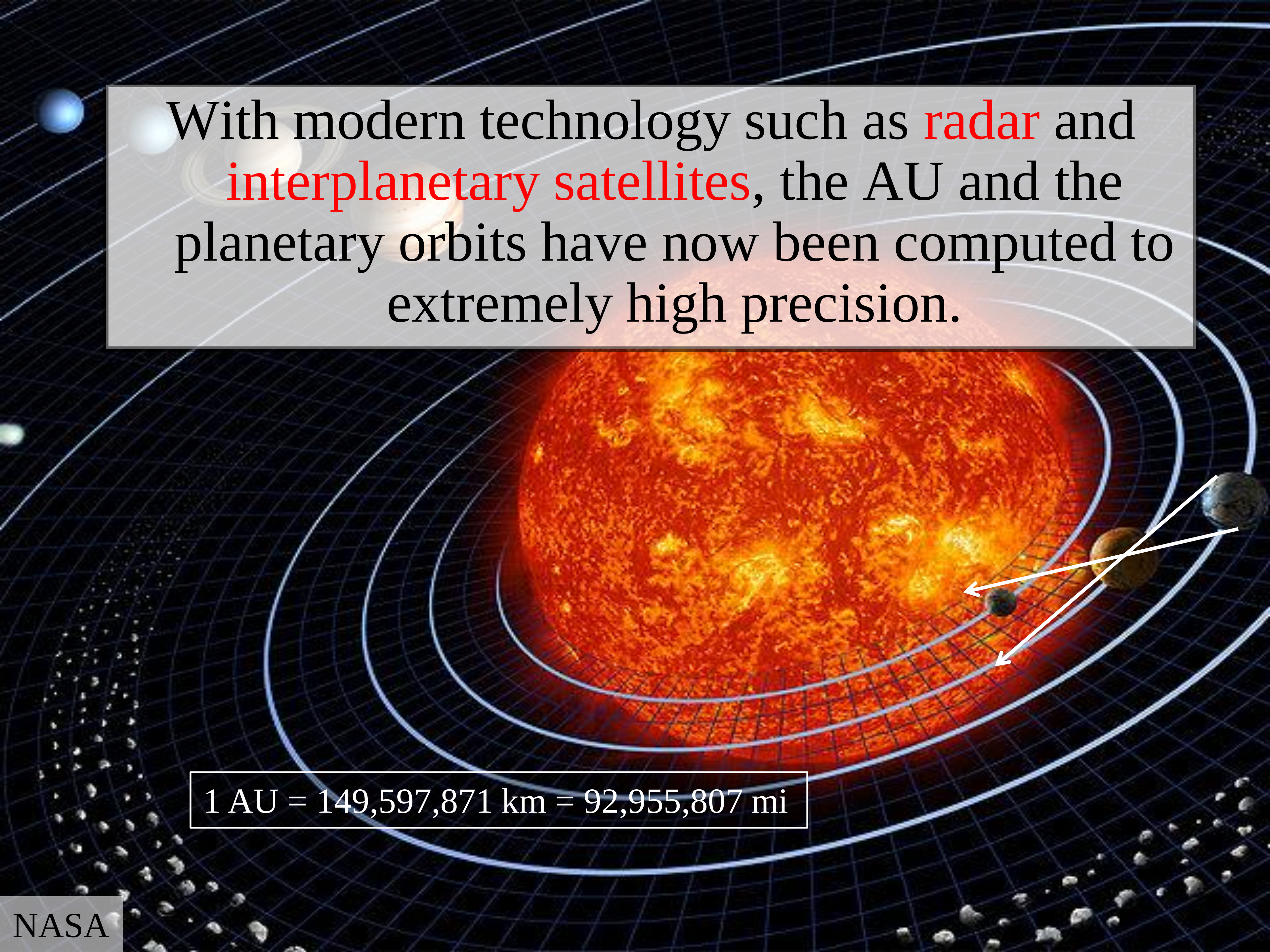
NASA

By measuring the parallax of the transit of Venus across the Sun simultaneously in several locations (including James Cook’s voyage!), the AU was computed reasonably accurately in the 18th century.



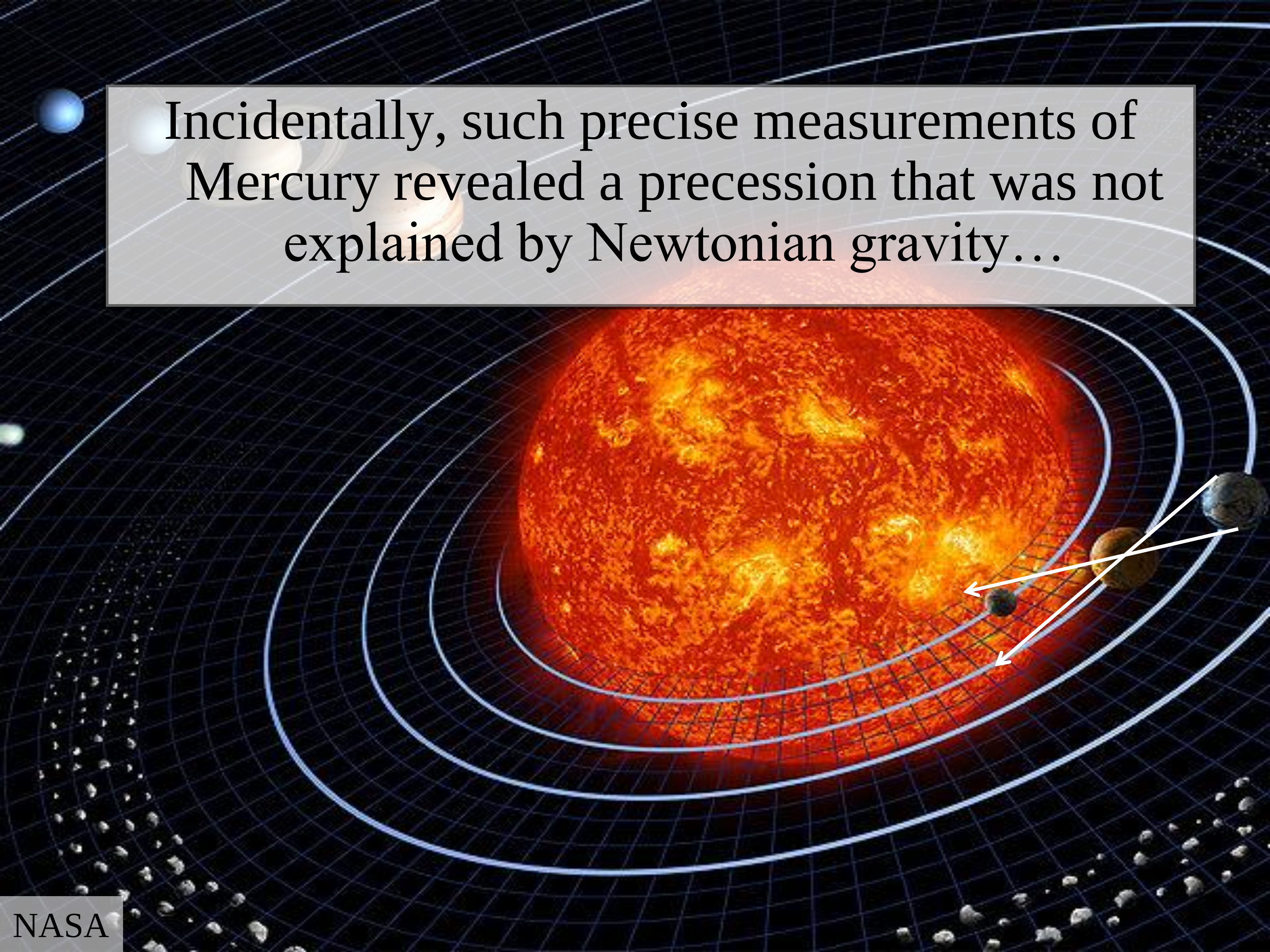
With modern technology such as radar and interplanetary satellites, the AU and the planetary orbits have now been computed to extremely high precision.

1 AU = 149,597,871 km = 92,955,807 mi



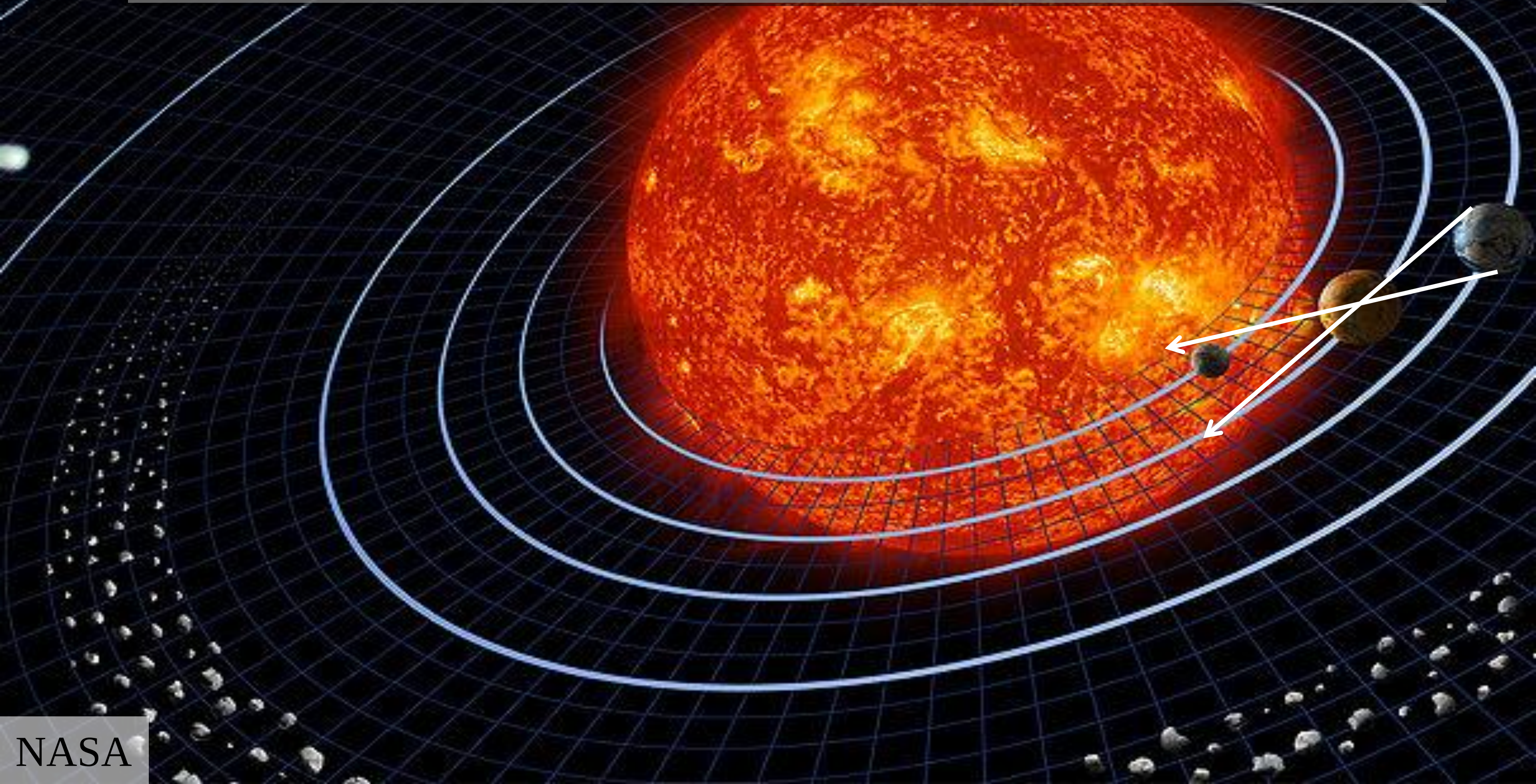
NASA

Incidentally, such precise measurements of Mercury revealed a precession that was not explained by Newtonian gravity…



NASA

… , and was one of the first experimental verifications of general relativity (which is needed in later rungs of the ladder).



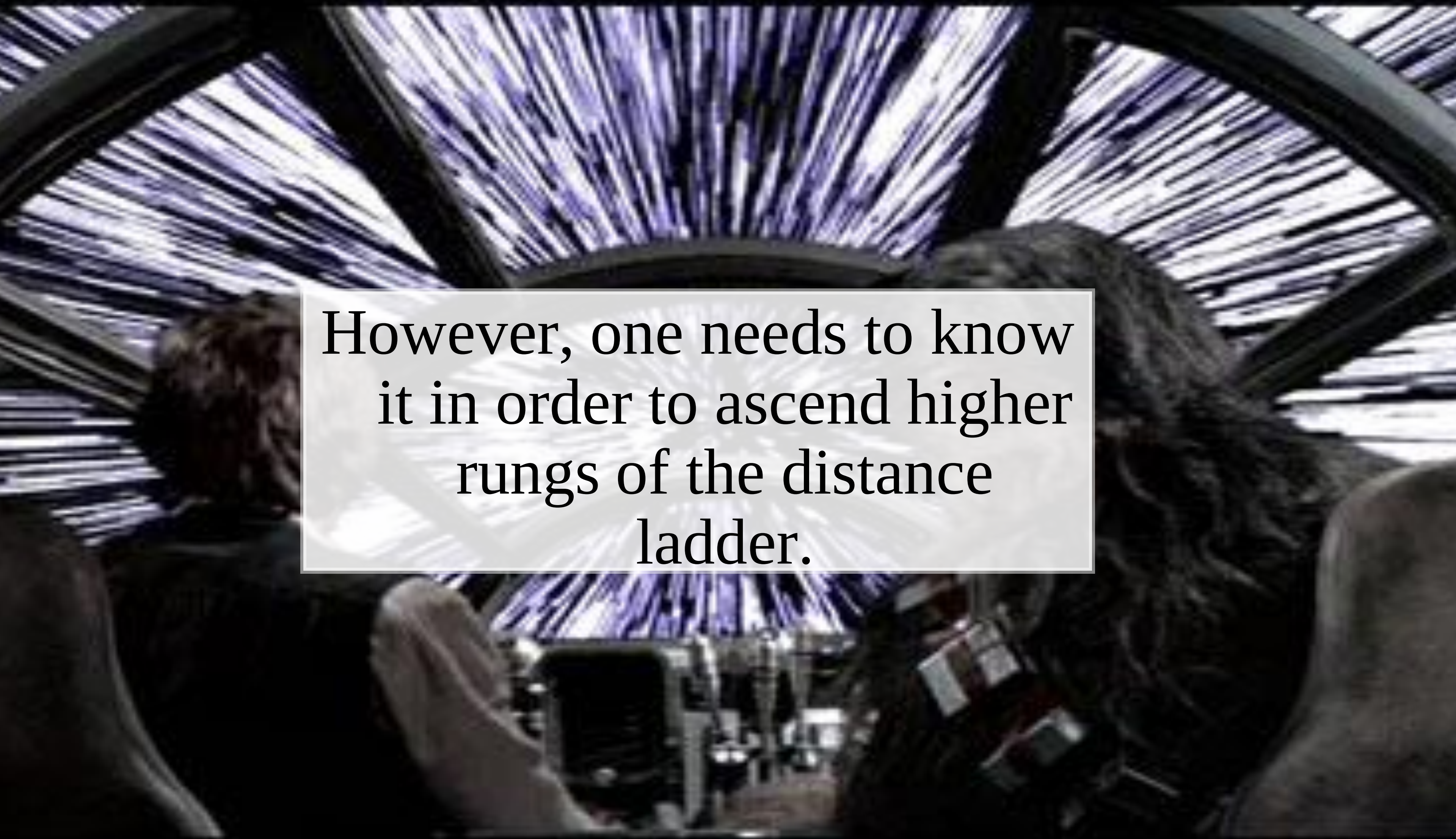
NASA

5th rung: the speed of light

Lucasfilm

Technically, the speed of light, *c*, is not a distance.

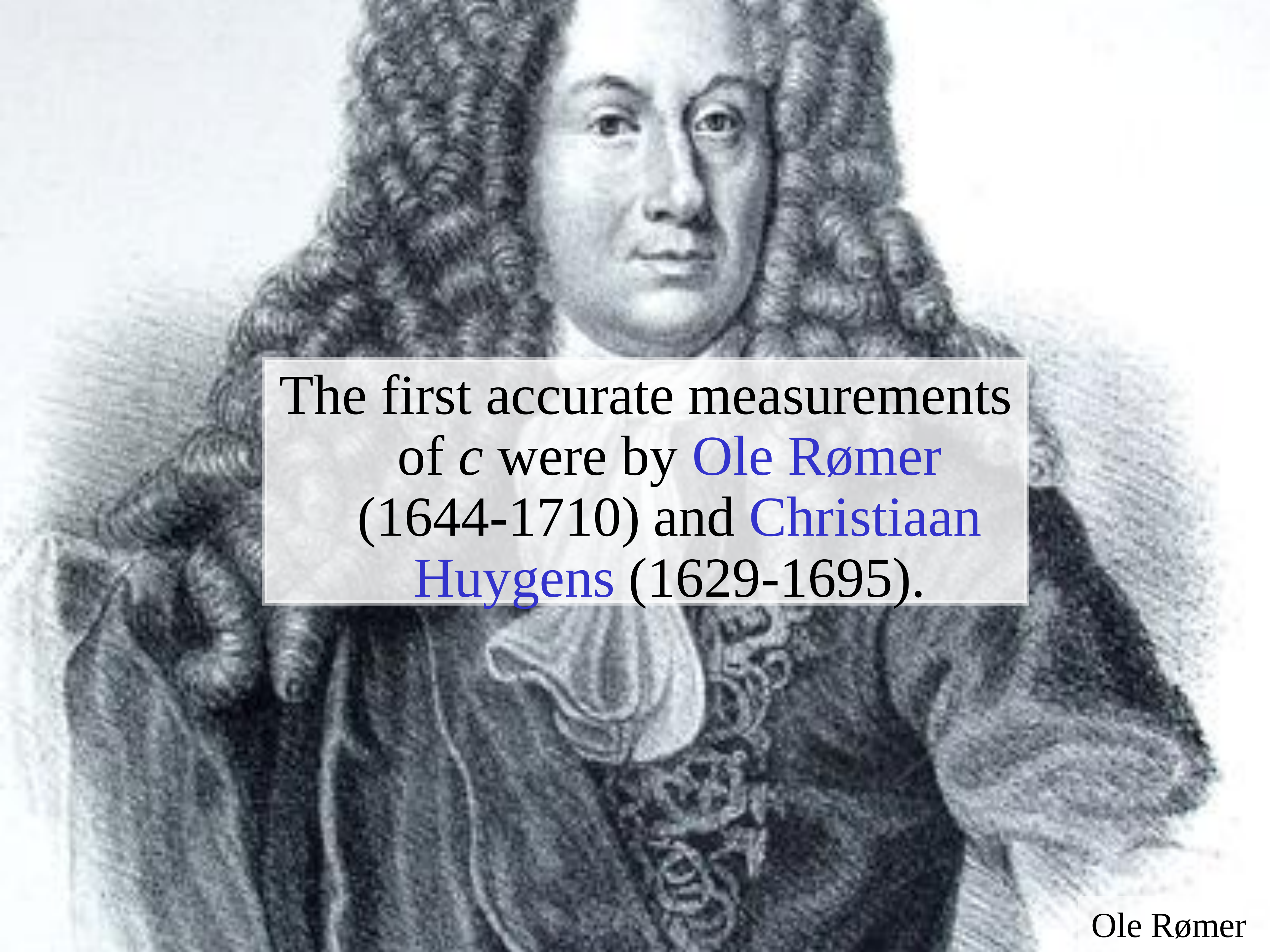
Lucasfilm



However, one needs to know it in order to ascend higher rungs of the distance ladder.

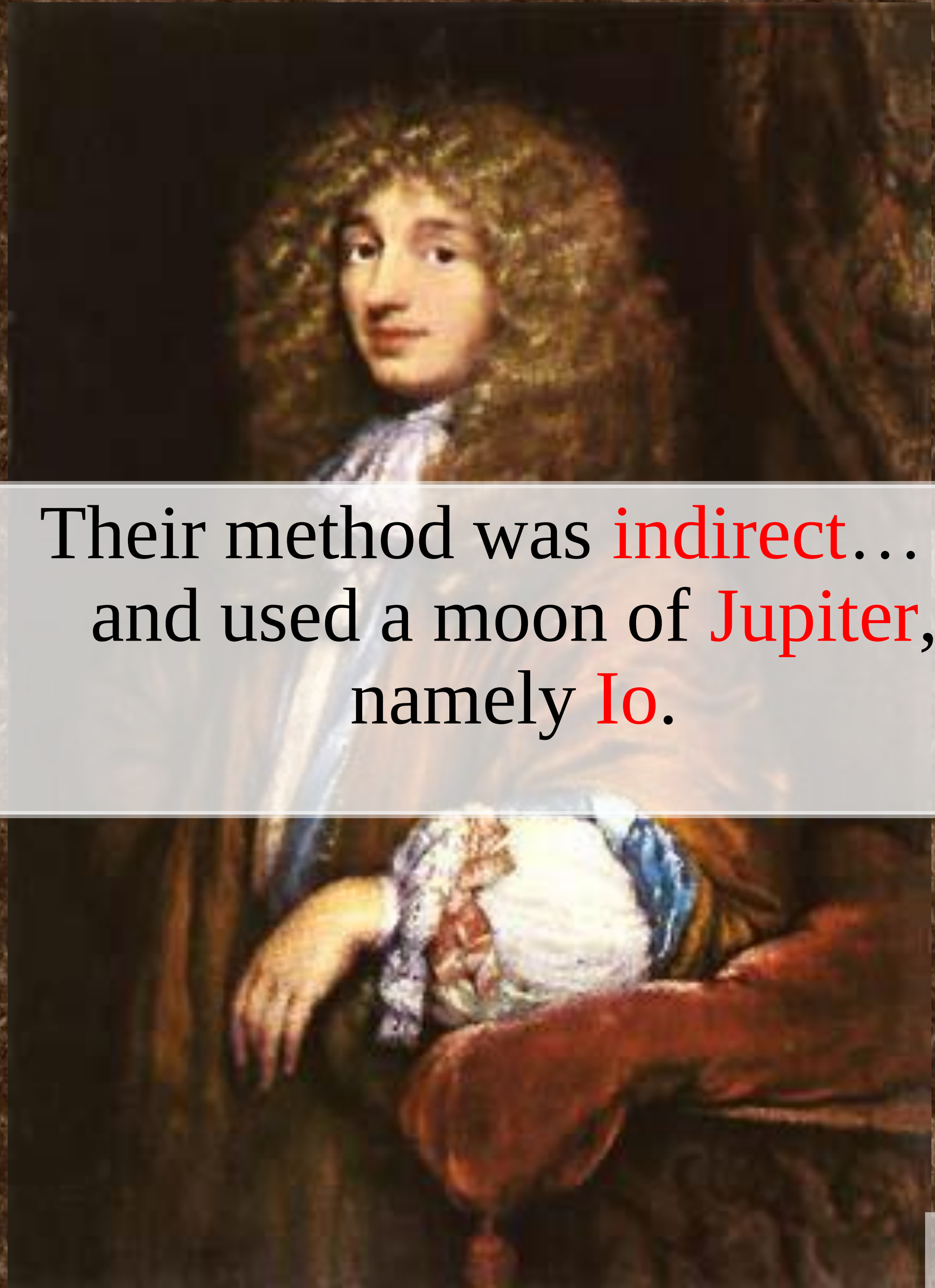
Lucasfilm

The first accurate measurements of *c* were by Ole Rømer (1644-1710) and Christiaan Huygens (1629-1695).

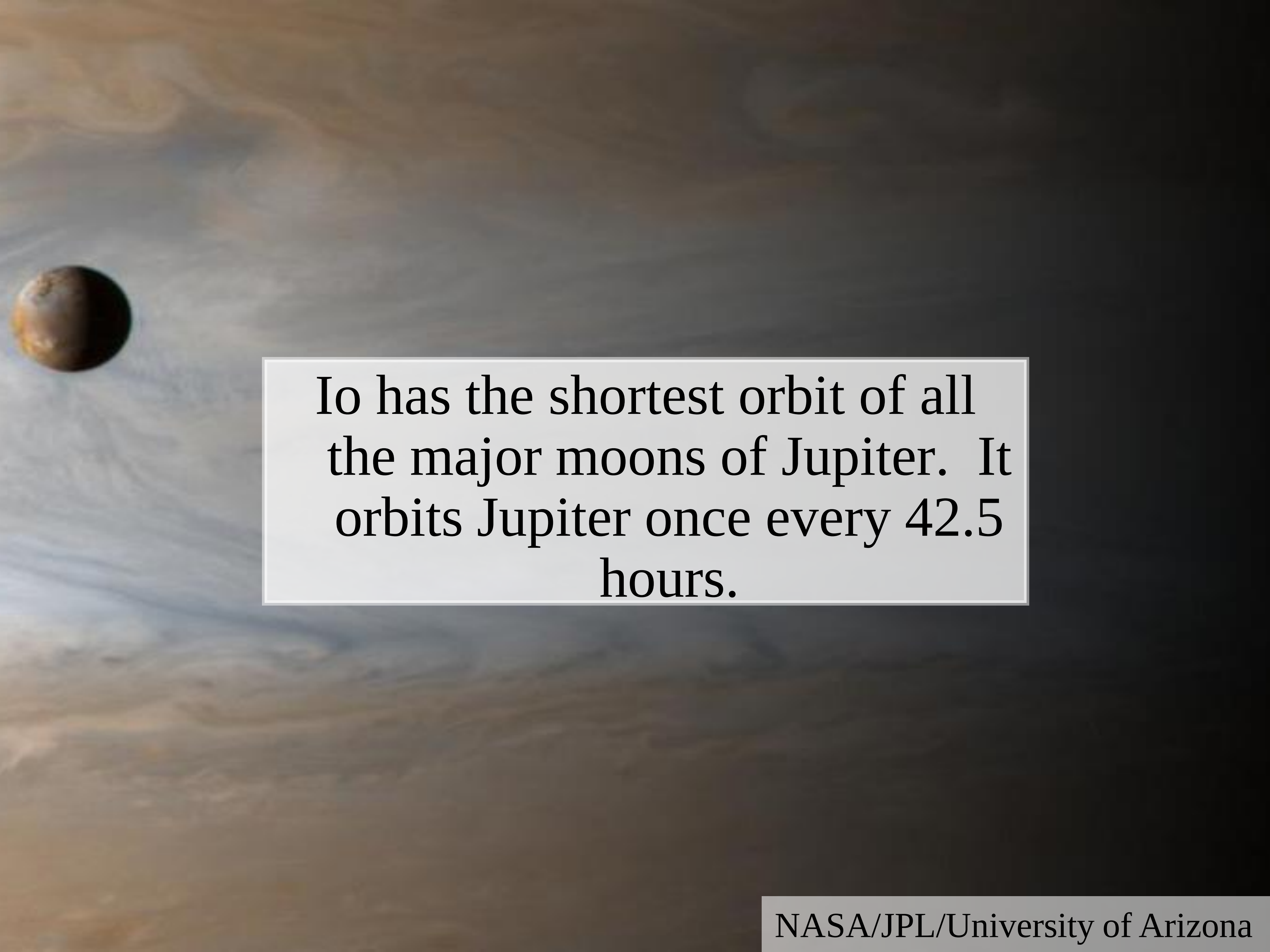


Ole Rømer

Their method was indirect…
and used a moon of Jupiter,
namely Io.

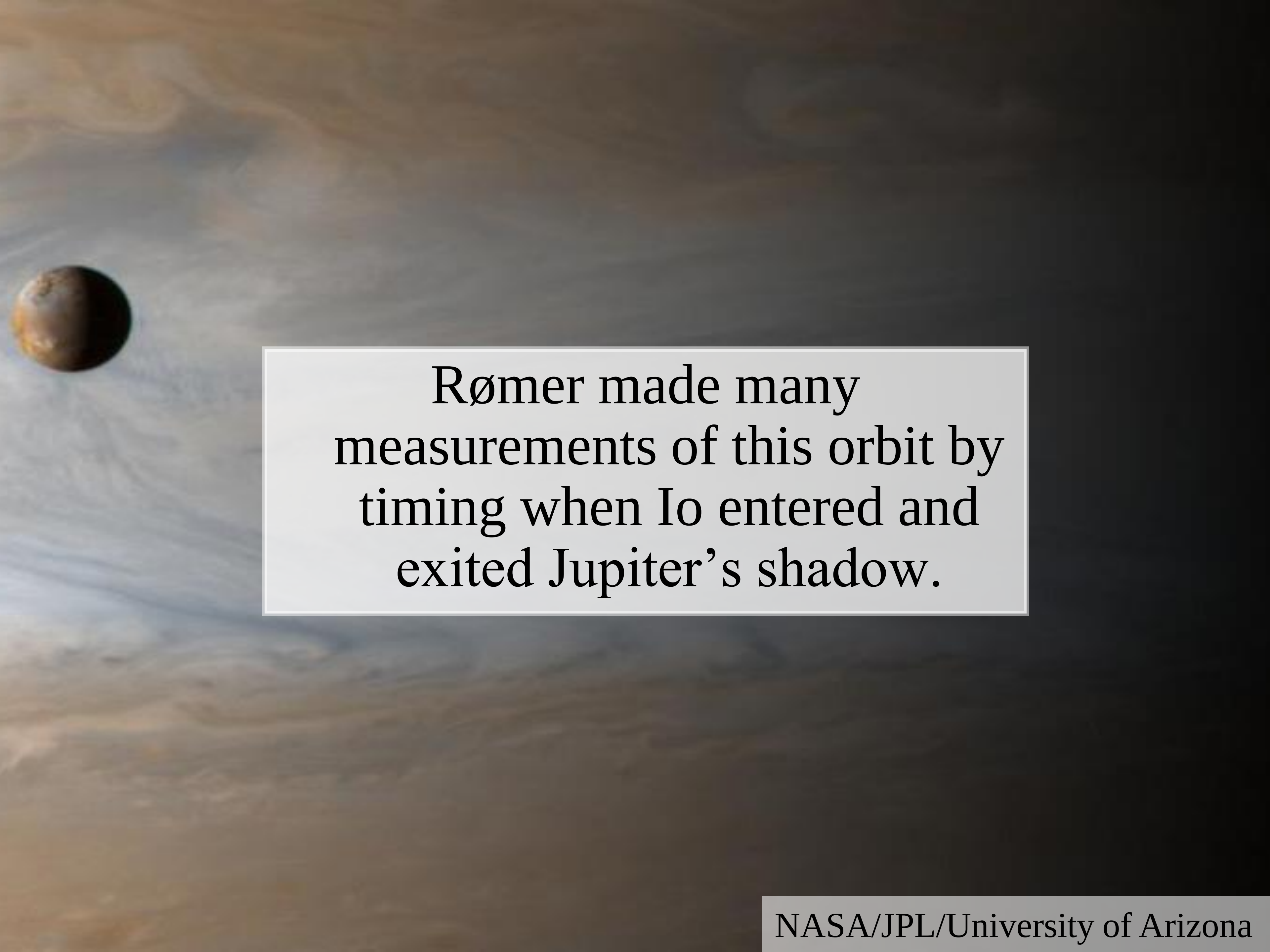


Christaan Huygens



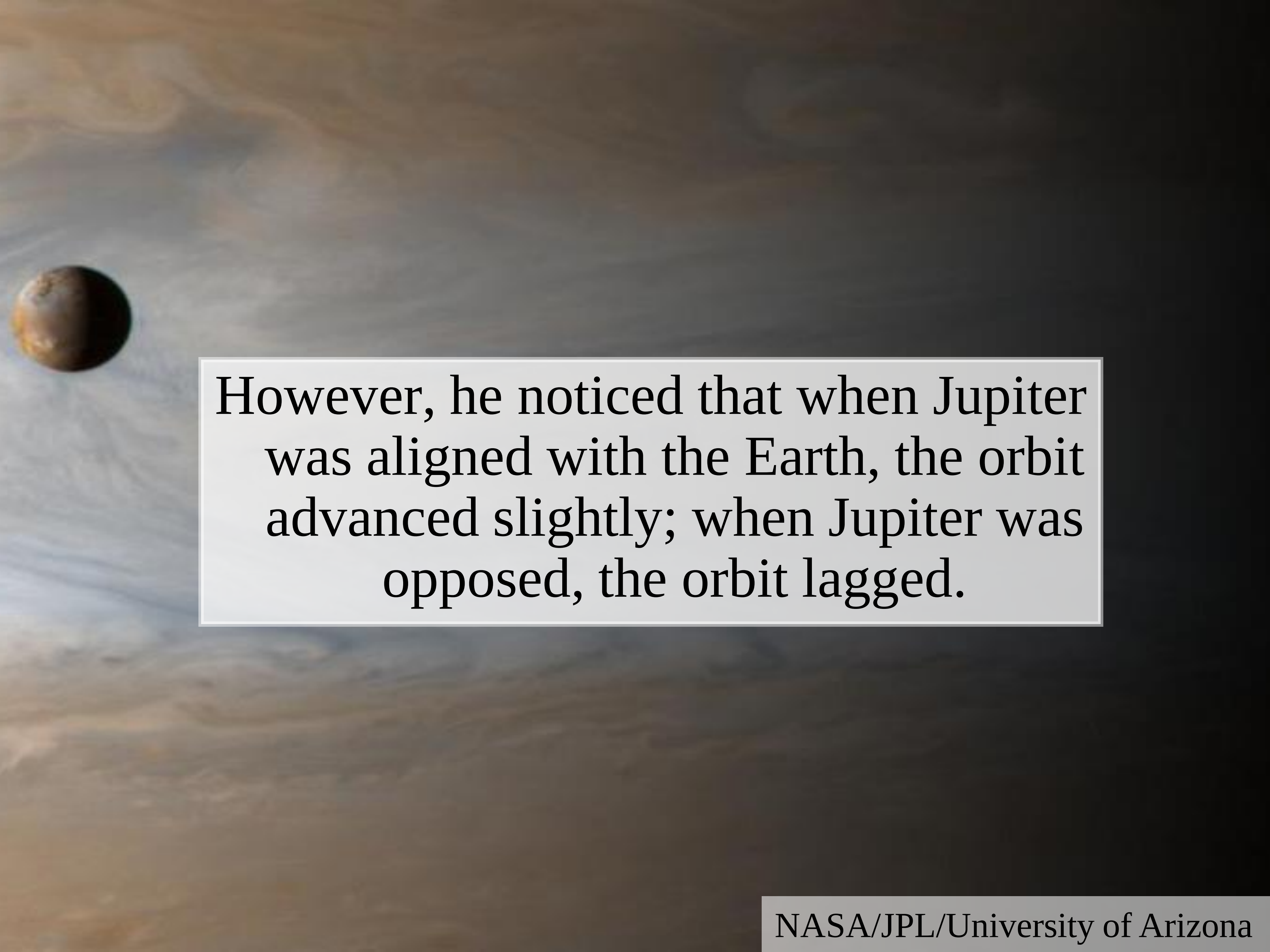
Io has the shortest orbit of all the major moons of Jupiter. It orbits Jupiter once every 42.5 hours.

NASA/JPL/University of Arizona



Rømer made many measurements of this orbit by timing when Io entered and exited Jupiter’s shadow.

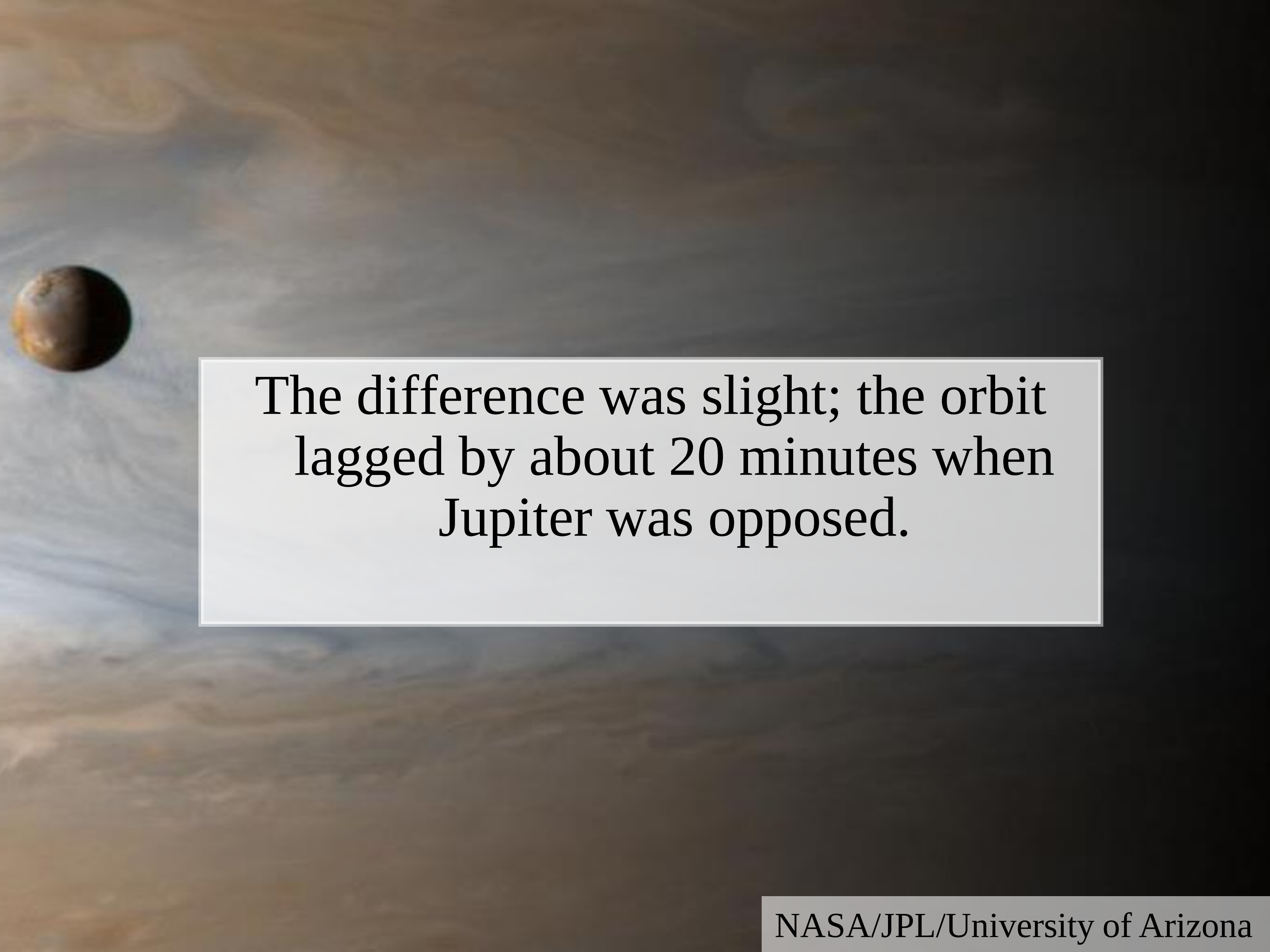
NASA/JPL/University of Arizona



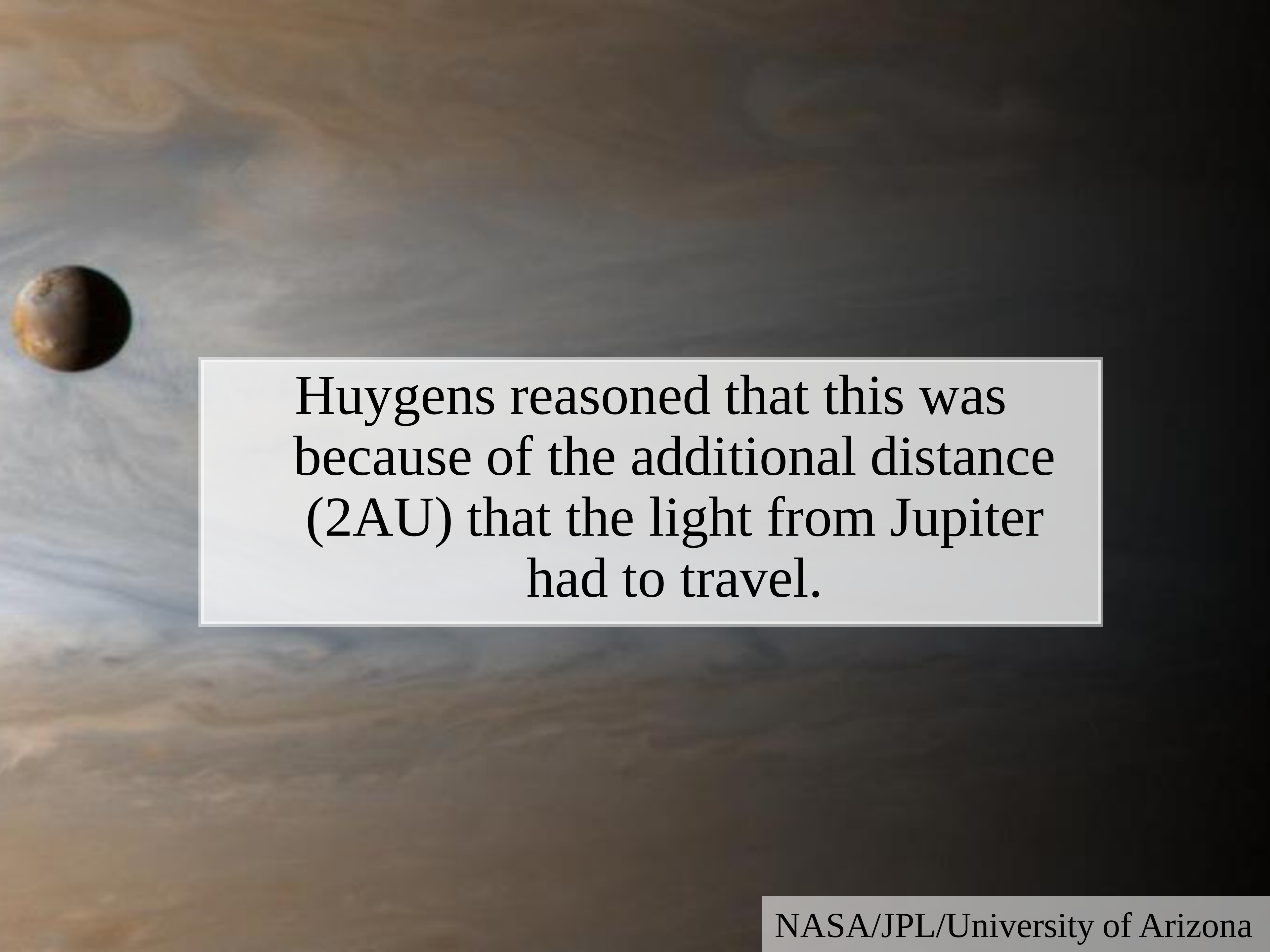
However, he noticed that when Jupiter was aligned with the Earth, the orbit advanced slightly; when Jupiter was opposed, the orbit lagged.

NASA/JPL/University of Arizona

The difference was slight; the orbit lagged by about 20 minutes when Jupiter was opposed.

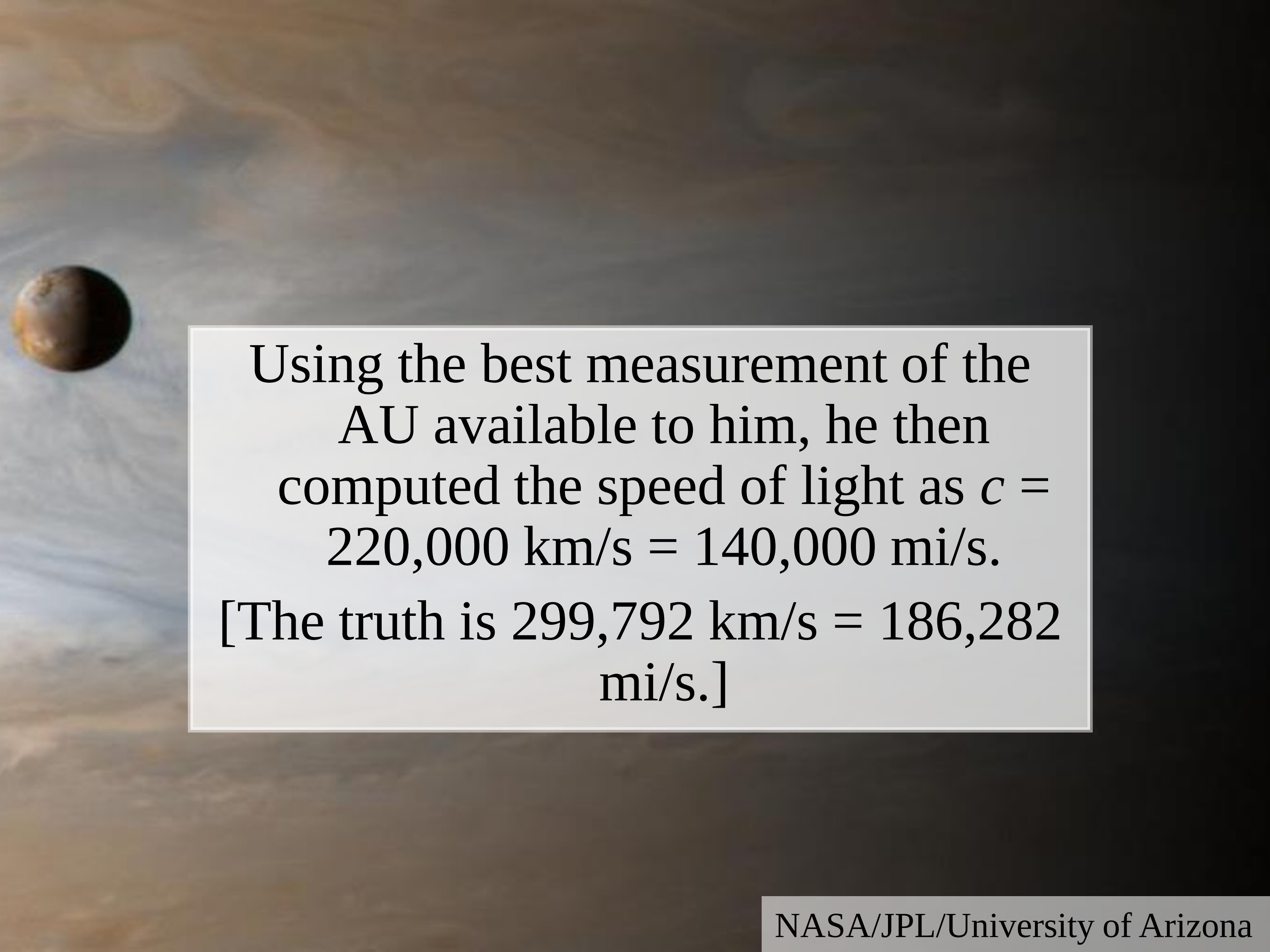


NASA/JPL/University of Arizona



Huygens reasoned that this was because of the additional distance (2AU) that the light from Jupiter had to travel.

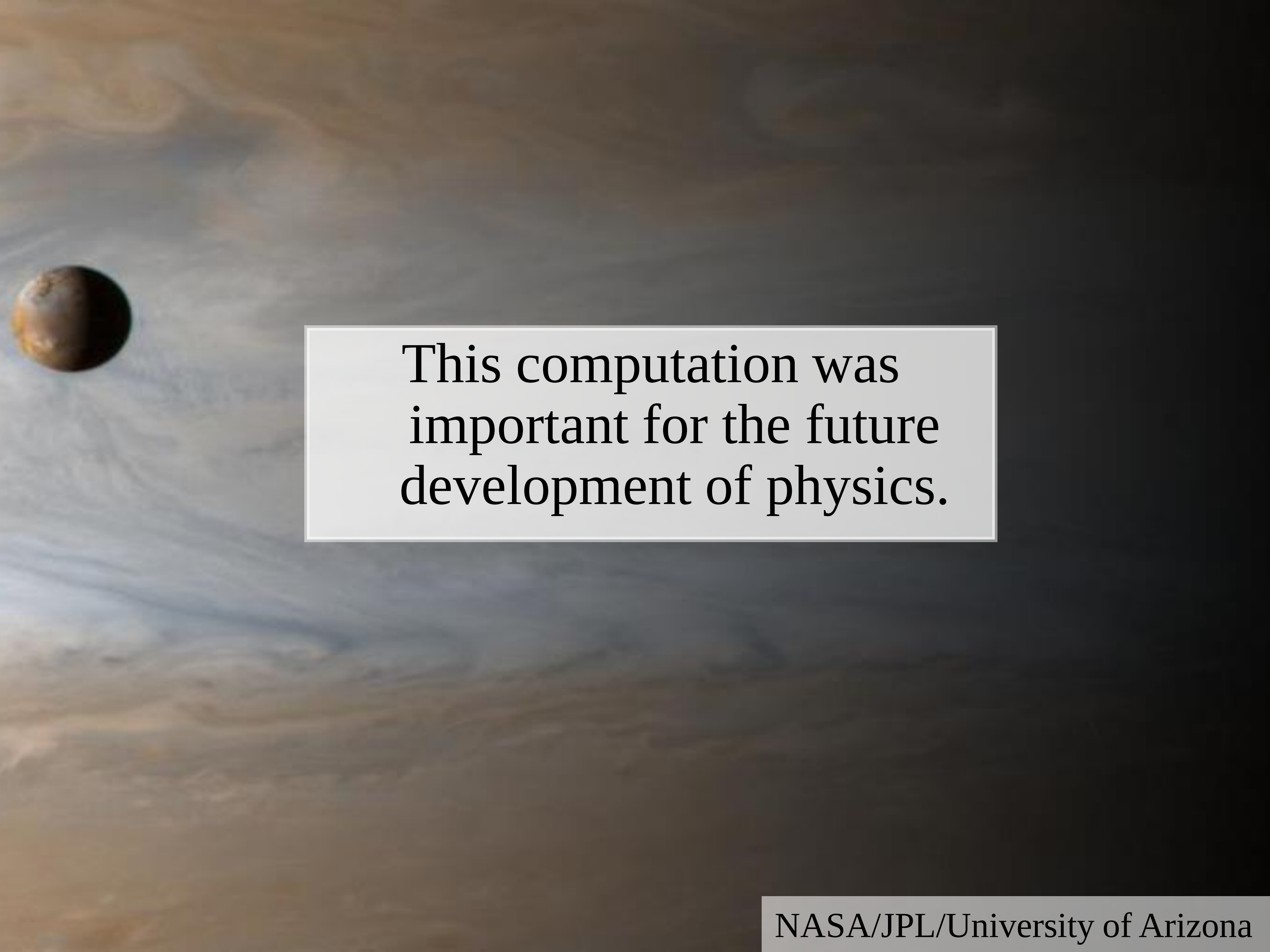
NASA/JPL/University of Arizona

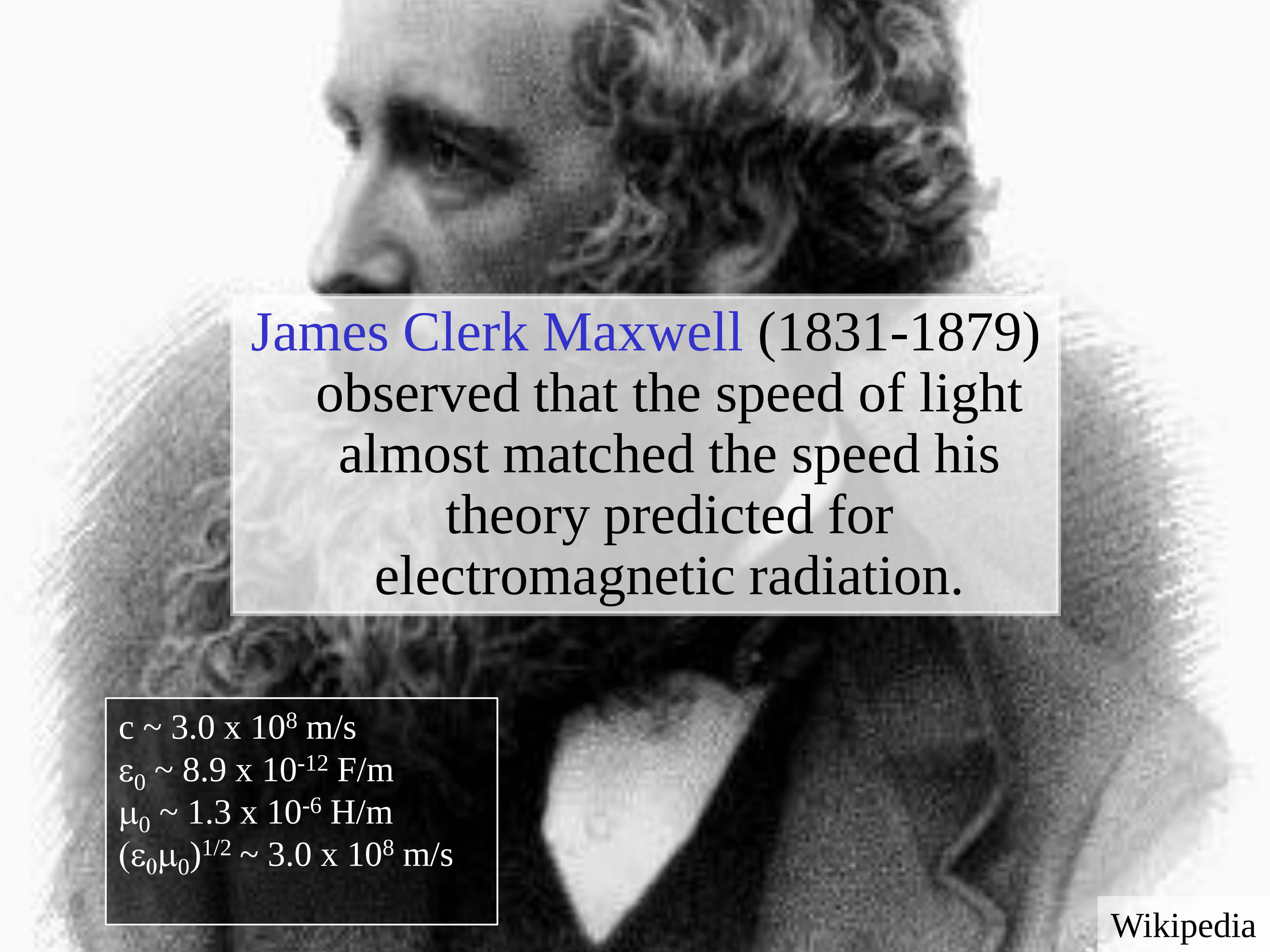


Using the best measurement of the AU available to him, he then computed the speed of light as *c* = 220,000 km/s = 140,000 mi/s.

[The truth is 299,792 km/s = 186,282 mi/s.]

NASA/JPL/University of Arizona


This computation was important for the future development of physics.
NASA/JPL/University of Arizona



James Clerk Maxwell (1831-1879) observed that the speed of light almost matched the speed his theory predicted for electromagnetic radiation.

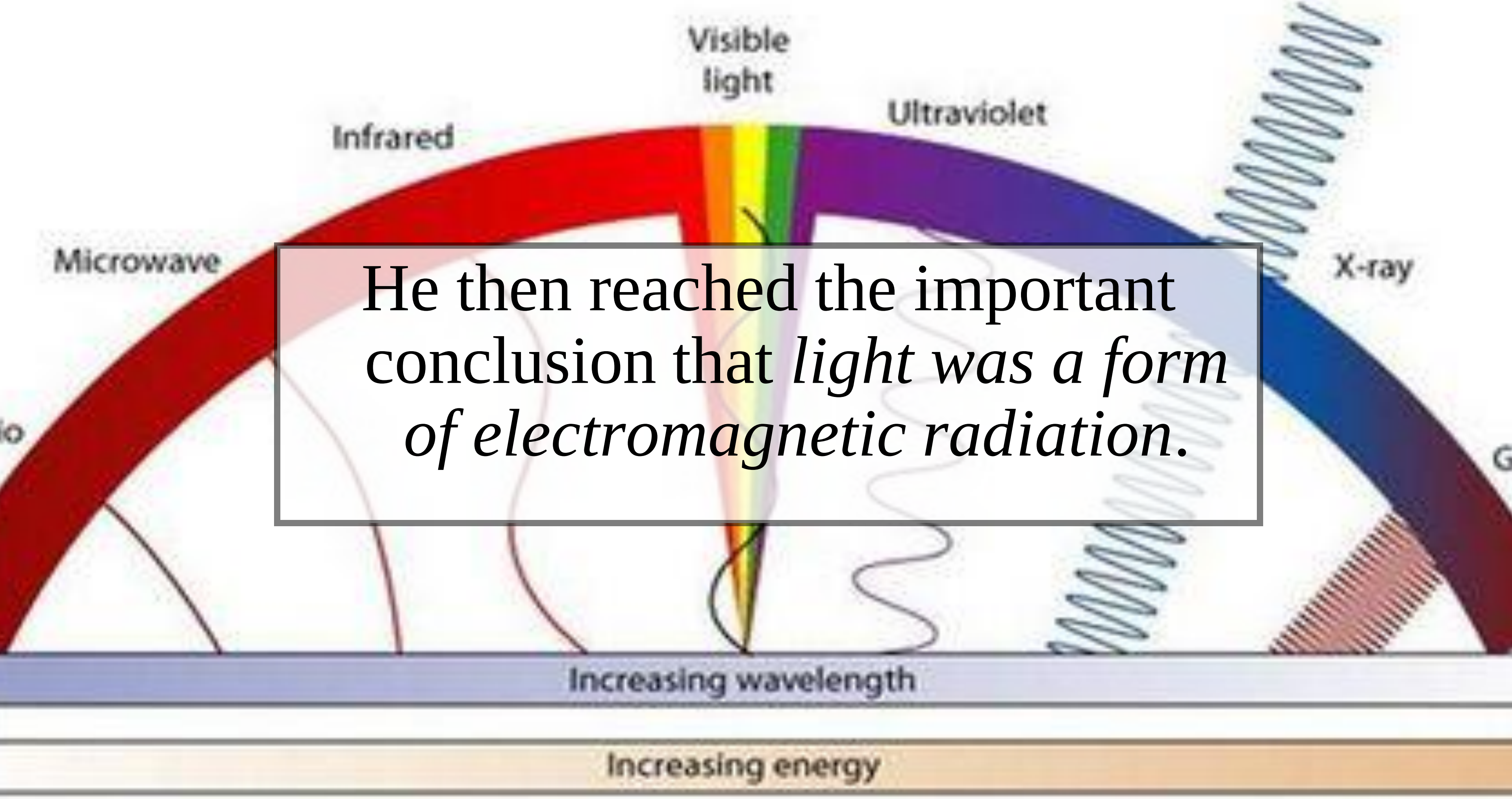
$c \sim 3.0 \times 10^{8}$ m/s

$\varepsilon_0 \sim 8.9 \times 10^{-12}$ F/m

$\mu_0 \sim 1.3 \times 10^{-6}$ H/m

$(\varepsilon_0\mu_0)^{1/2} \sim 3.0 \times 10^{8}$ m/s

Wikipedia



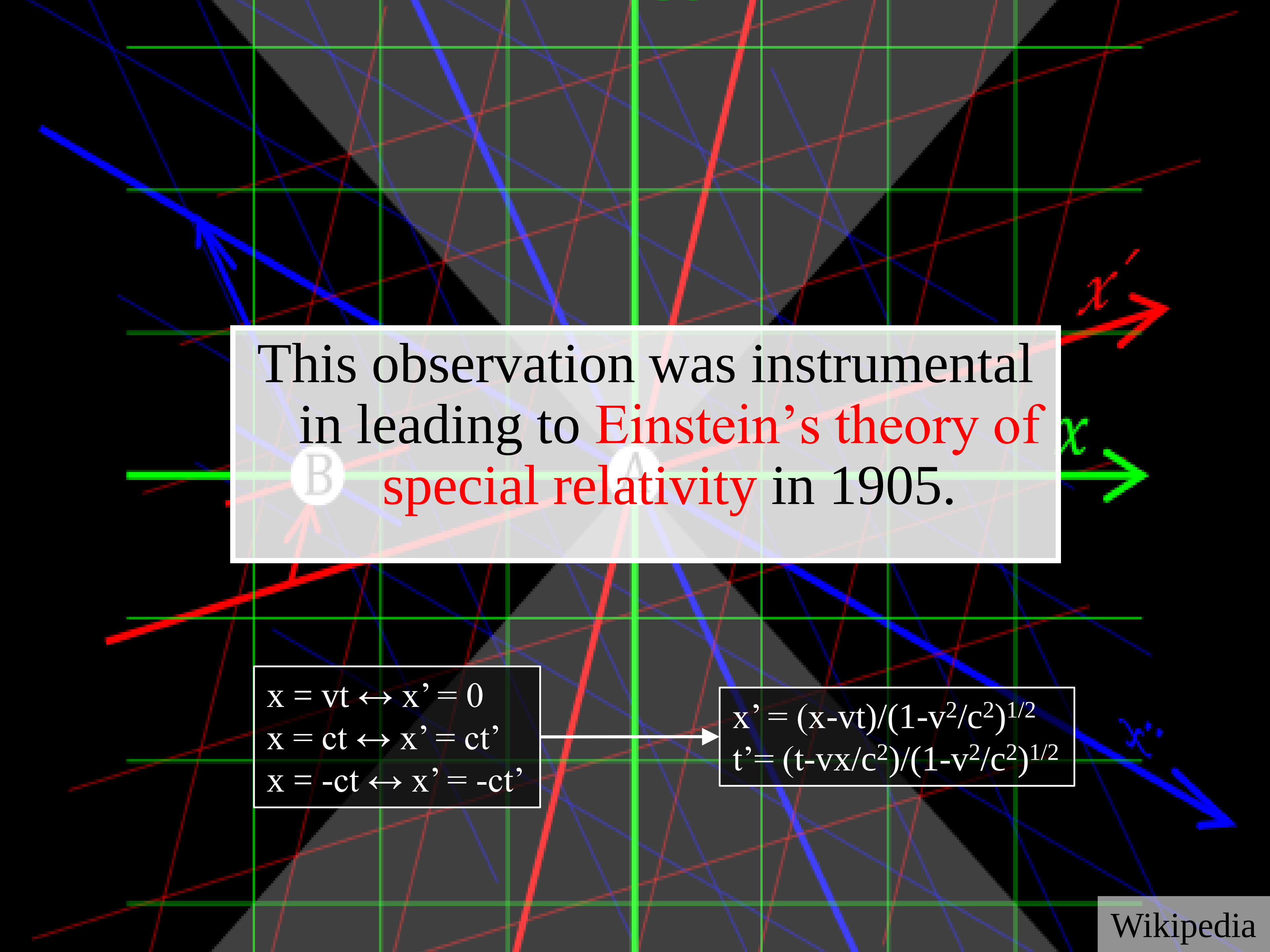
He then reached the important conclusion that *light was a form of electromagnetic radiation*.

Science Learning Hub, University of Waikato, NZ

This observation was instrumental in leading to Einstein's theory of special relativity in 1905.

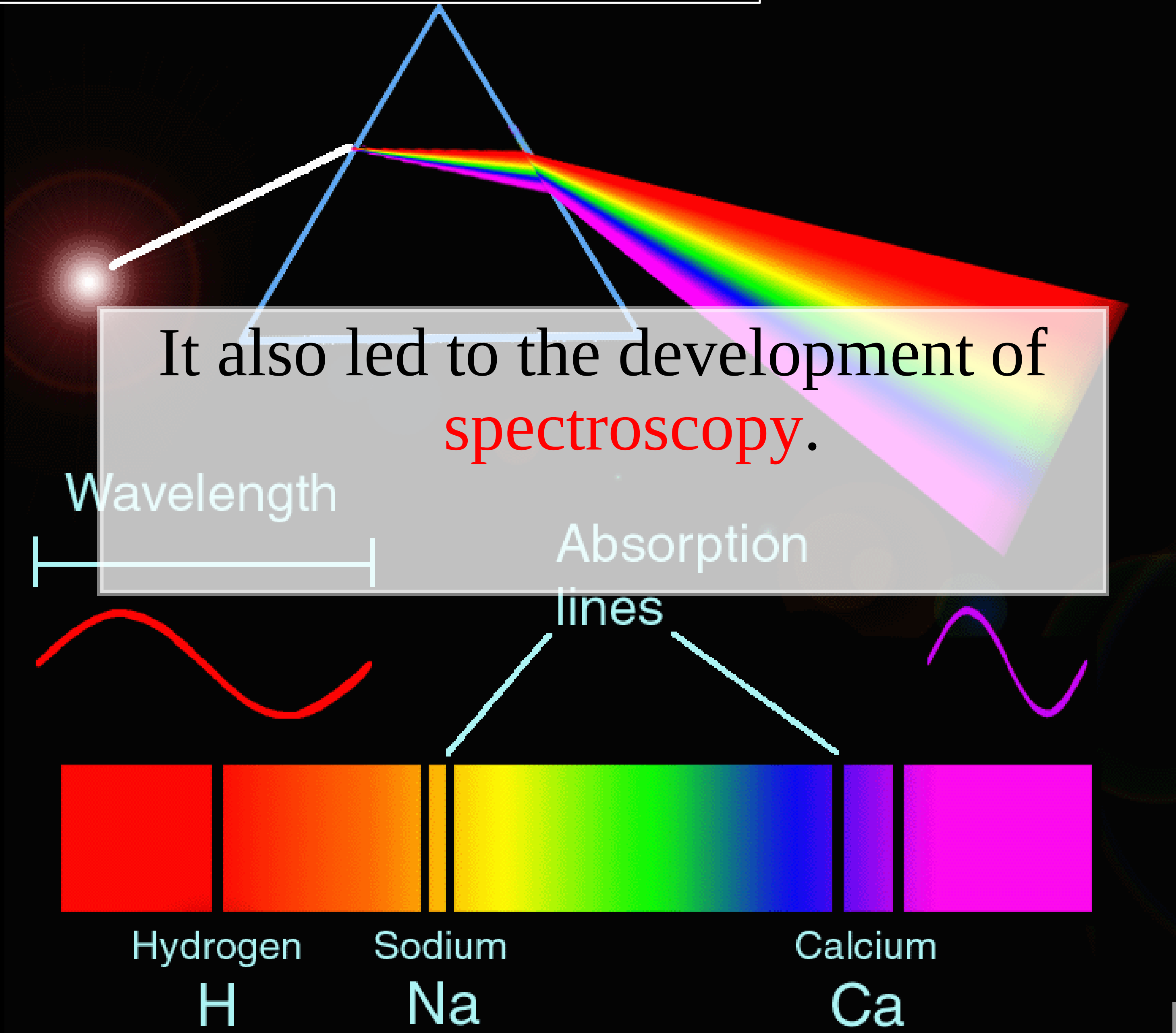
$x = vt \leftrightarrow x' = 0$
$x = ct \leftrightarrow x' = ct'$
$x = -ct \leftrightarrow x' = -ct'$

$x' = (x-vt)/(1-v^2/c^2)^{1/2}$
$t' = (t-vx/c^2)/(1-v^2/c^2)^{1/2}$



Wikipedia

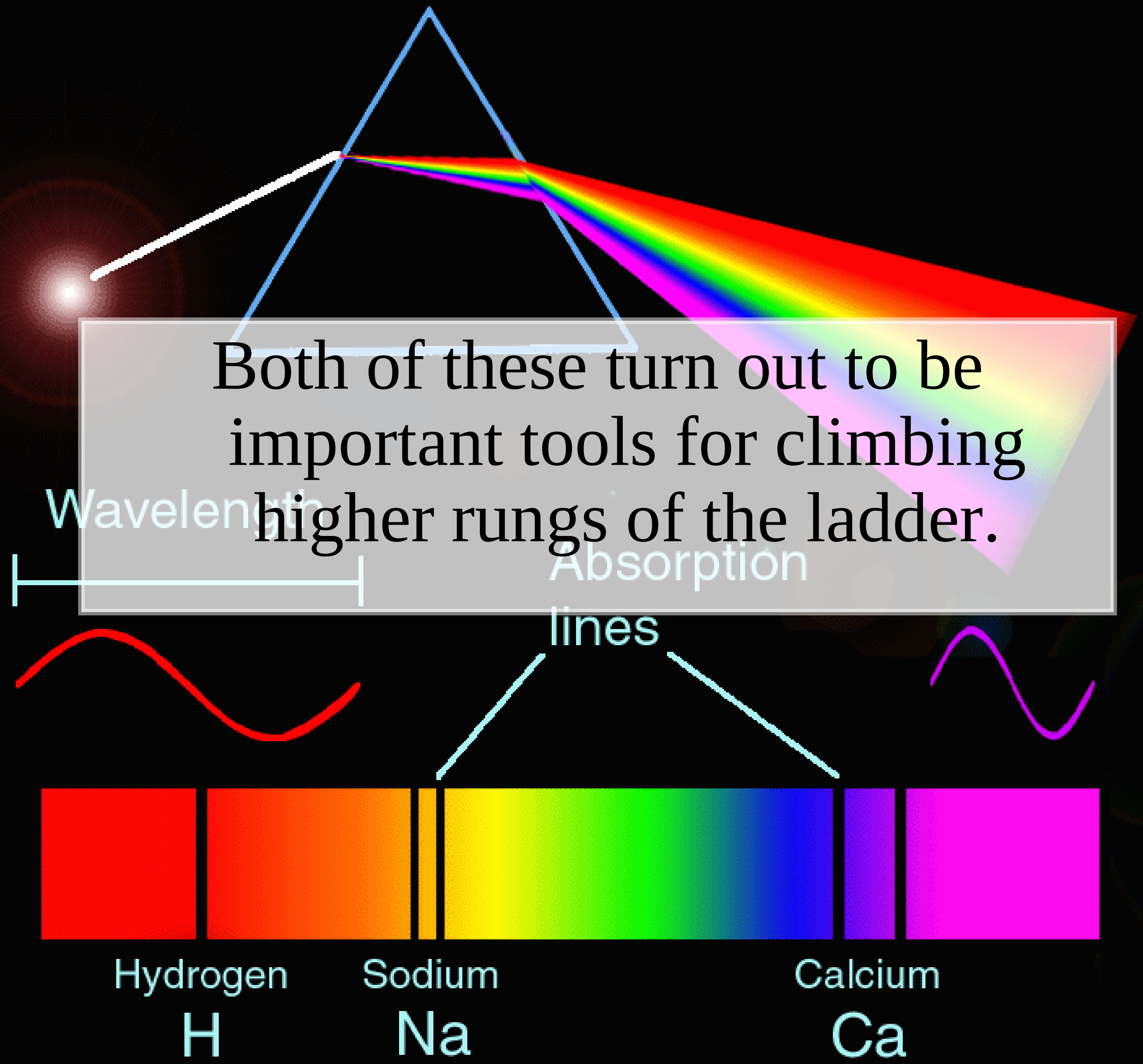
First spectroscope: 1814 (Joseph von Fraunhofer)



It also led to the development of spectroscopy.

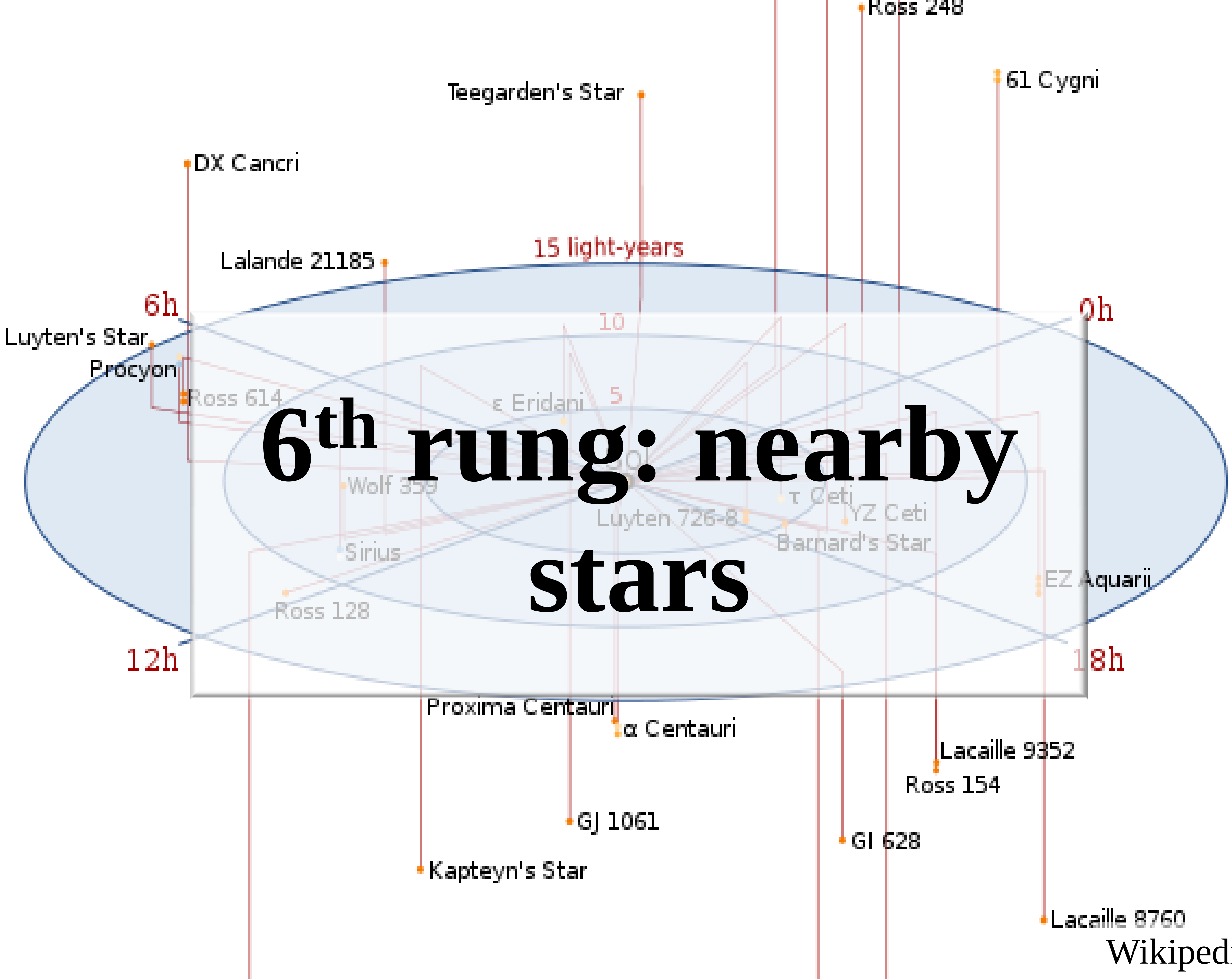
Ian Short

Both of these turn out to be important tools for climbing higher rungs of the ladder.



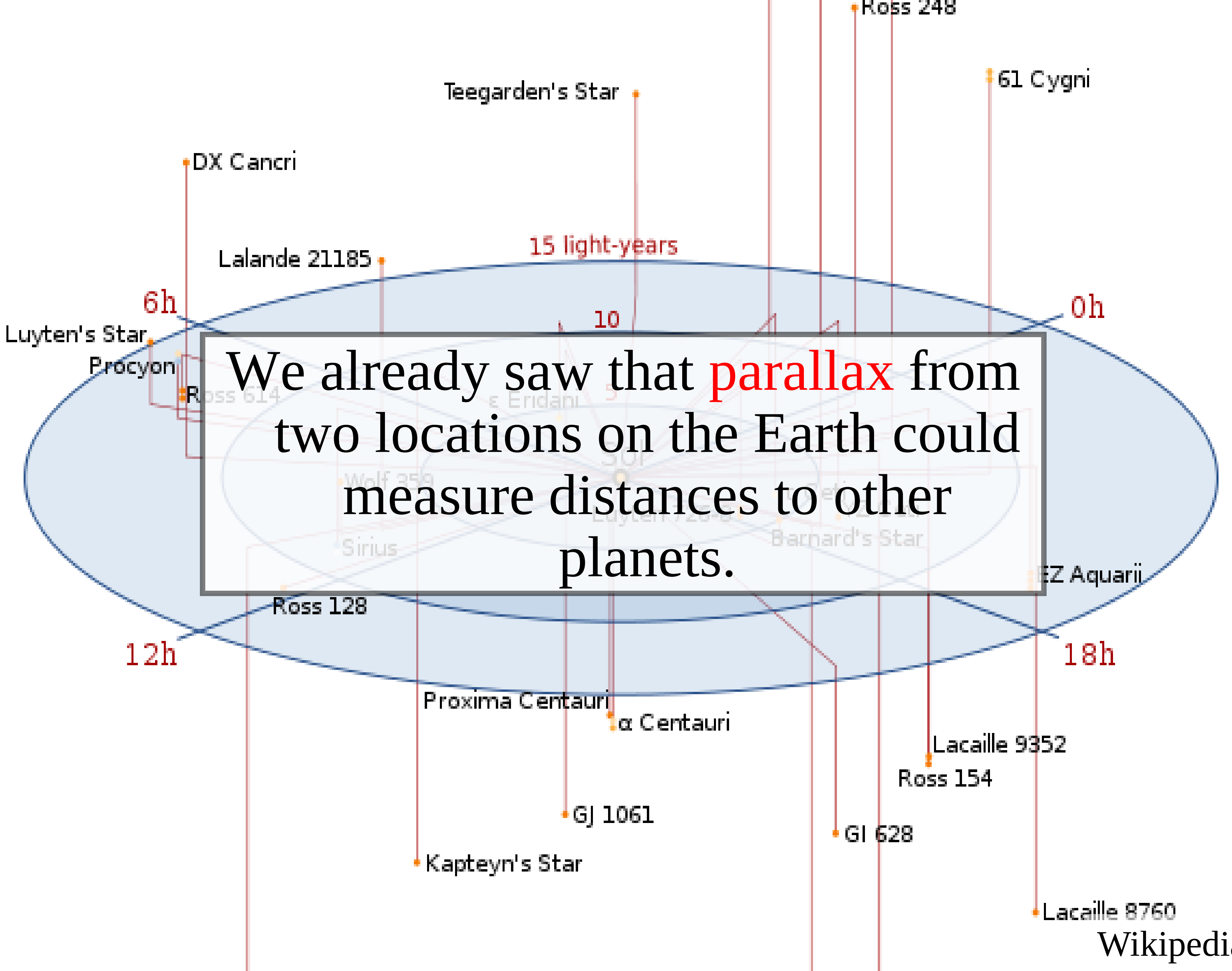
Ian Short

6th rung: nearby stars

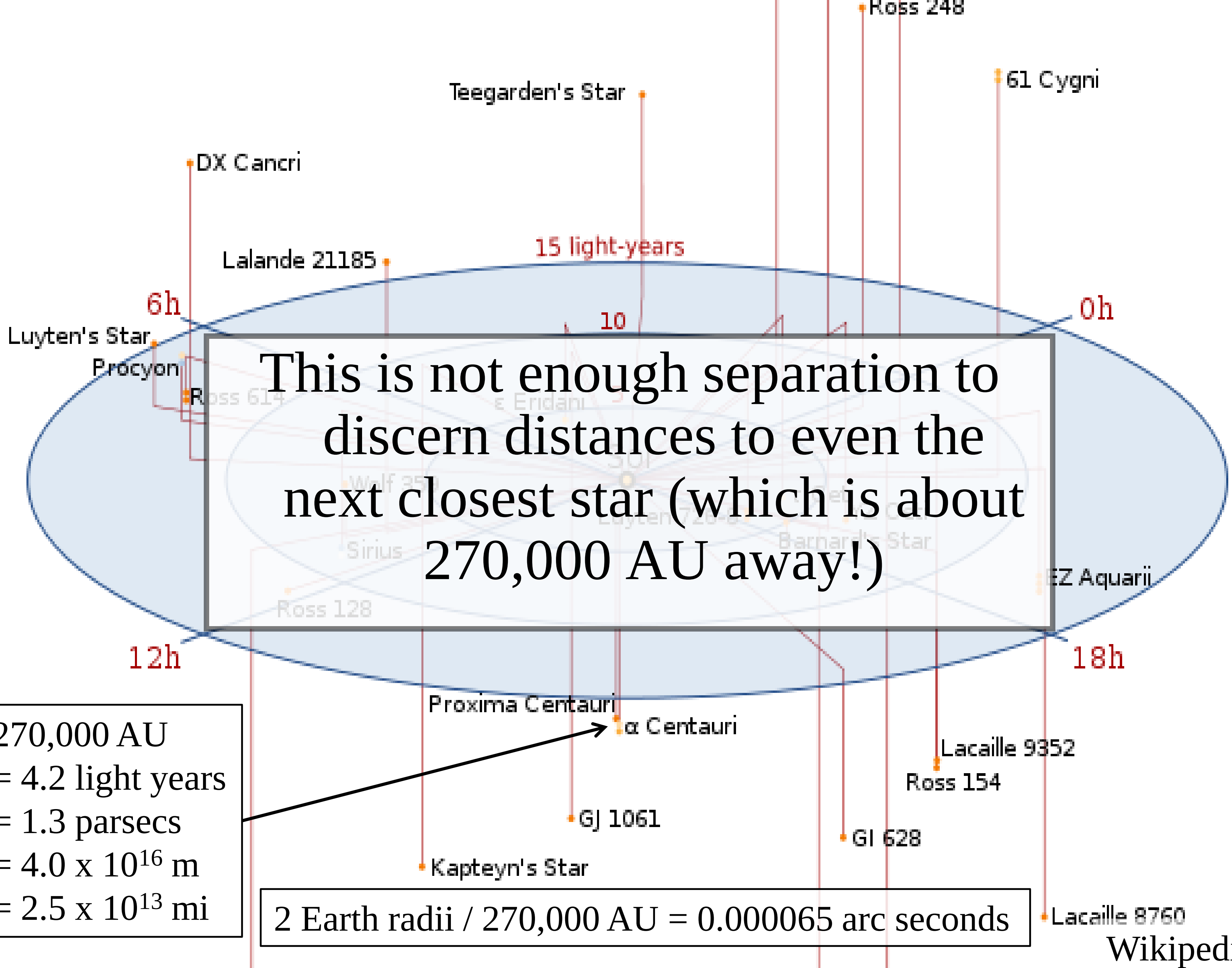


Wikipedia

We already saw that parallax from two locations on the Earth could measure distances to other planets.



Wikipedia



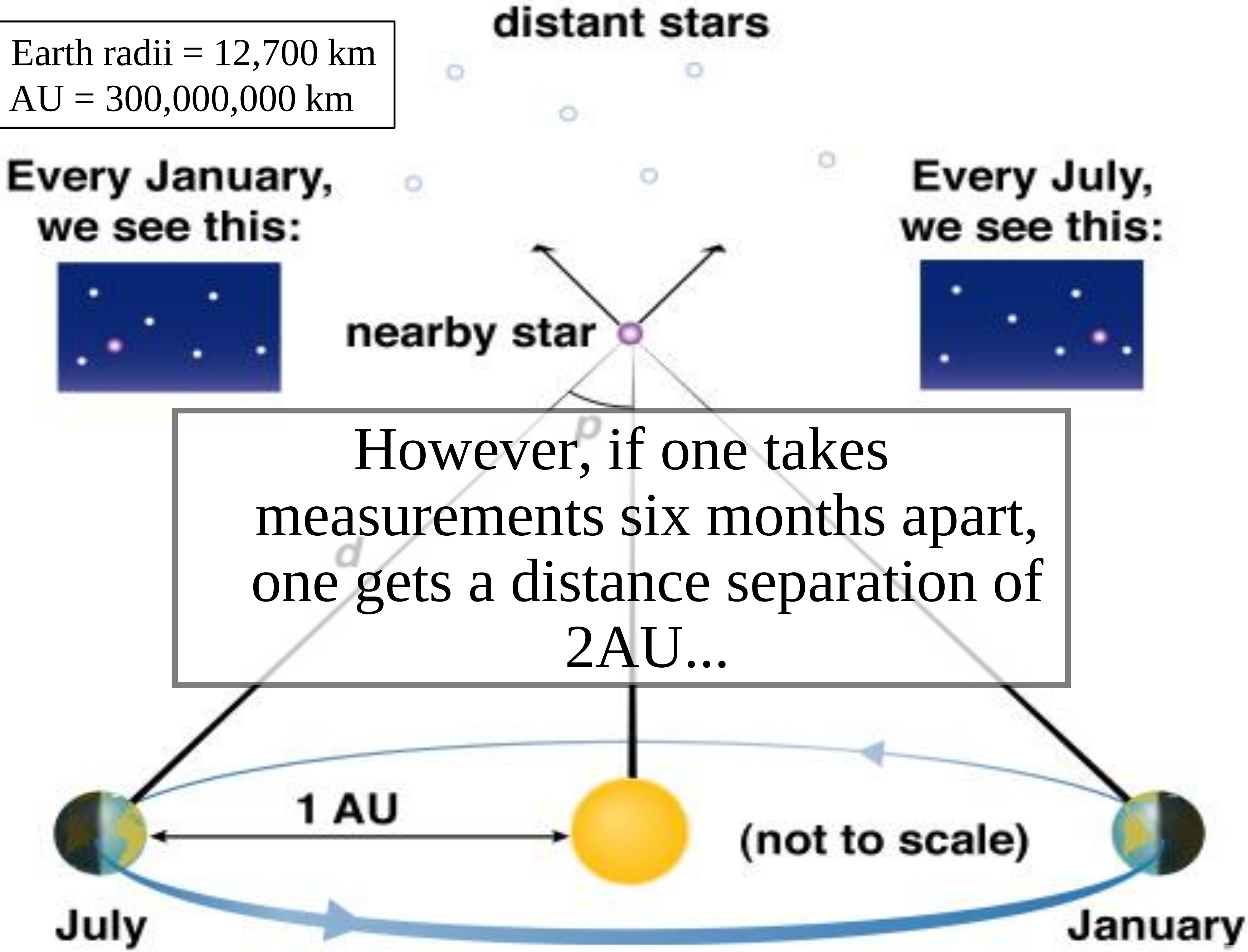
This is not enough separation to discern distances to even the next closest star (which is about 270,000 AU away!)

270,000 AU
= 4.2 light years
= 1.3 parsecs
= 4.0×10^{16} m
= 2.5×10^{13} mi

2 Earth radii / 270,000 AU = 0.000065 arc seconds

Wikipedia

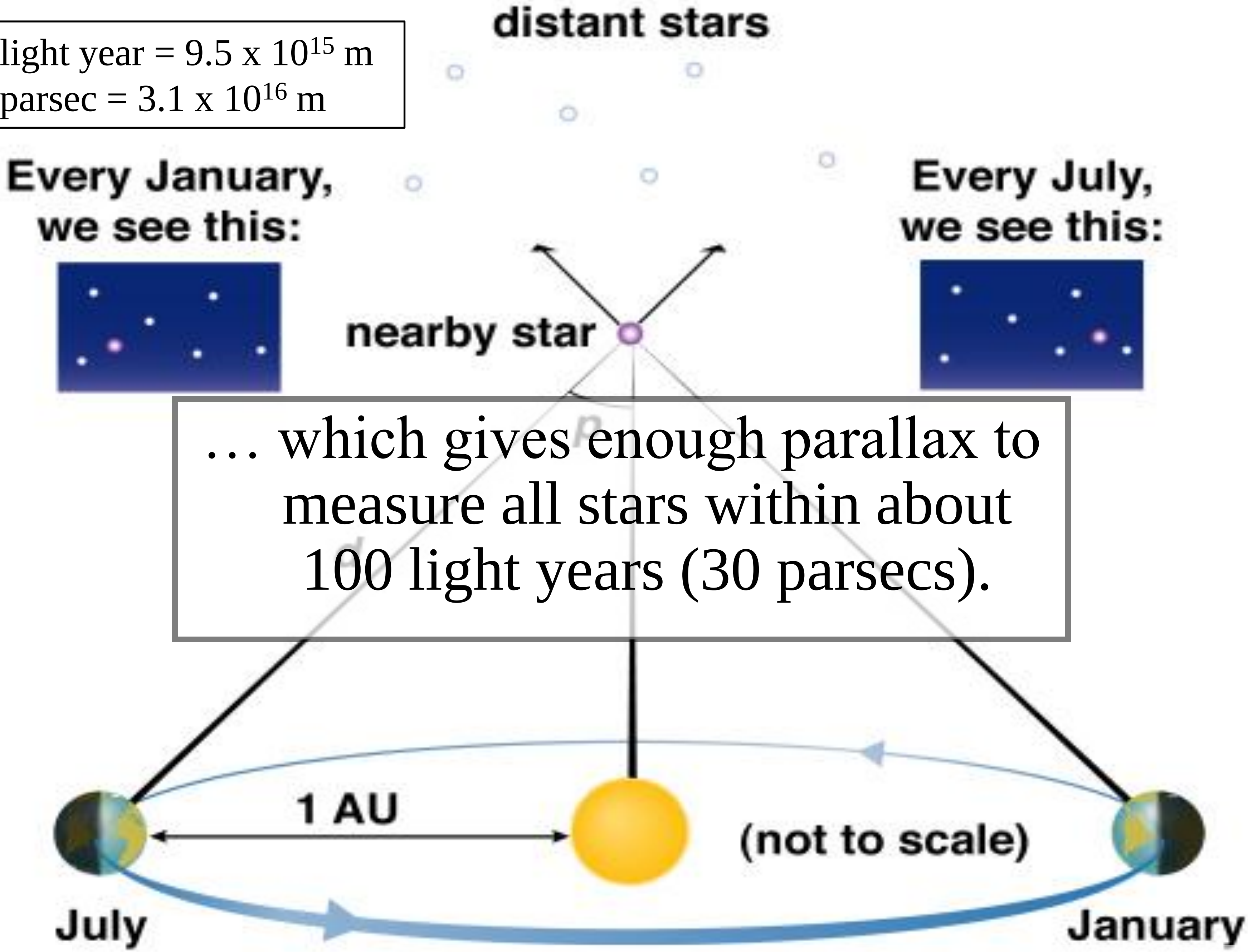
2 Earth radii = 12,700 km
2 AU = 300,000,000 km



However, if one takes measurements six months apart, one gets a distance separation of 2AU...

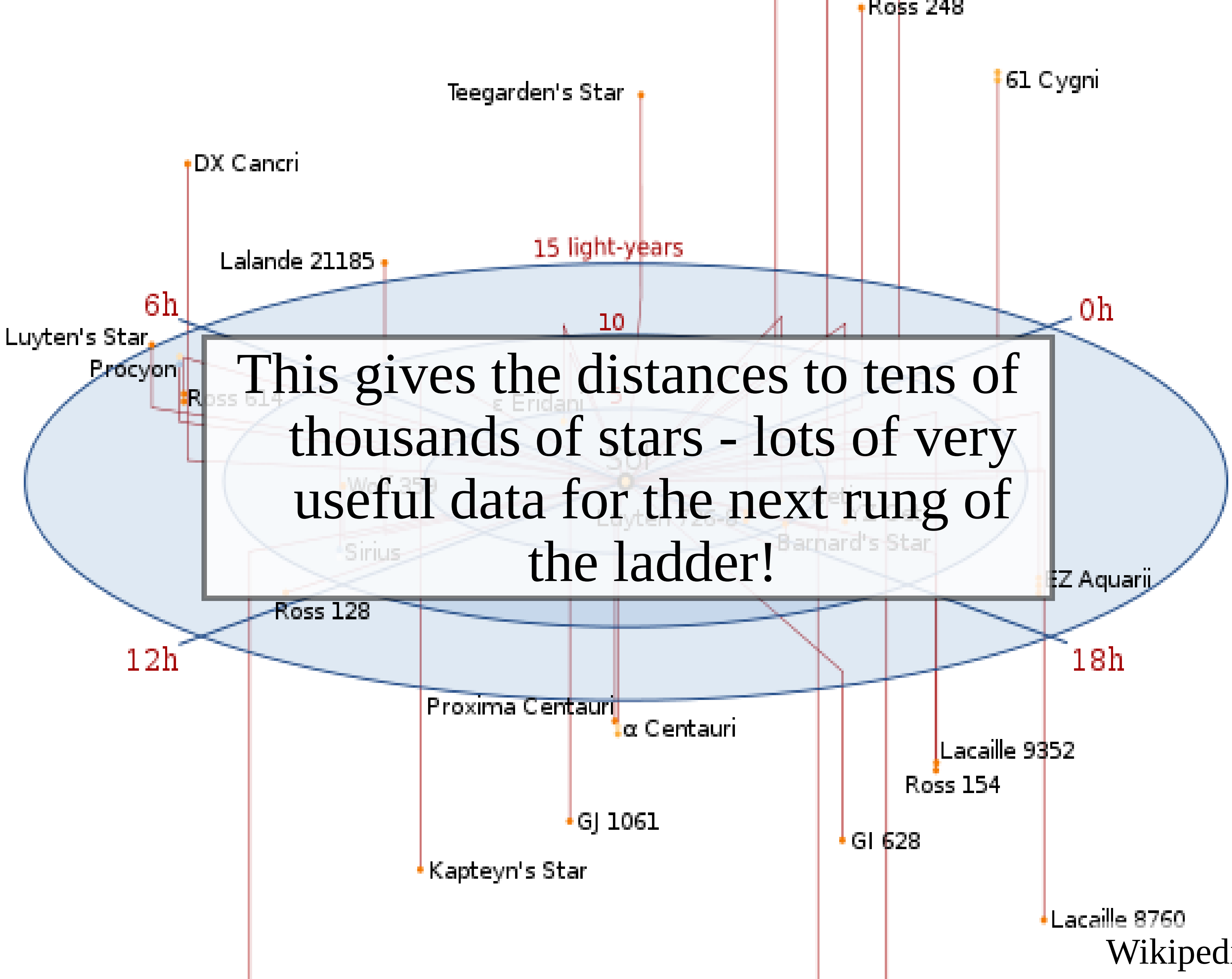
From “The Essential Cosmic Perspective”, Bennett et al.

1 light year = 9.5 x 10^{15} m
1 parsec = 3.1 x 10^{16} m



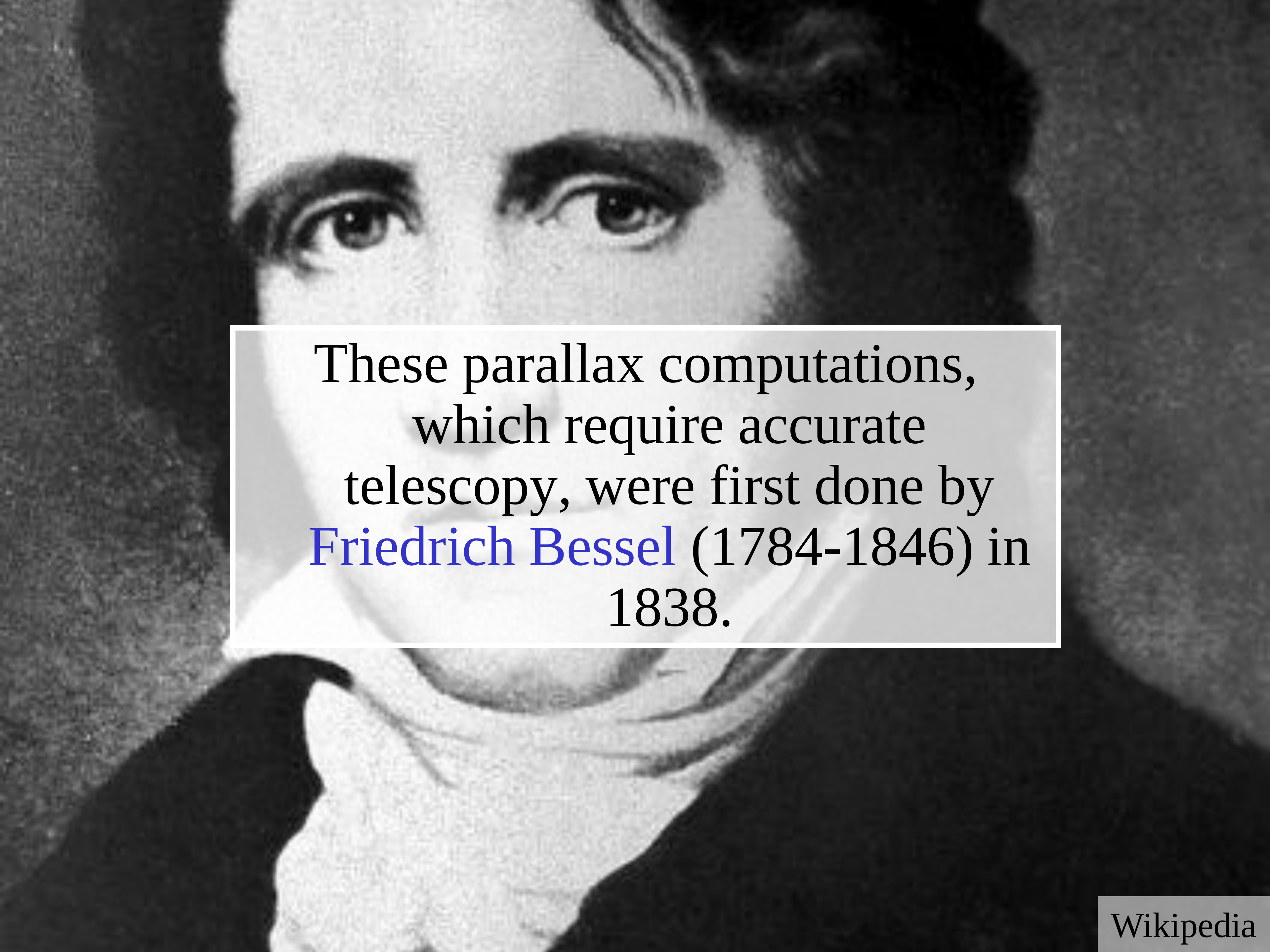
… which gives enough parallax to measure all stars within about 100 light years (30 parsecs).

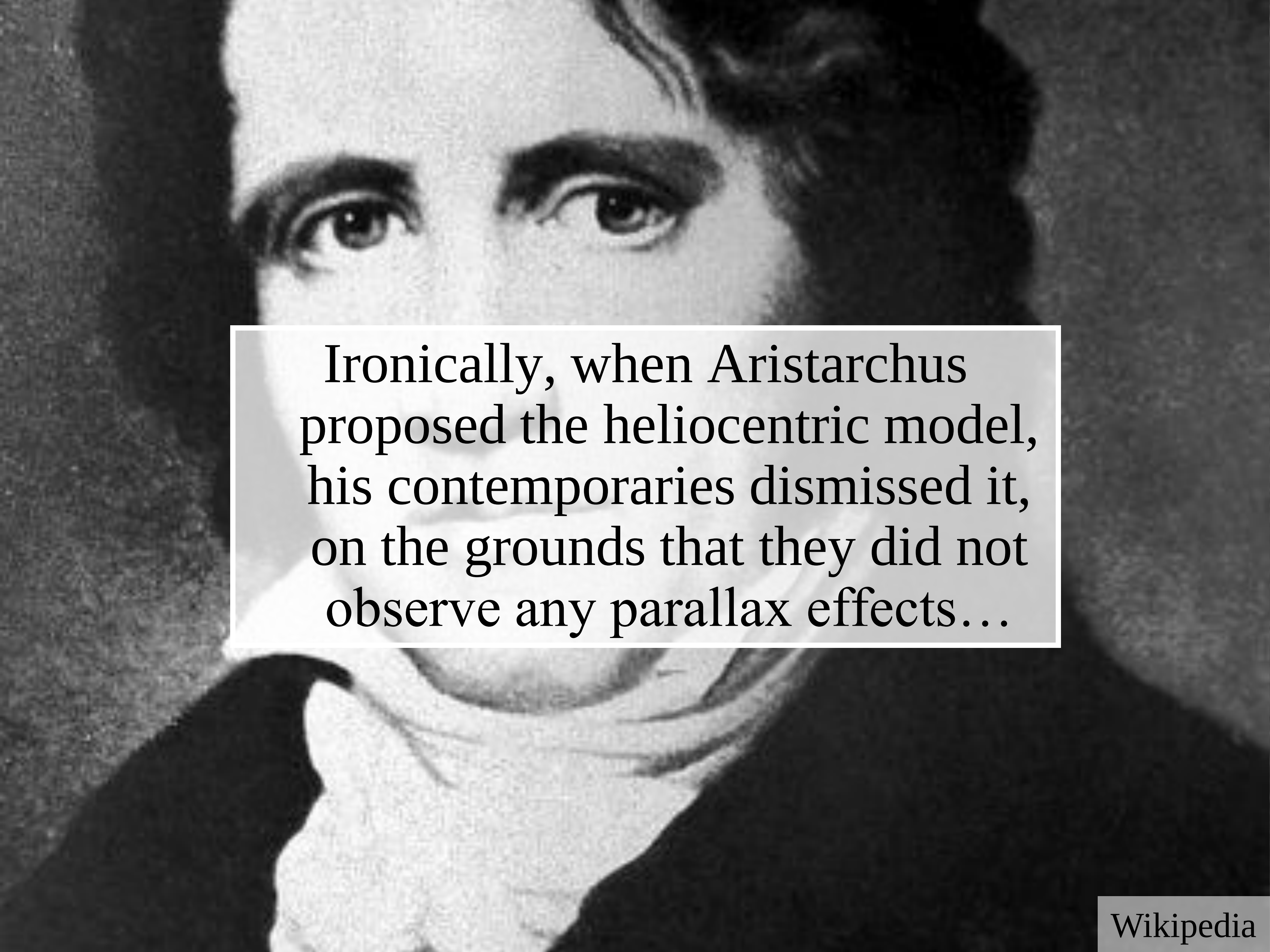
From "The Essential Cosmic Perspective", Bennett et al.



This gives the distances to tens of thousands of stars - lots of very useful data for the next rung of the ladder!

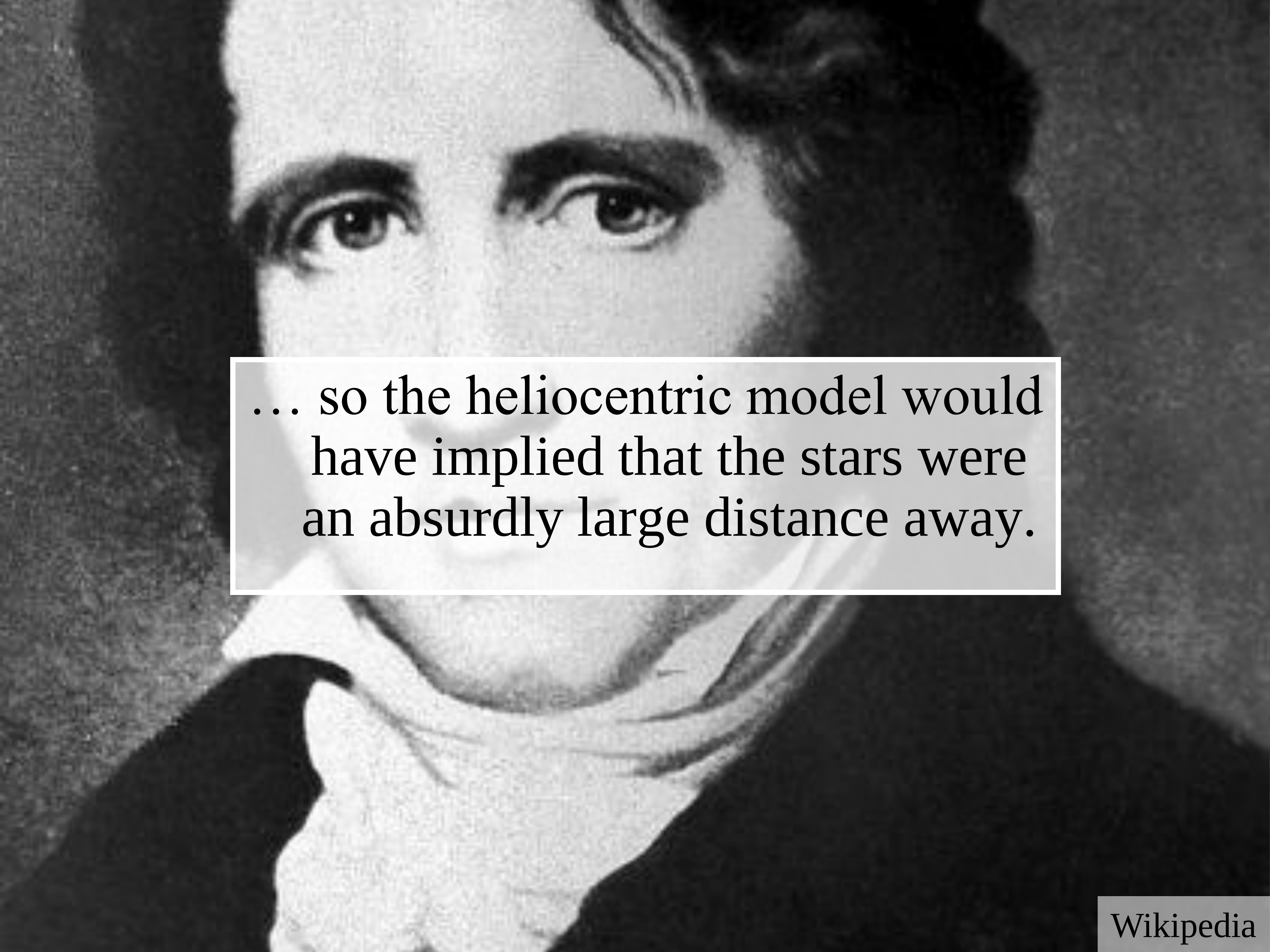
Wikipedia

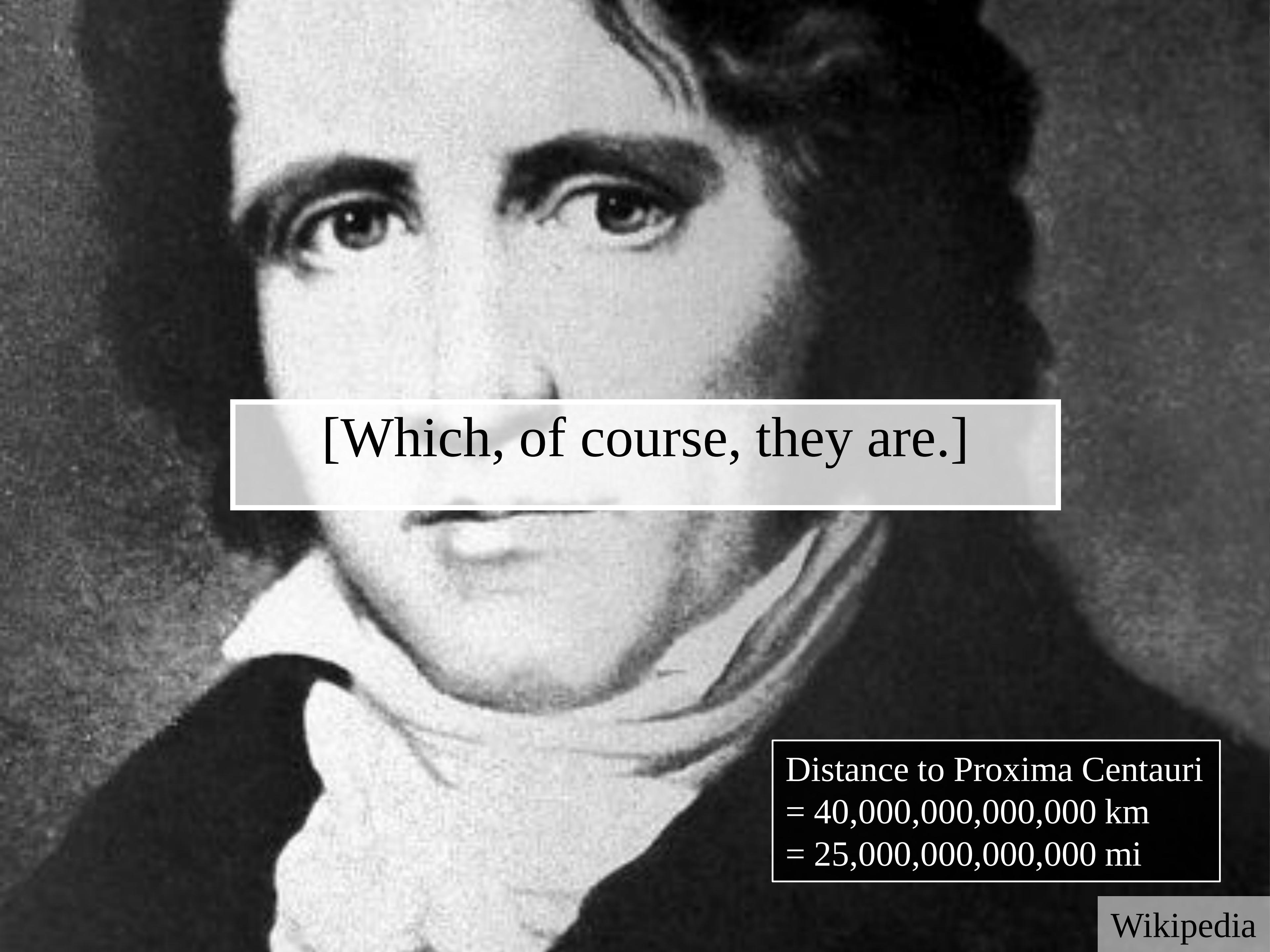

These parallax computations, which require accurate telescopy, were first done by Friedrich Bessel (1784-1846) in 1838.
Wikipedia



Ironically, when Aristarchus proposed the heliocentric model, his contemporaries dismissed it, on the grounds that they did not observe any parallax effects…

Wikipedia

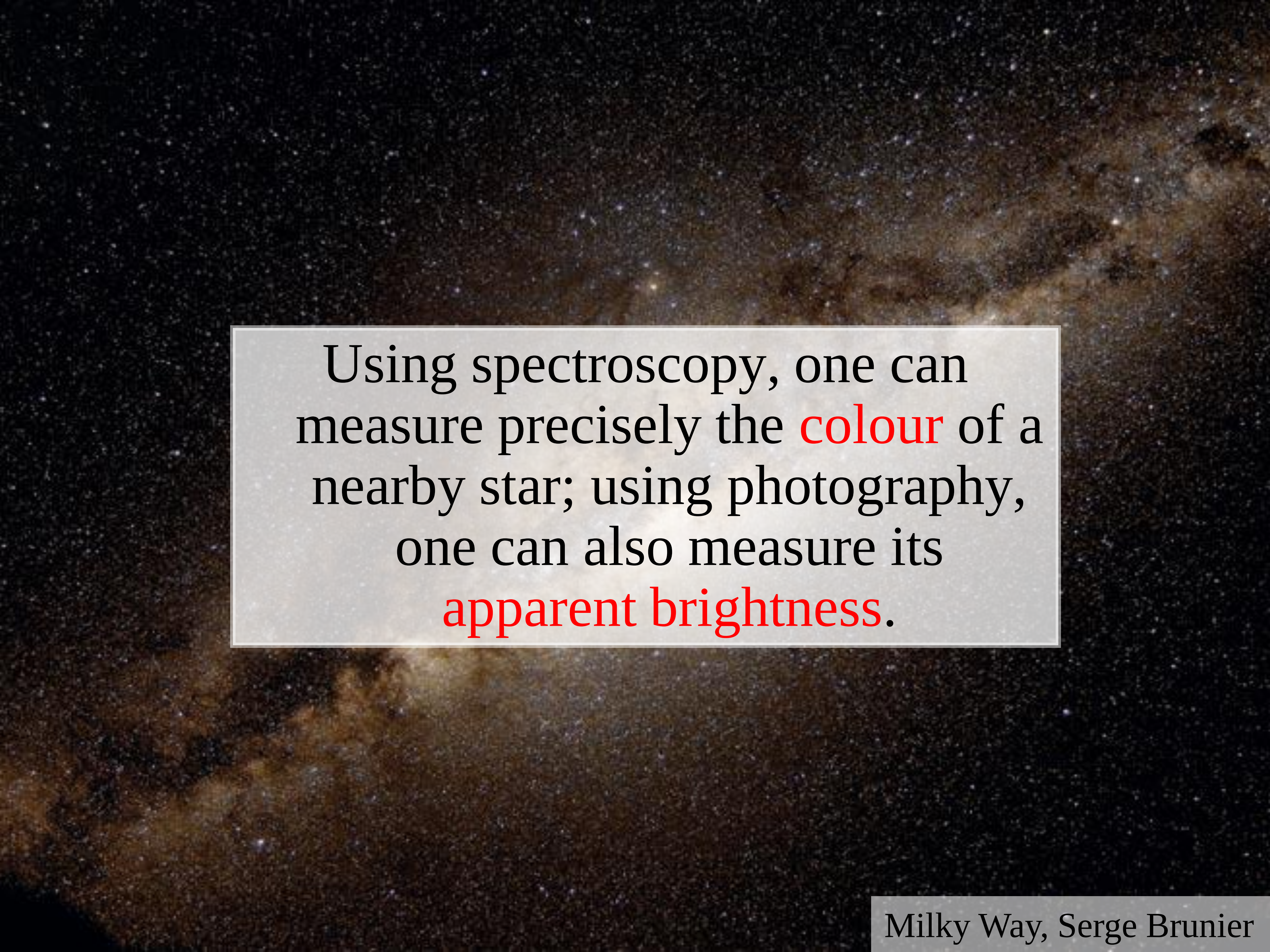

… so the heliocentric model would have implied that the stars were an absurdly large distance away.
Wikipedia


[Which, of course, they are.]
Distance to Proxima Centauri
= 40,000,000,000,000 km
= 25,000,000,000,000 mi
Wikipedia

7^{th} rung: the Milky Way

Milky Way, Serge Brunier

Milky Way, Serge Brunier



Milky Way, Serge Brunier

Using the apparent brightness, the distance, and inverse square law, one can compute the absolute brightness of these stars.

$$M = m - 5(\log_{10} D_L - 1)$$

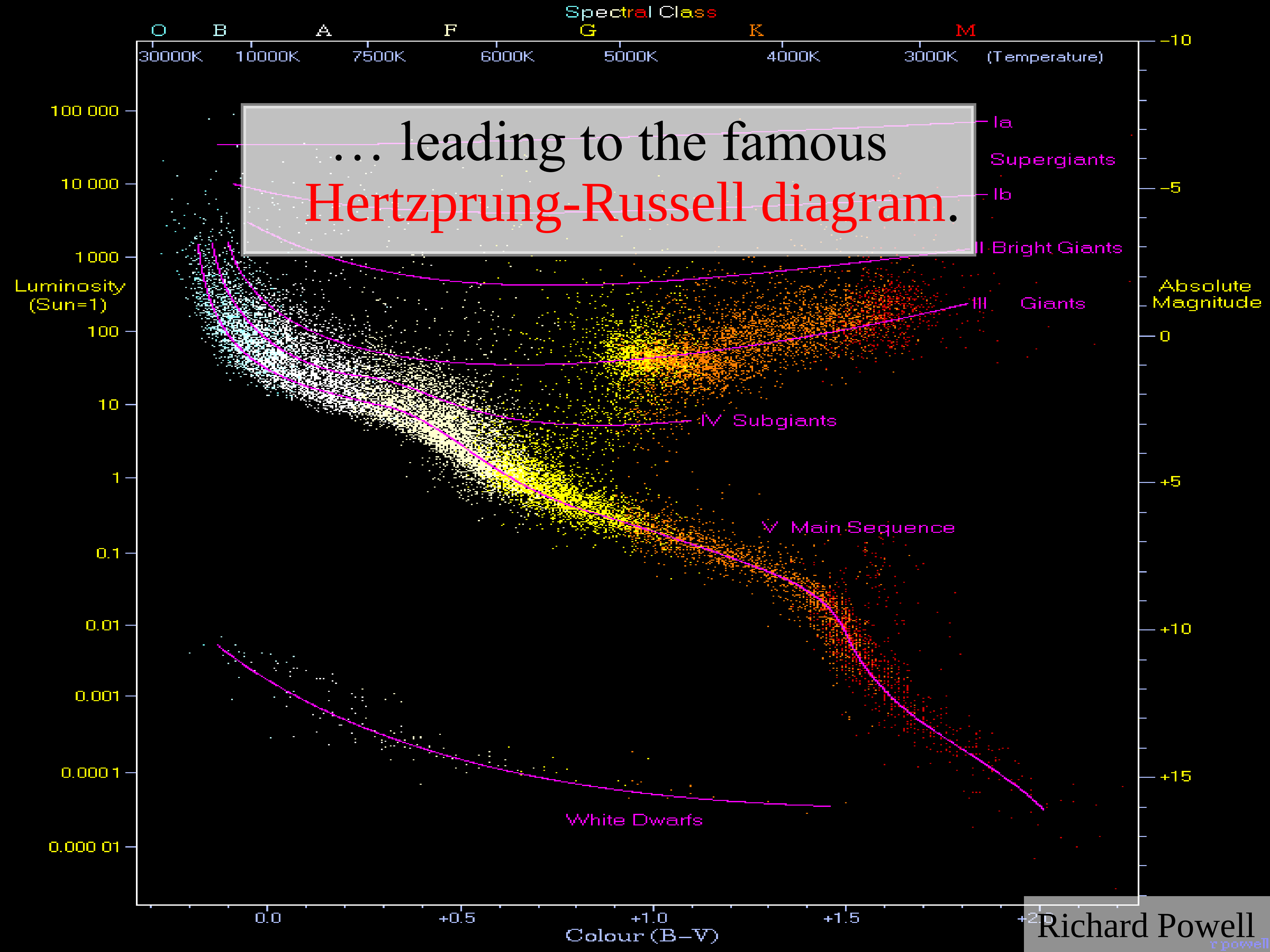
Milky Way, Serge Brunier

Leiden Observatory

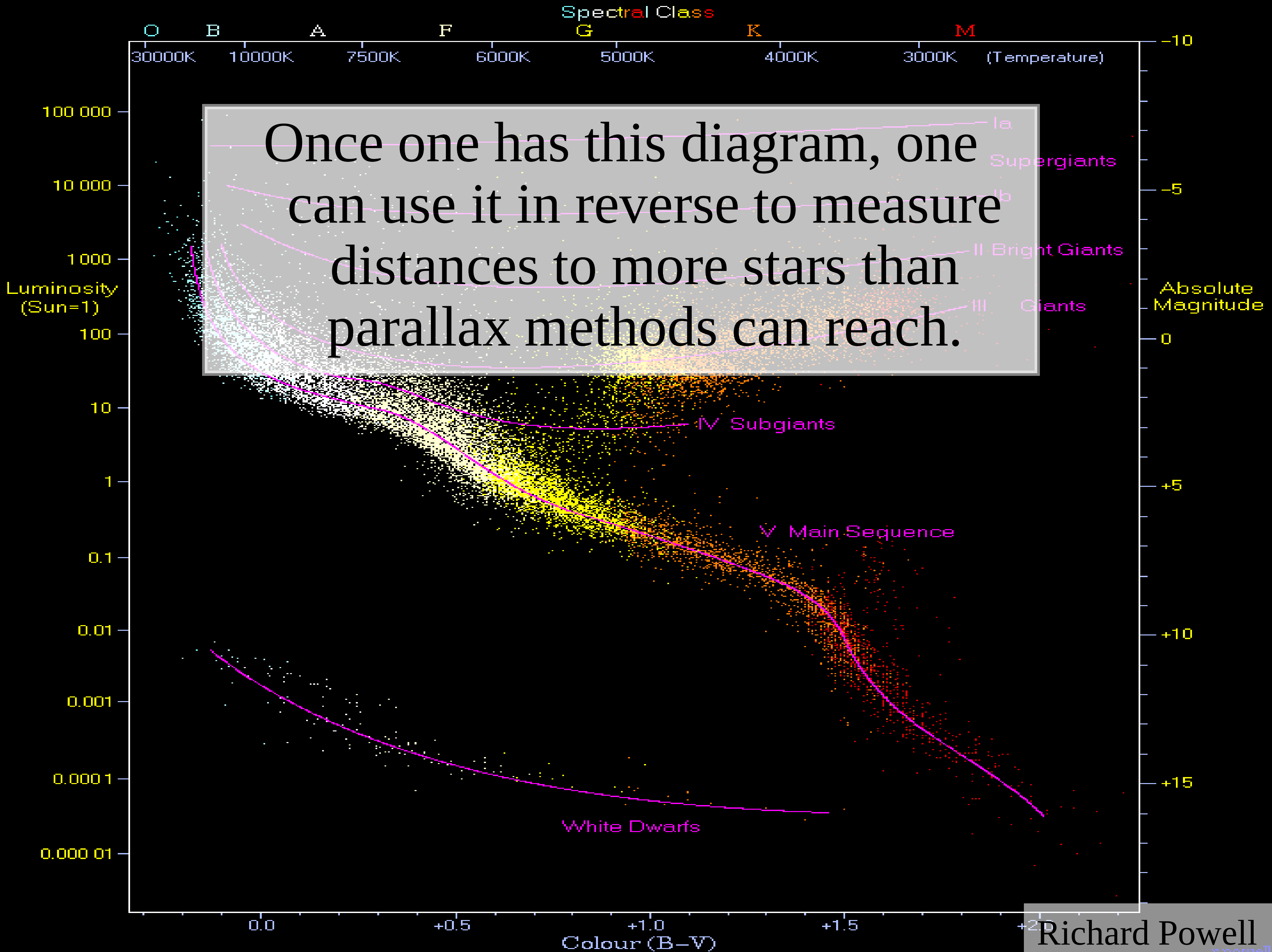
University of Chicago/Yerkes Observatory

Ejnar Hertzsprung (1873-1967) and Henry Russell (1877-1957) plotted this absolute brightness against color for thousands of nearby stars in 1905-1915…

… leading to the famous Hertzprung-Russell diagram.



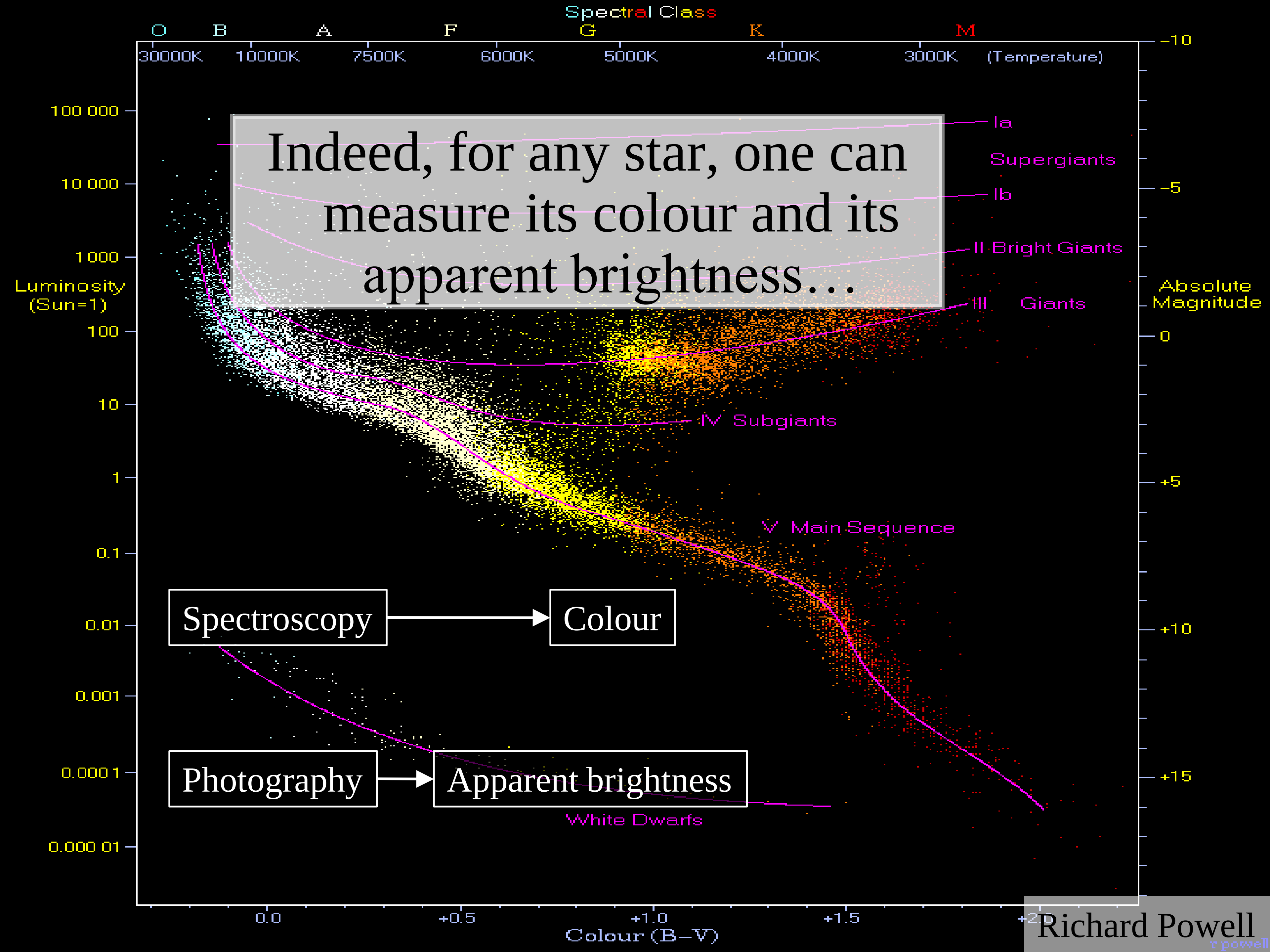
Richard Powell



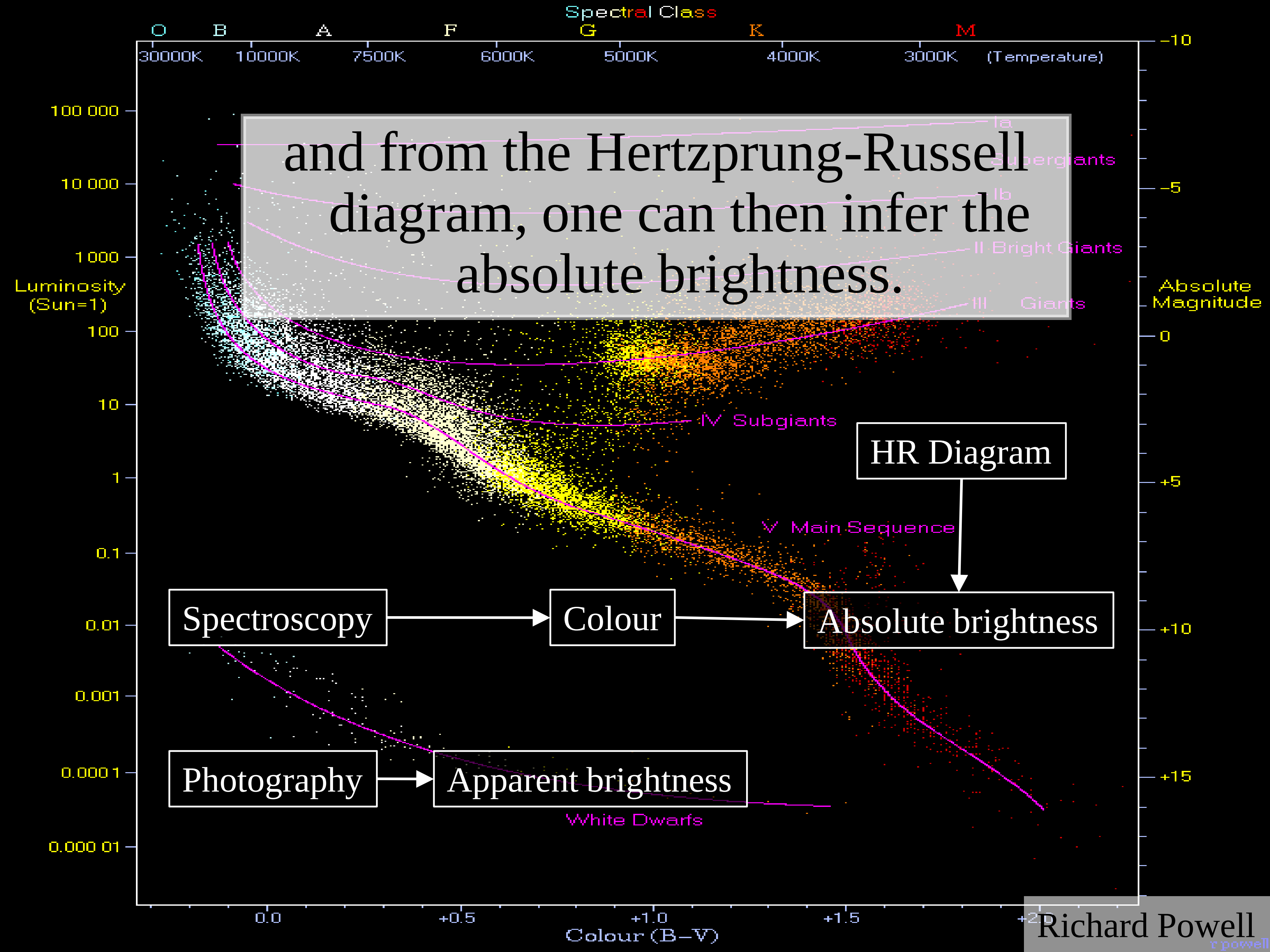
Once one has this diagram, one can use it in reverse to measure distances to more stars than parallax methods can reach.

Richard Powell

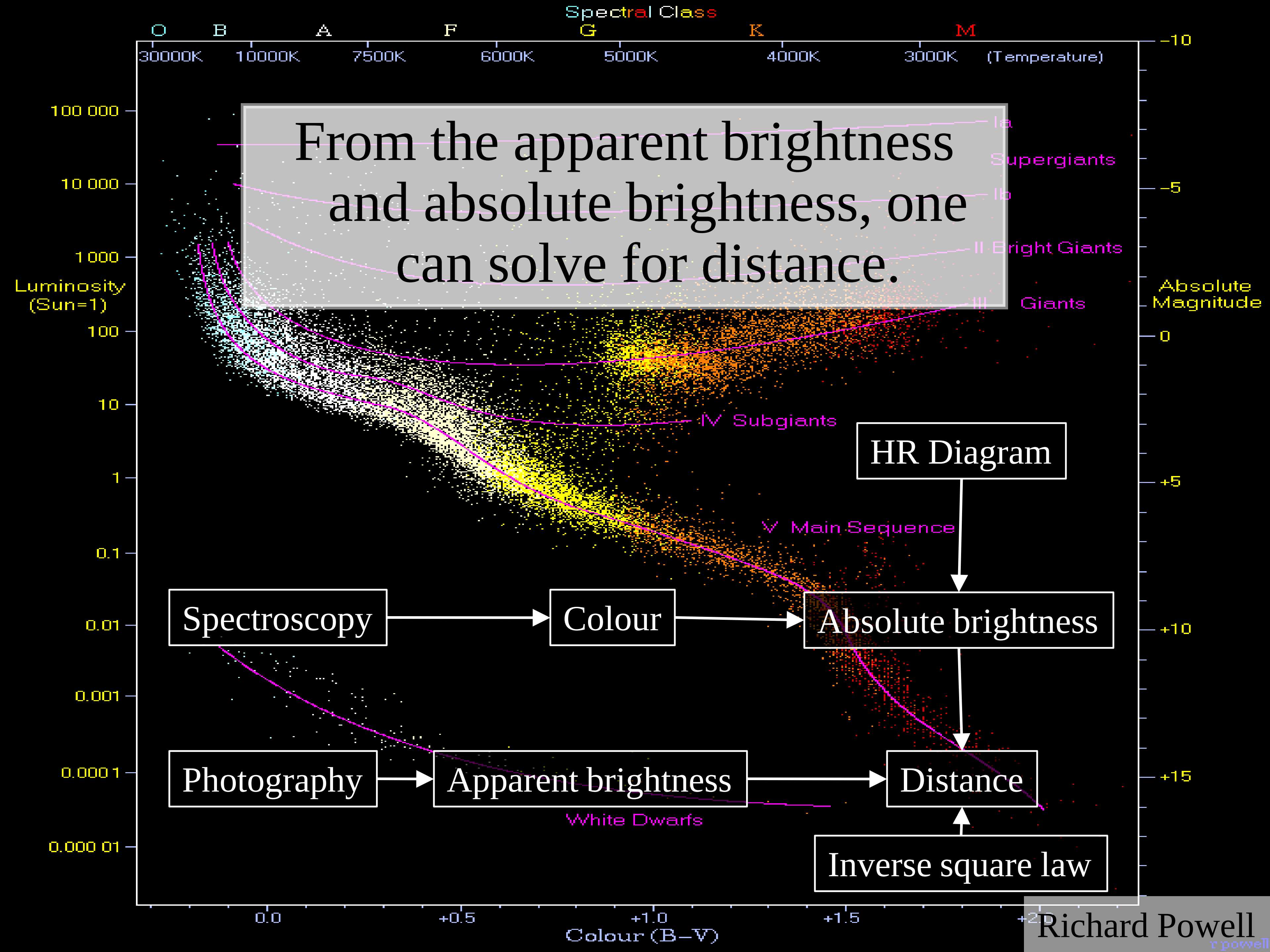
Indeed, for any star, one can measure its colour and its apparent brightness…



Richard Powell



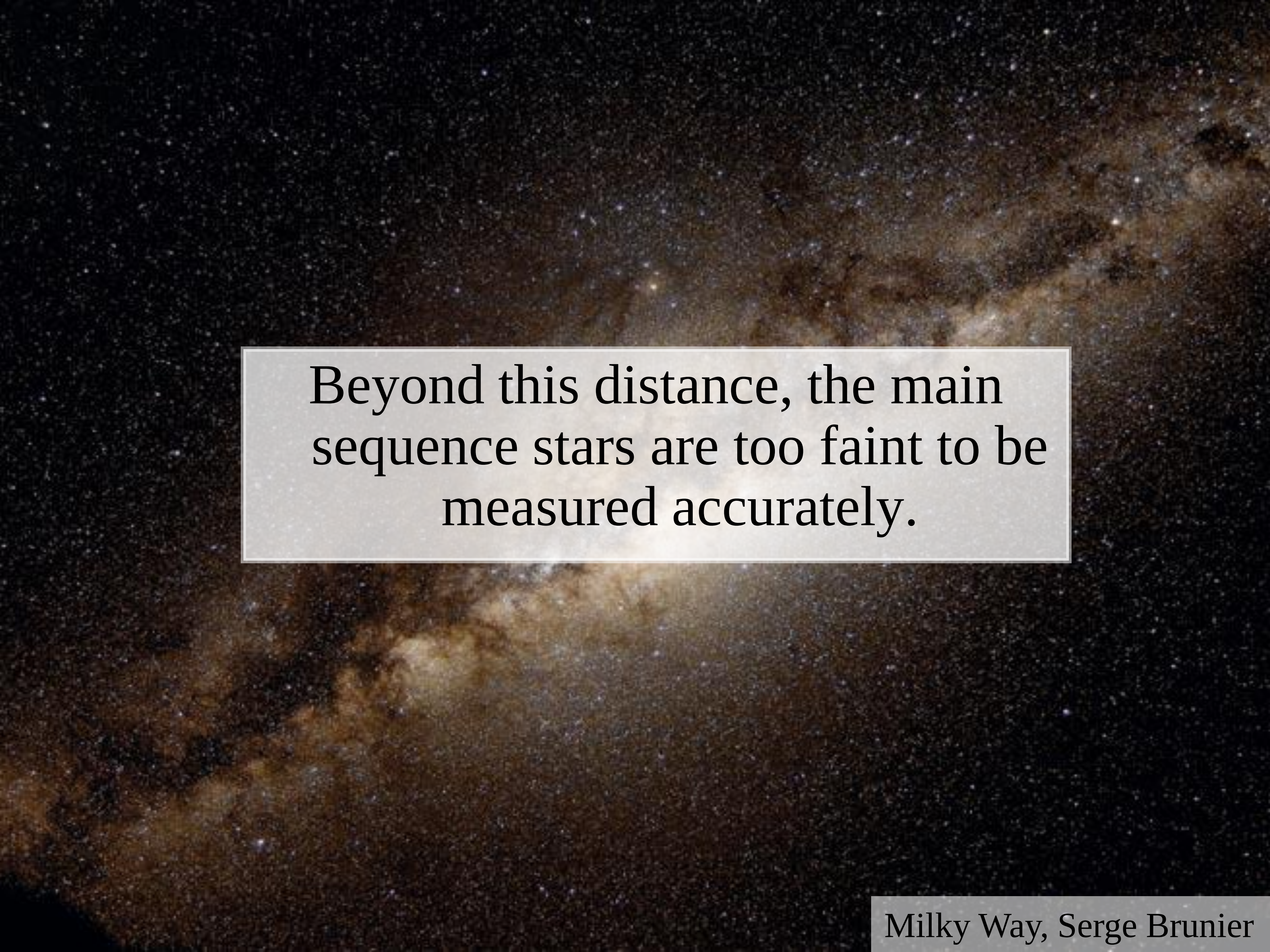
and from the Hertzprung-Russell diagram, one can then infer the absolute brightness.
HR Diagram
Spectroscopy
Colour
Absolute brightness
Photography
Apparent brightness
Spectral Class
O B A F G K M
30000K 10000K 7500K 6000K 5000K 4000K 3000K (Temperature)
Luminosity (Sun=1)
100 000
10 000
1 000
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
Absolute Magnitude
−10
−5
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0 +0.5 +1.0 +1.5 +2.0
Colour (B−V)
Richard Powell


Spectral Class
O
B
A
F
G
K
M
30000K
10000K
7500K
6000K
5000K
4000K
3000K
(Temperature)
From the apparent brightness and absolute brightness, one can solve for distance.
Luminosity (Sun=1)
100 000
10 000
1 000
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
Absolute Magnitude
−10
−5
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
HR Diagram
Spectroscopy
Colour
Absolute brightness
Photography
Apparent brightness
Distance
Inverse square law
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B−V)
Richard Powell

This technique (main sequence fitting) works out to about 300,000 light years (covering the entire galaxy!)

300,000 light years = 2.8 x 10^{21} m = 1.8 x 10^{18} mi
Diameter of Milky Way = 100,000 light years

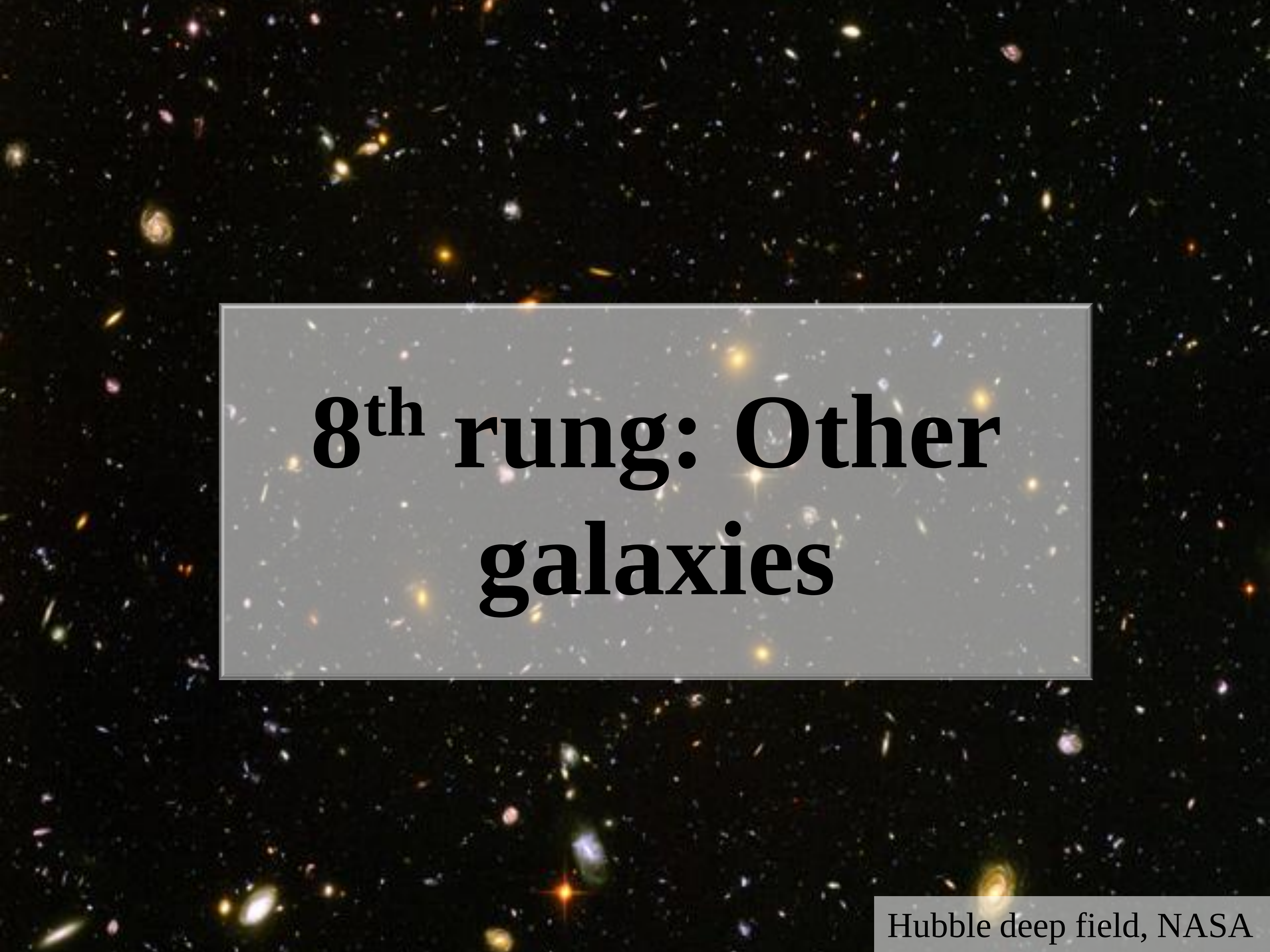
Milky Way, Serge Brunier



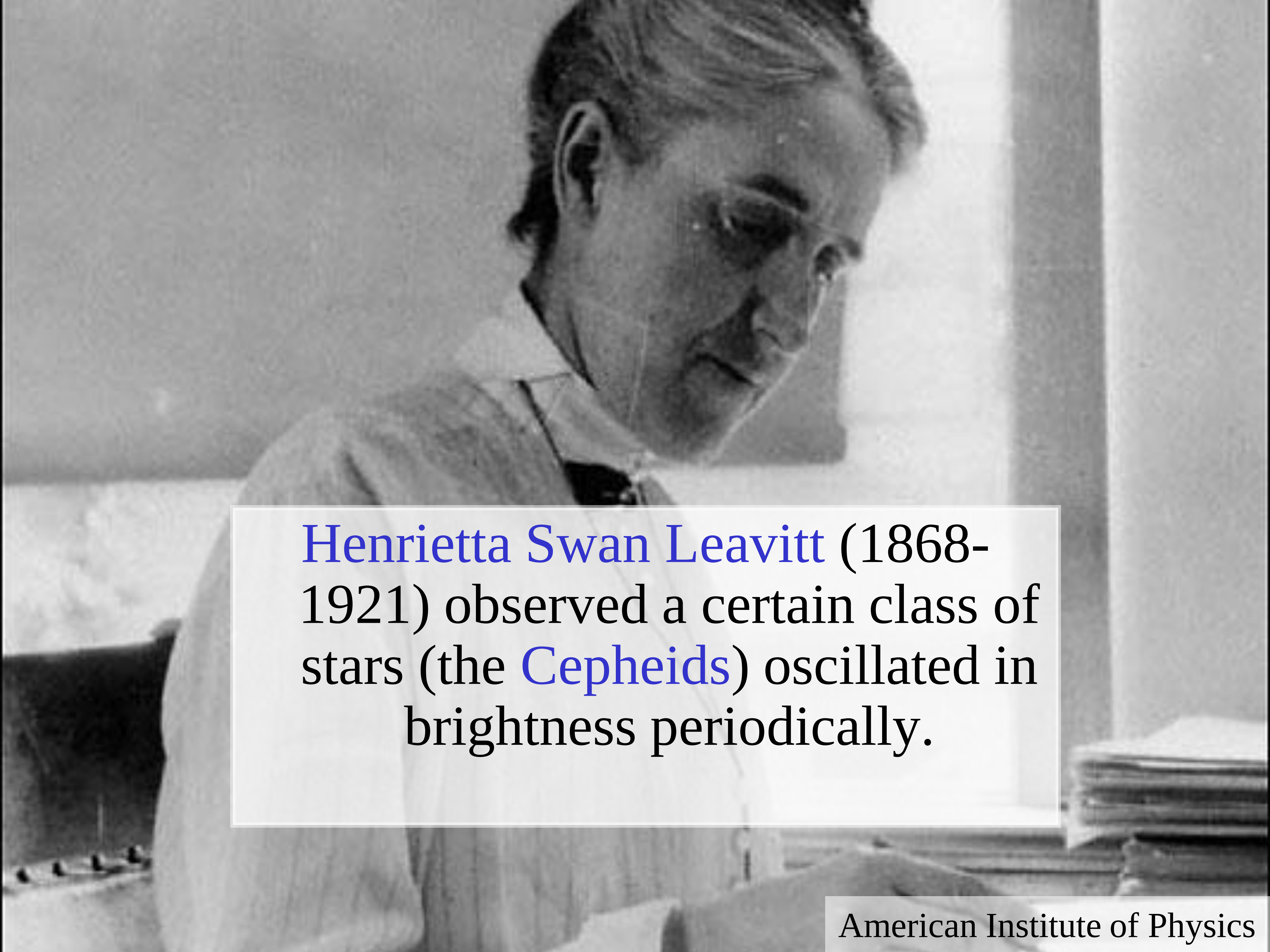
Beyond this distance, the main sequence stars are too faint to be measured accurately.

Milky Way, Serge Brunier

8^{th} rung: Other galaxies

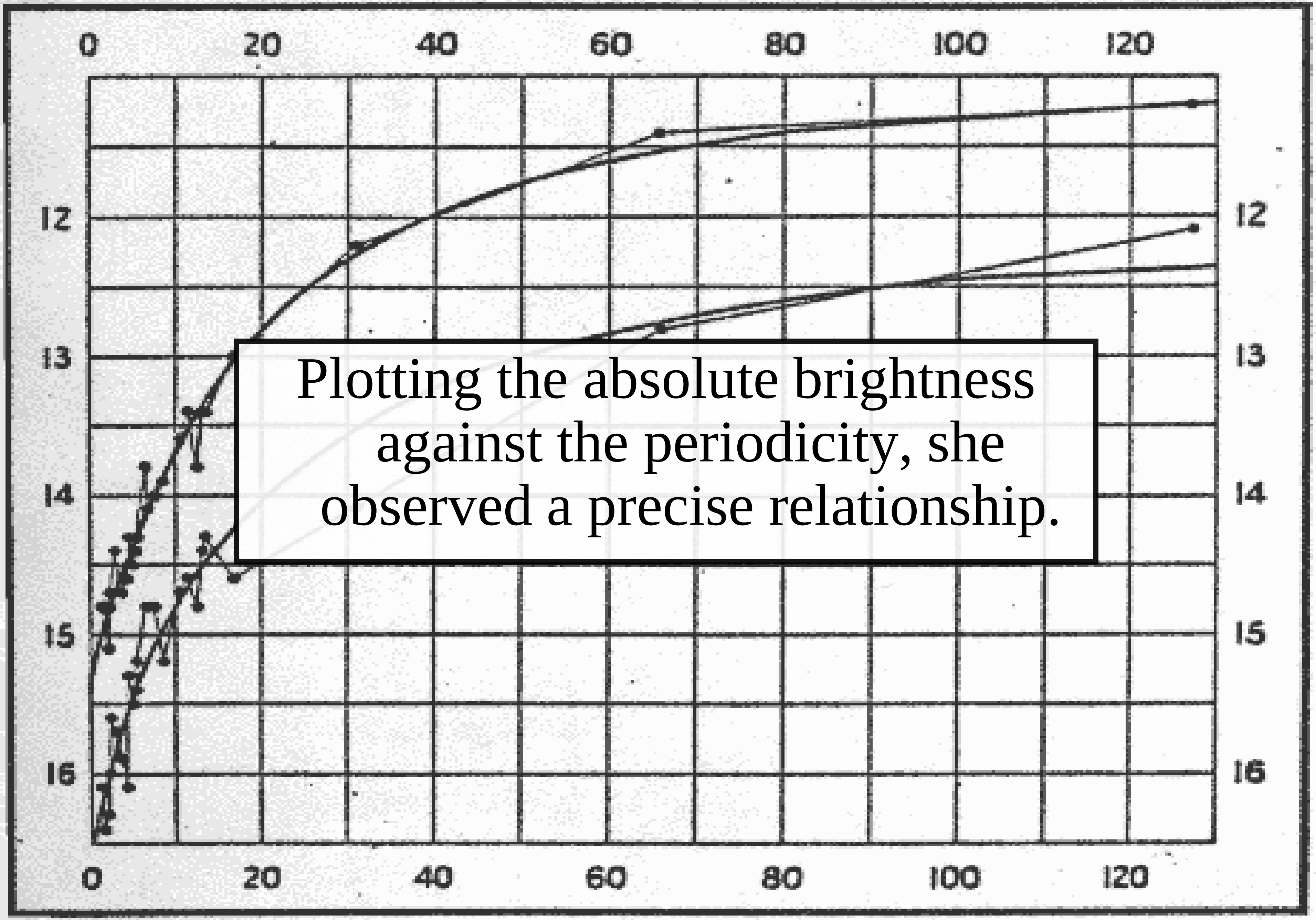


Hubble deep field, NASA



Henrietta Swan Leavitt (1868-1921) observed a certain class of stars (the Cepheids) oscillated in brightness periodically.

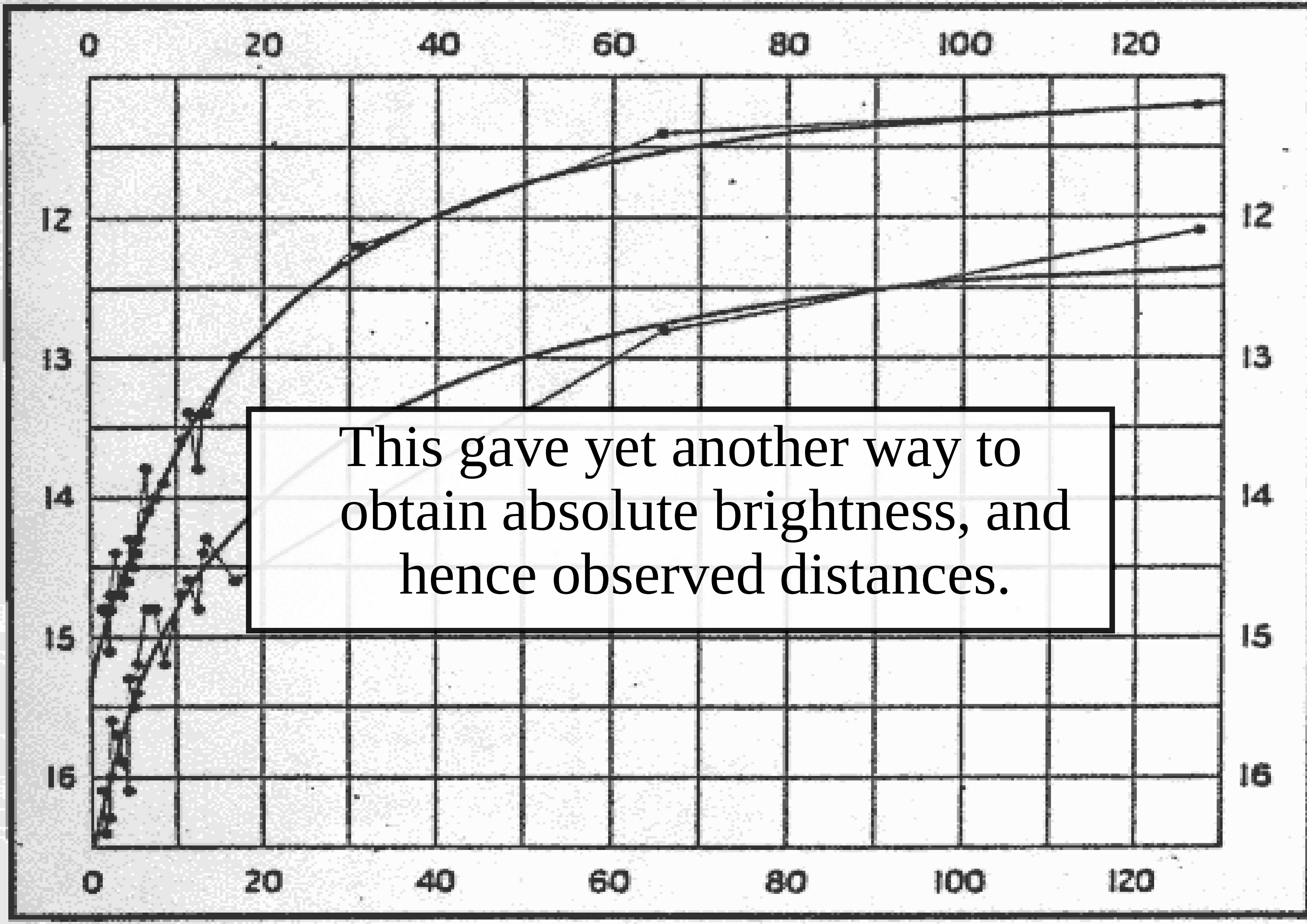
American Institute of Physics



Plotting the absolute brightness against the periodicity, she observed a precise relationship.

Fig. 1.

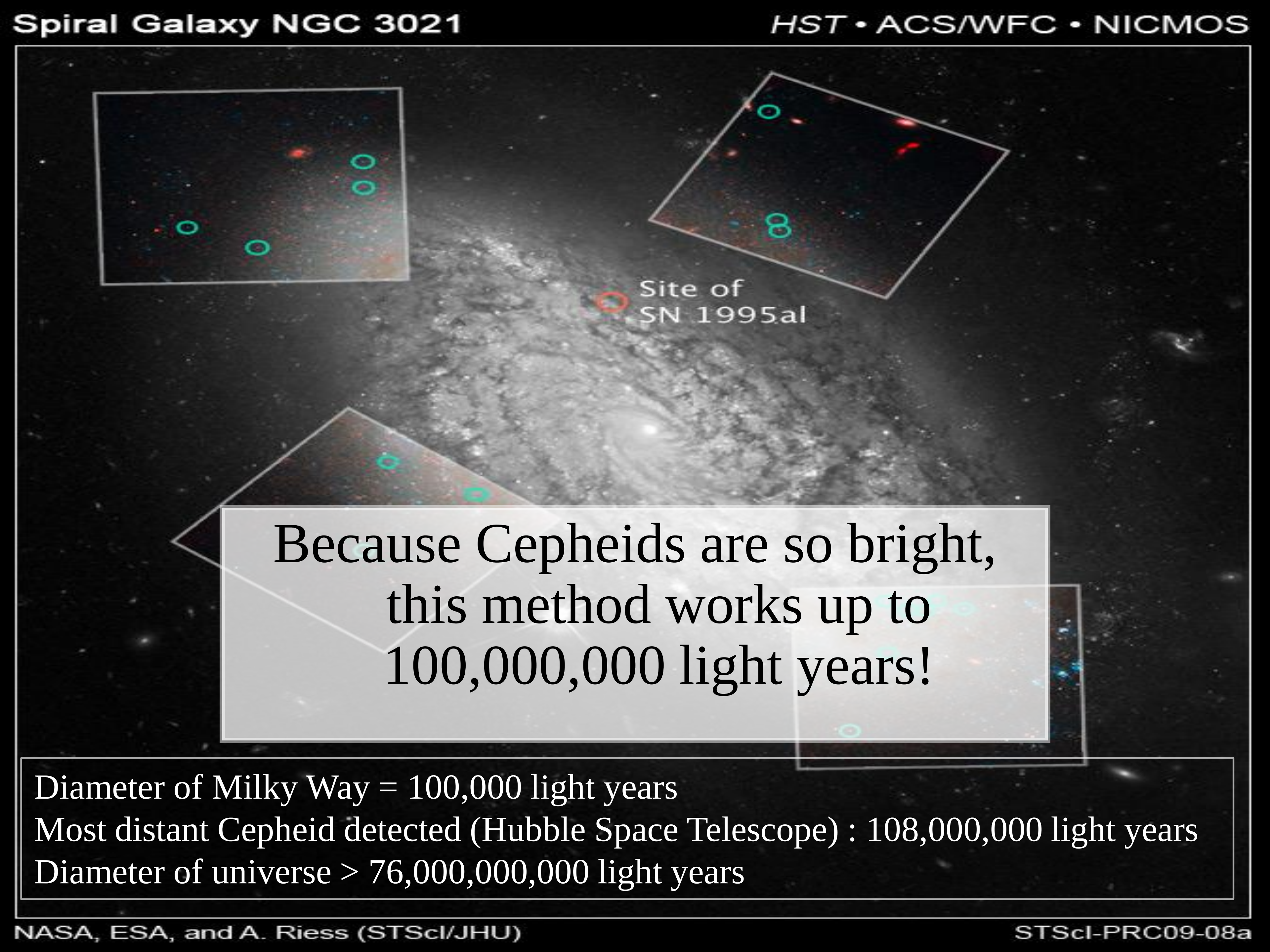
Henrietta Swan Leavitt, 1912



This gave yet another way to obtain absolute brightness, and hence observed distances.

FIG. 1.

Henrietta Swan Leavitt, 1912

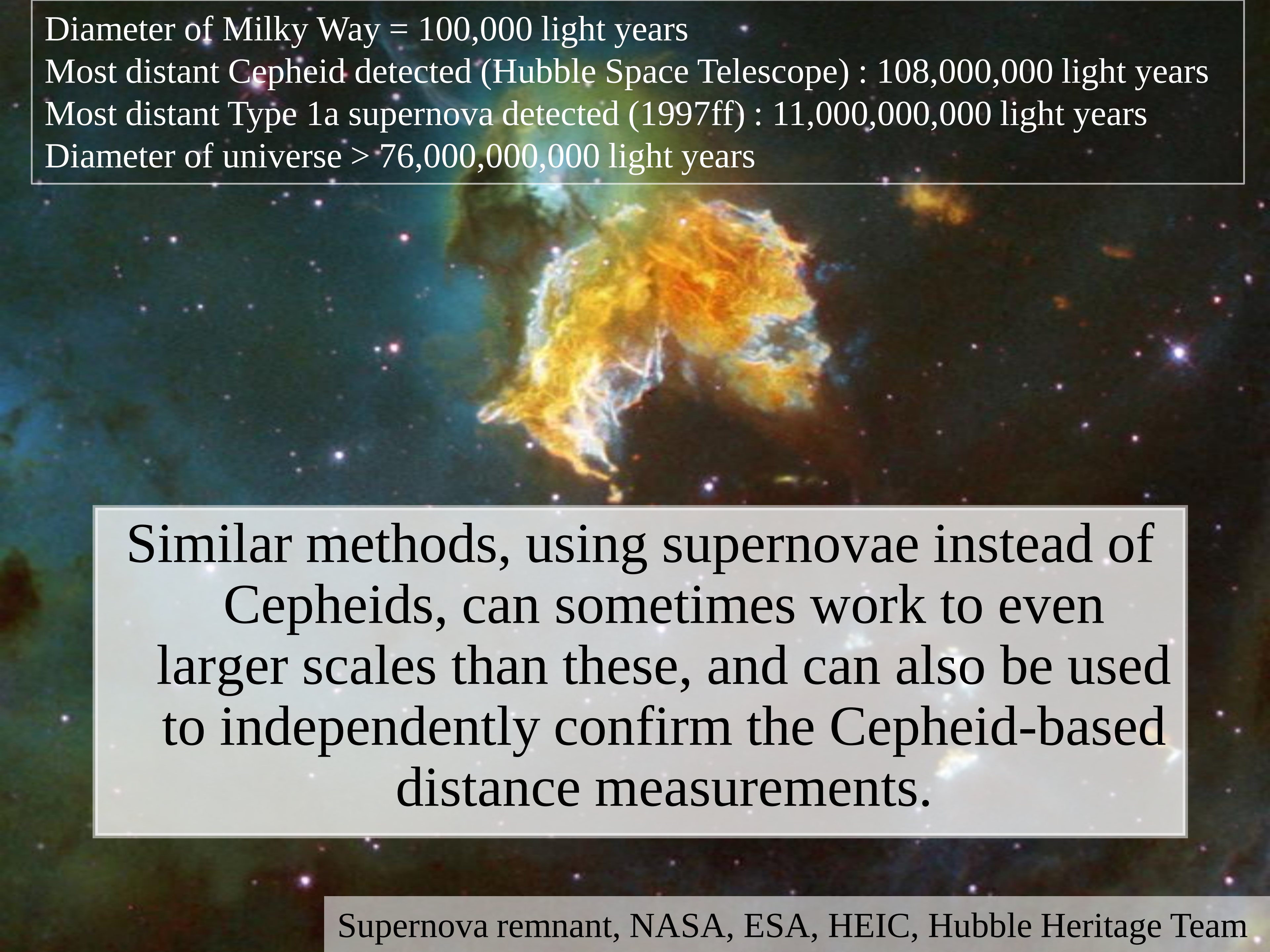


Because Cepheids are so bright, this method works up to 100,000,000 light years!

Diameter of Milky Way = 100,000 light years
Most distant Cepheid detected (Hubble Space Telescope) : 108,000,000 light years
Diameter of universe > 76,000,000,000 light years

Most galaxies are fortunate to have at least one Cepheid in them, so we know the distances to all galaxies out to a reasonably large distance.

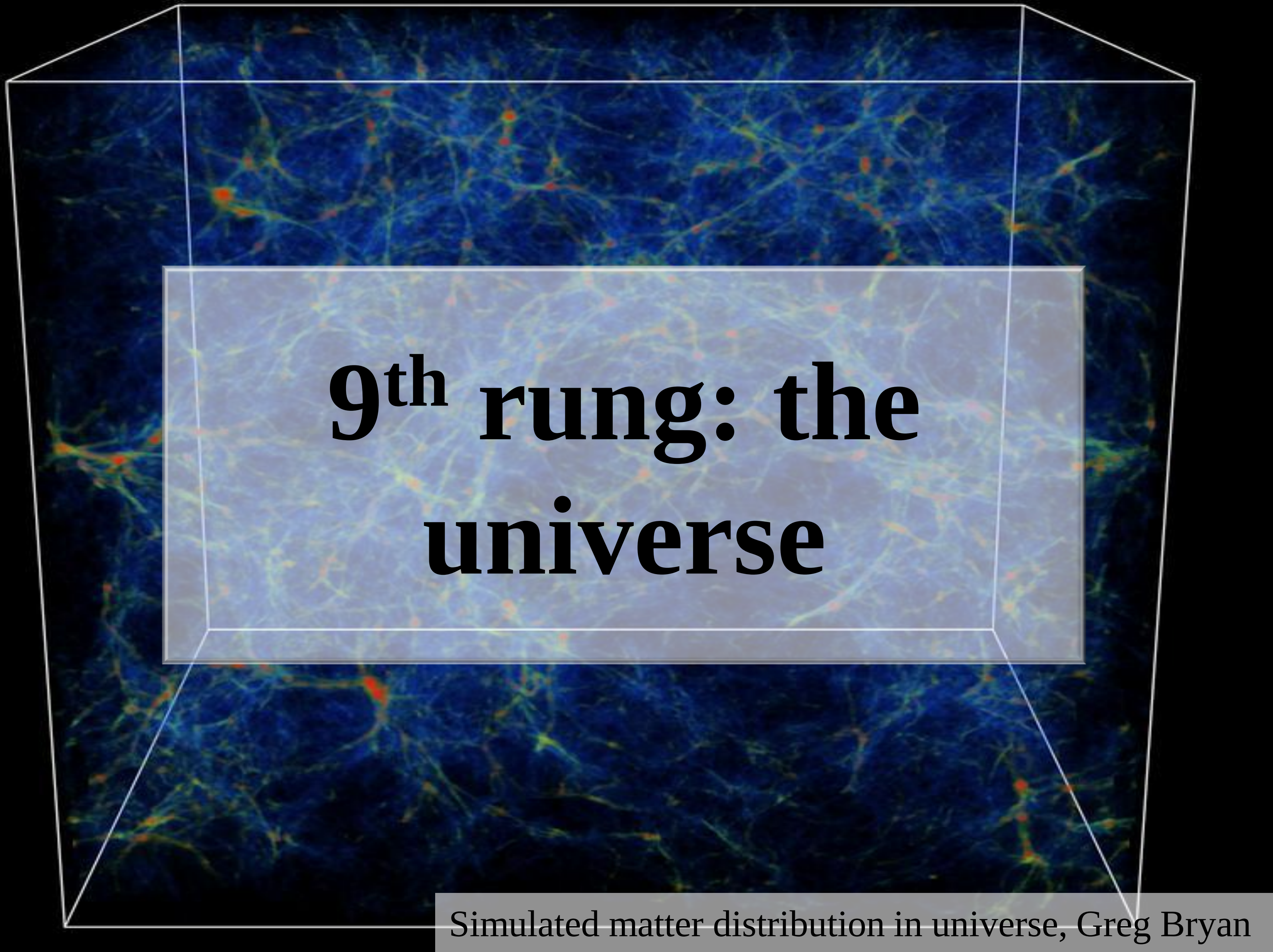
Diameter of Milky Way = 100,000 light years
Most distant Cepheid detected (Hubble Space Telescope) : 108,000,000 light years
Most distant Type 1a supernova detected (1997ff) : 11,000,000,000 light years
Diameter of universe > 76,000,000,000 light years



Similar methods, using supernovae instead of Cepheids, can sometimes work to even larger scales than these, and can also be used to independently confirm the Cepheid-based distance measurements.

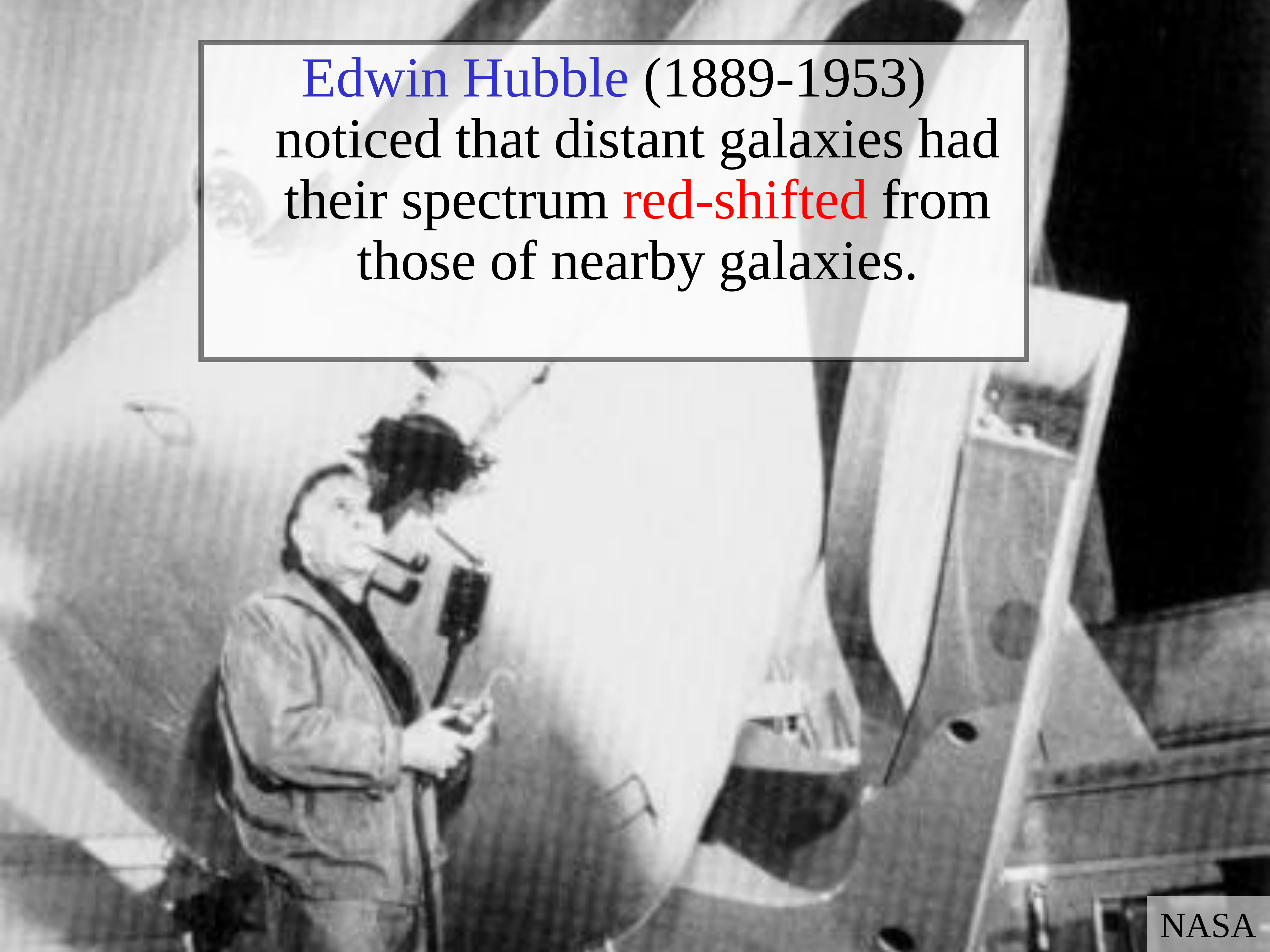
Supernova remnant, NASA, ESA, HEIC, Hubble Heritage Team

9th rung: the universe

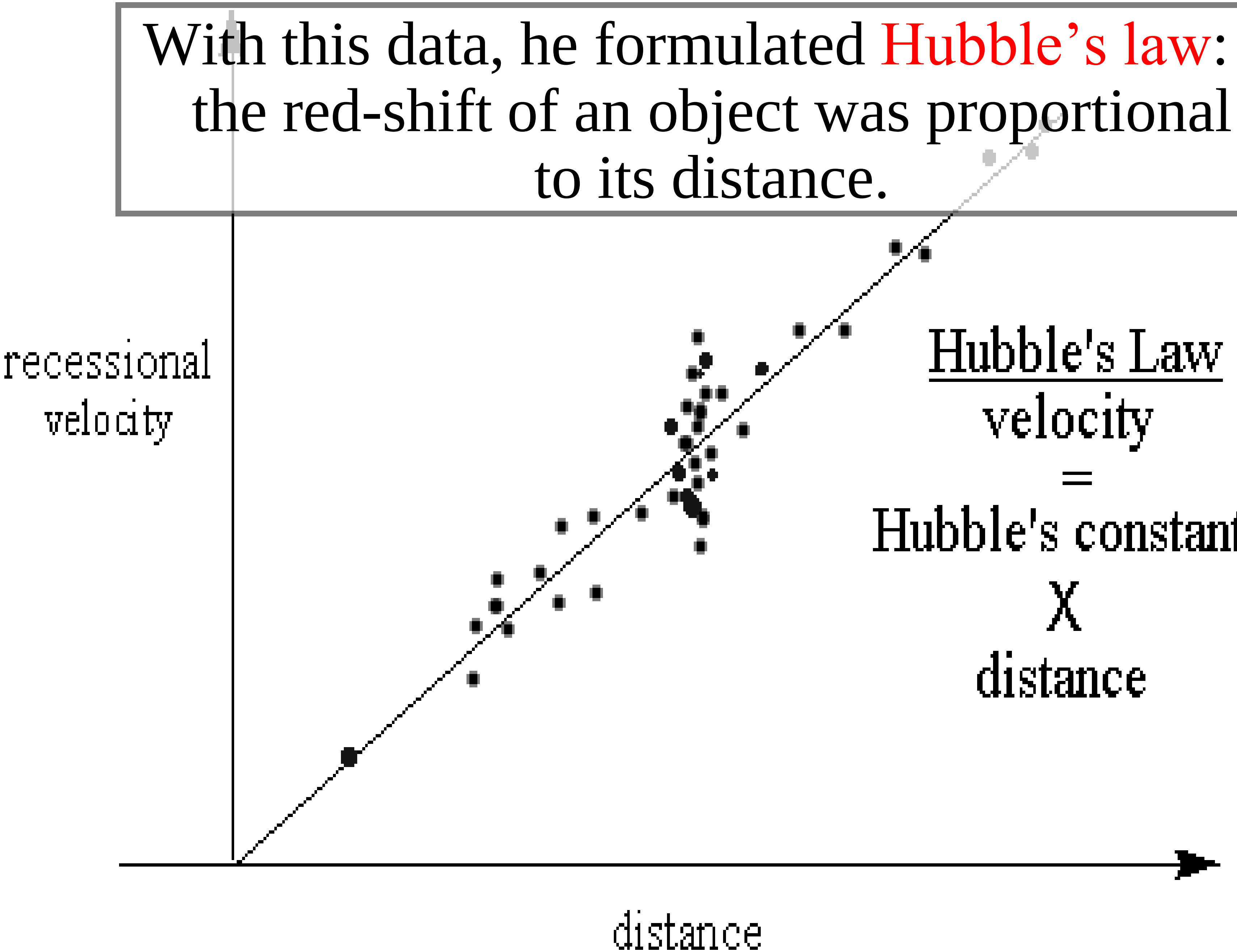


Simulated matter distribution in universe, Greg Bryan

Edwin Hubble (1889-1953) noticed that distant galaxies had their spectrum red-shifted from those of nearby galaxies.

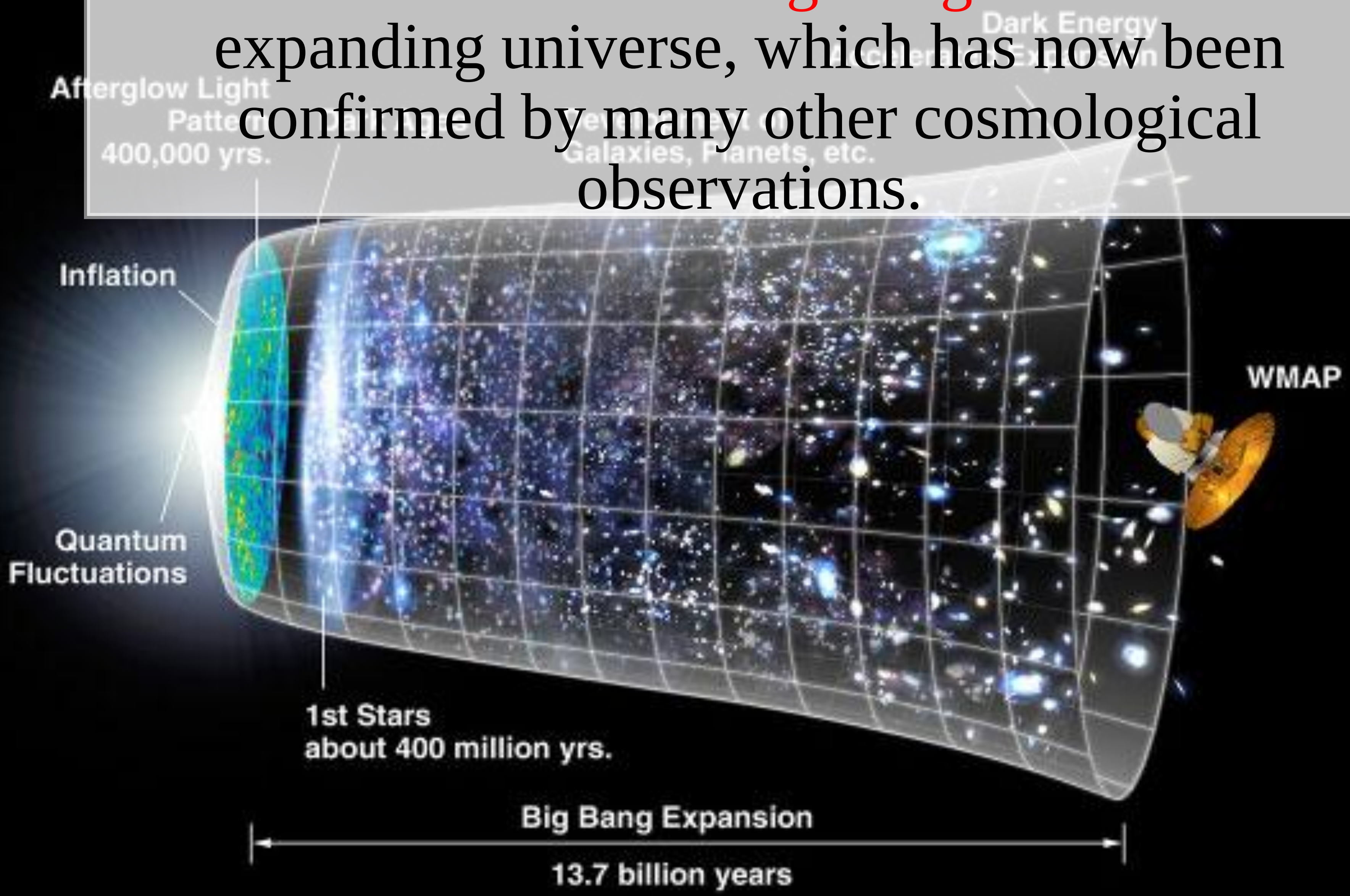


With this data, he formulated Hubble's law: the red-shift of an object was proportional to its distance.



NASA

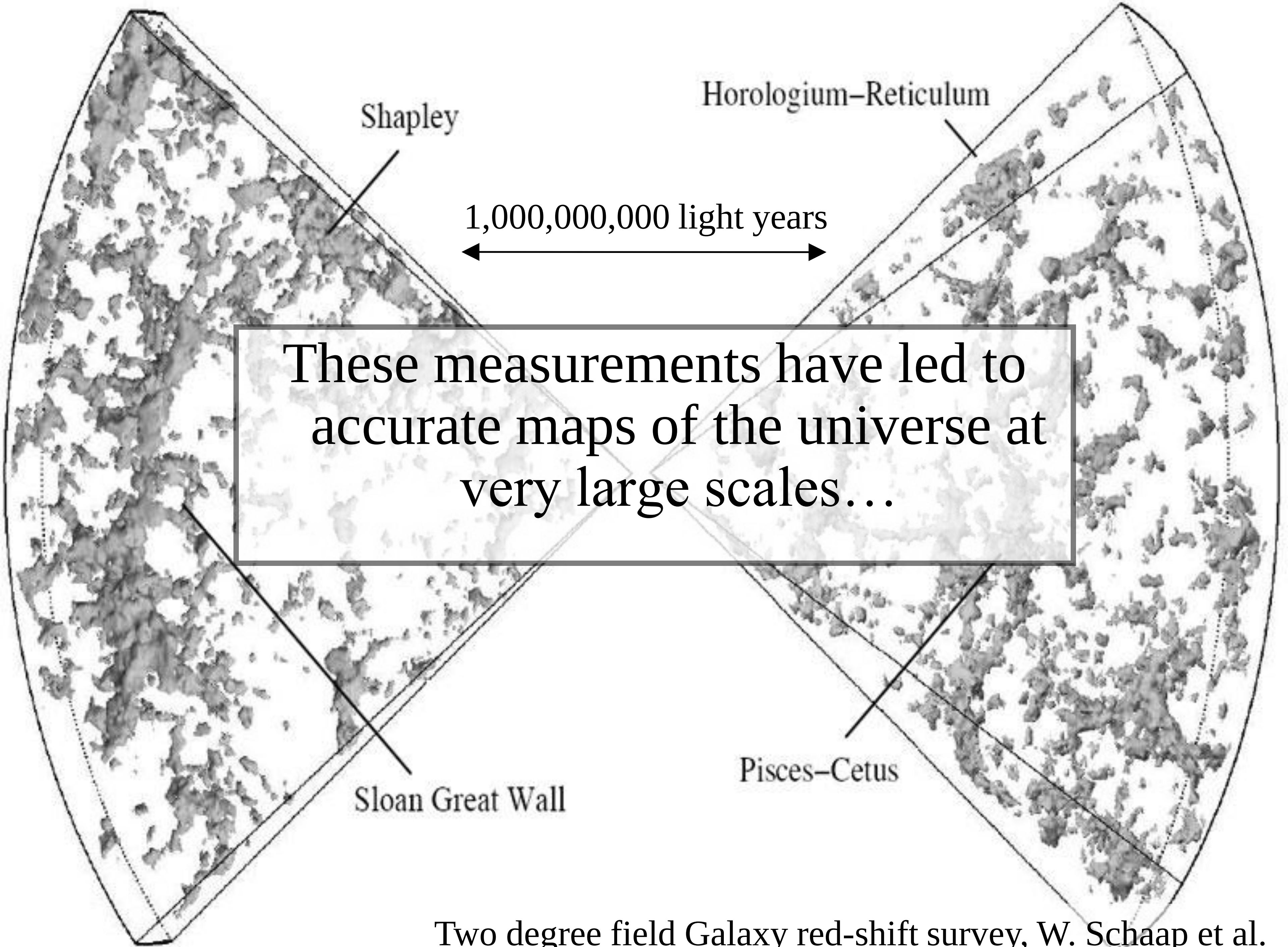
This led to the famous Big Bang model of the expanding universe, which has now been confirmed by many other cosmological observations.



NASA, WMAP

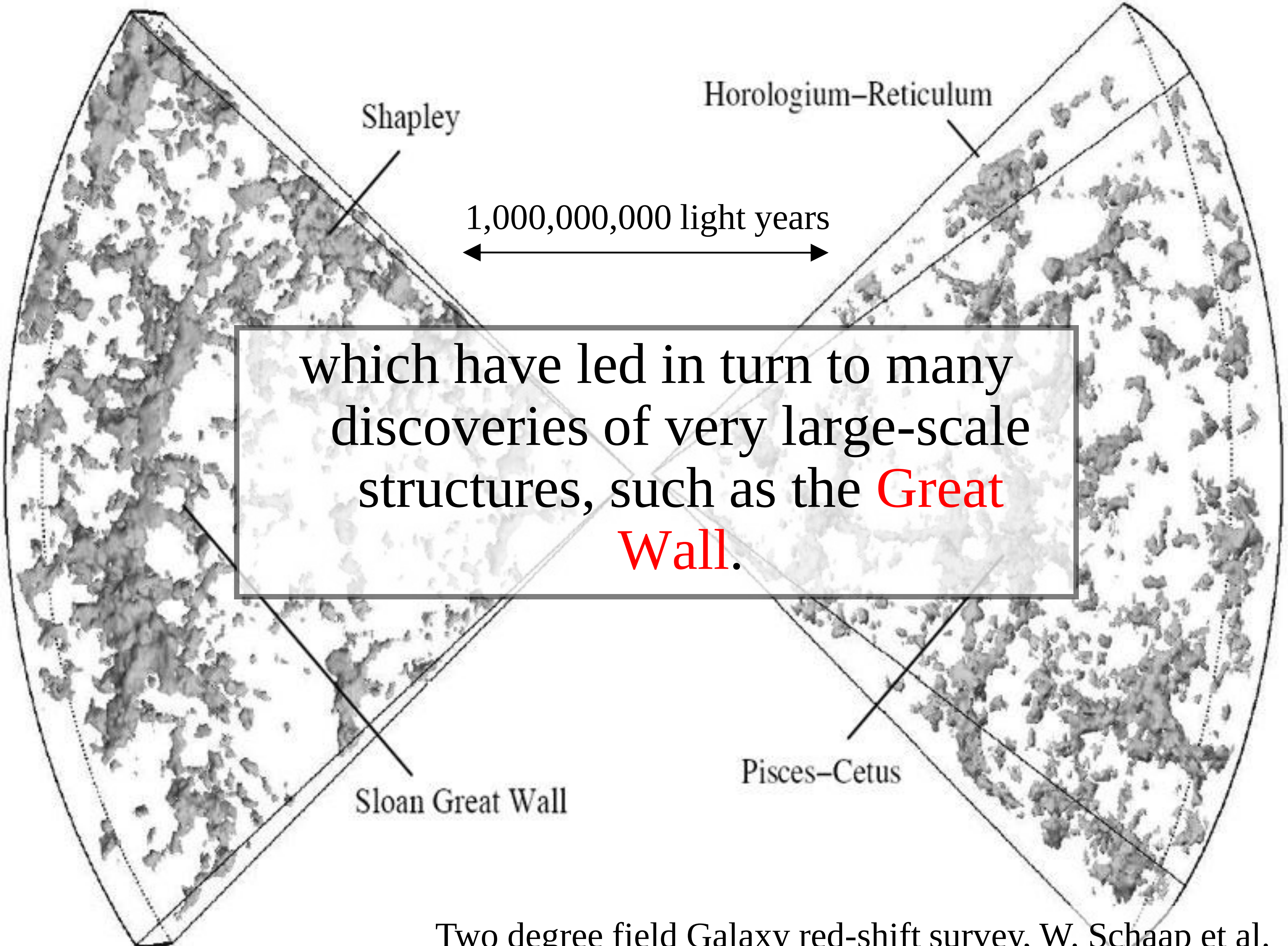
But it also gave a way to measure distances even at extremely large scales… by first measuring the red-shift and then applying Hubble's law.

Hubble deep field, NASA



These measurements have led to accurate maps of the universe at very large scales…

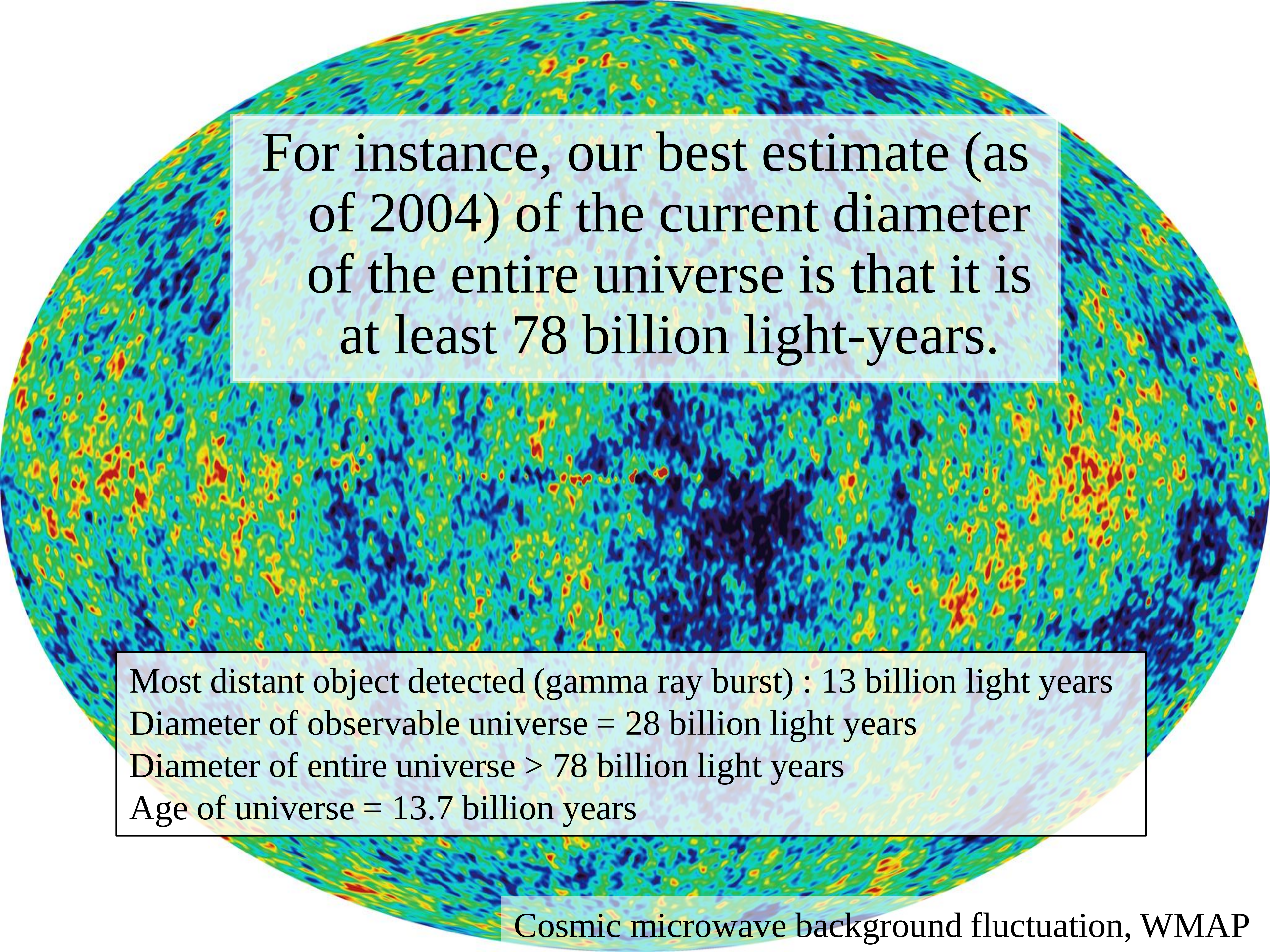
Two degree field Galaxy red-shift survey, W. Schaap et al.



which have led in turn to many discoveries of very large-scale structures, such as the Great Wall.

Two degree field Galaxy red-shift survey, W. Schaap et al.

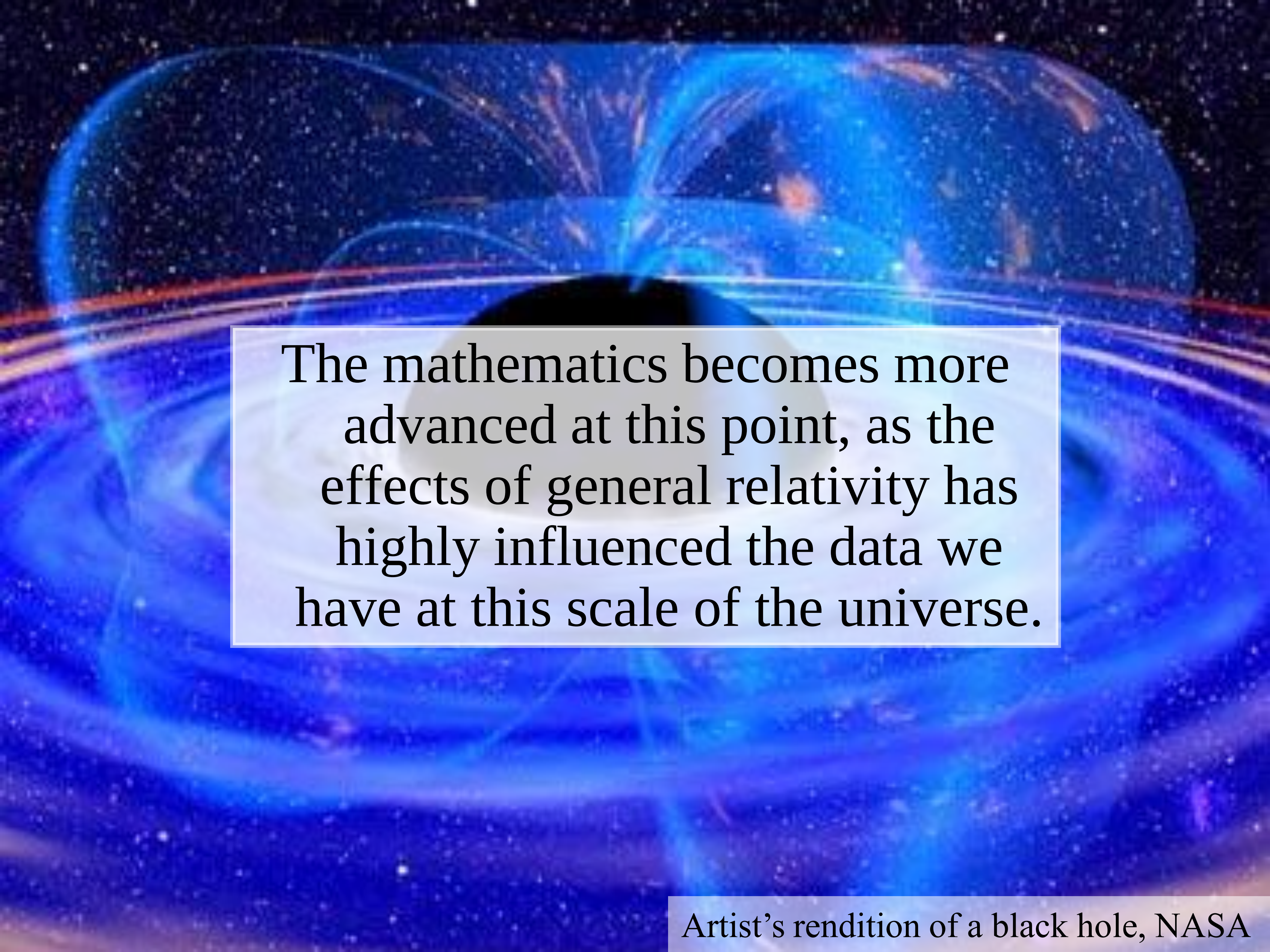
For instance, our best estimate (as of 2004) of the current diameter of the entire universe is that it is at least 78 billion light-years.



Most distant object detected (gamma ray burst) : 13 billion light years
Diameter of observable universe = 28 billion light years
Diameter of entire universe > 78 billion light years
Age of universe = 13.7 billion years

Cosmic microwave background fluctuation, WMAP

The mathematics becomes more advanced at this point, as the effects of general relativity has highly influenced the data we have at this scale of the universe.

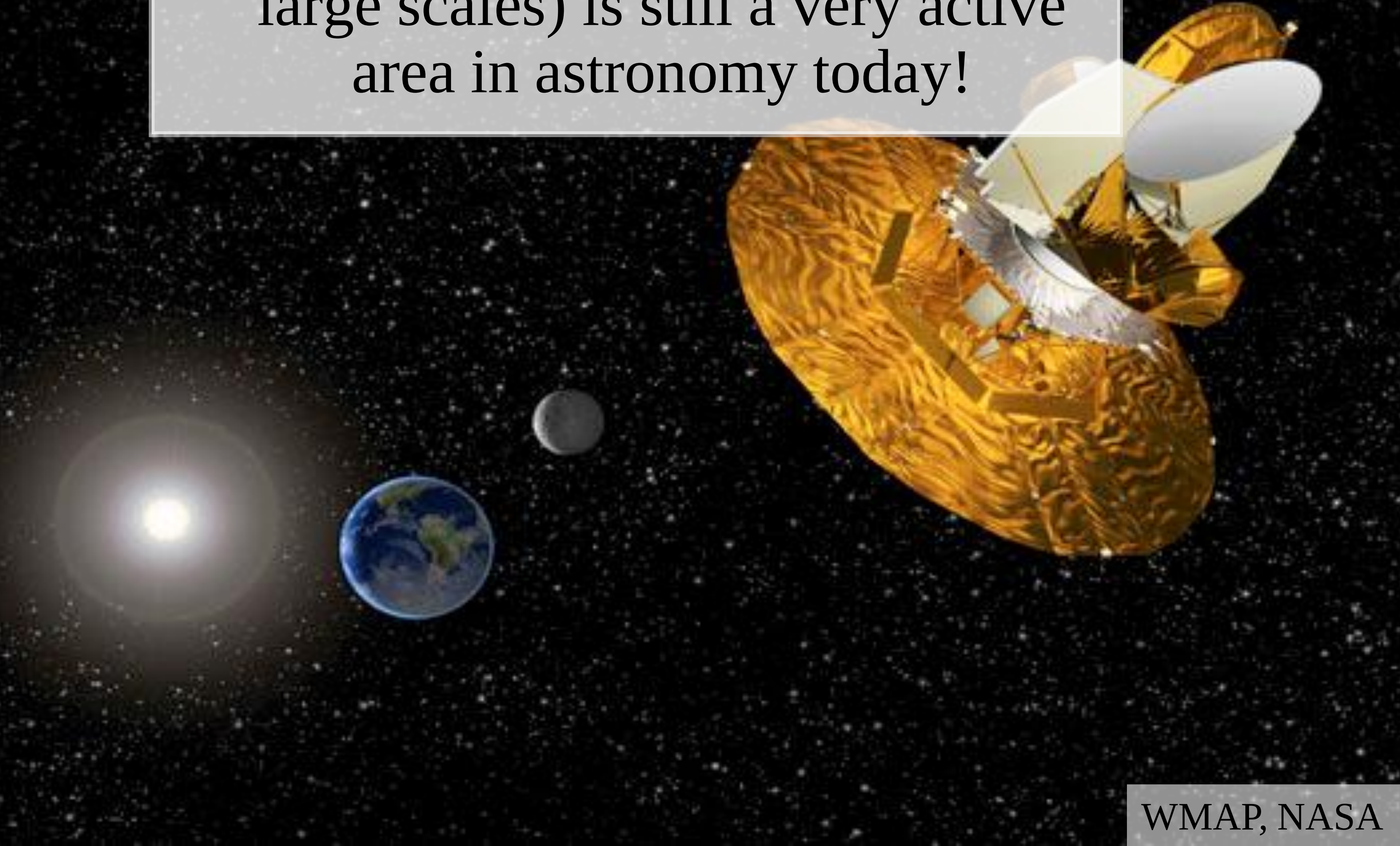


Artist’s rendition of a black hole, NASA

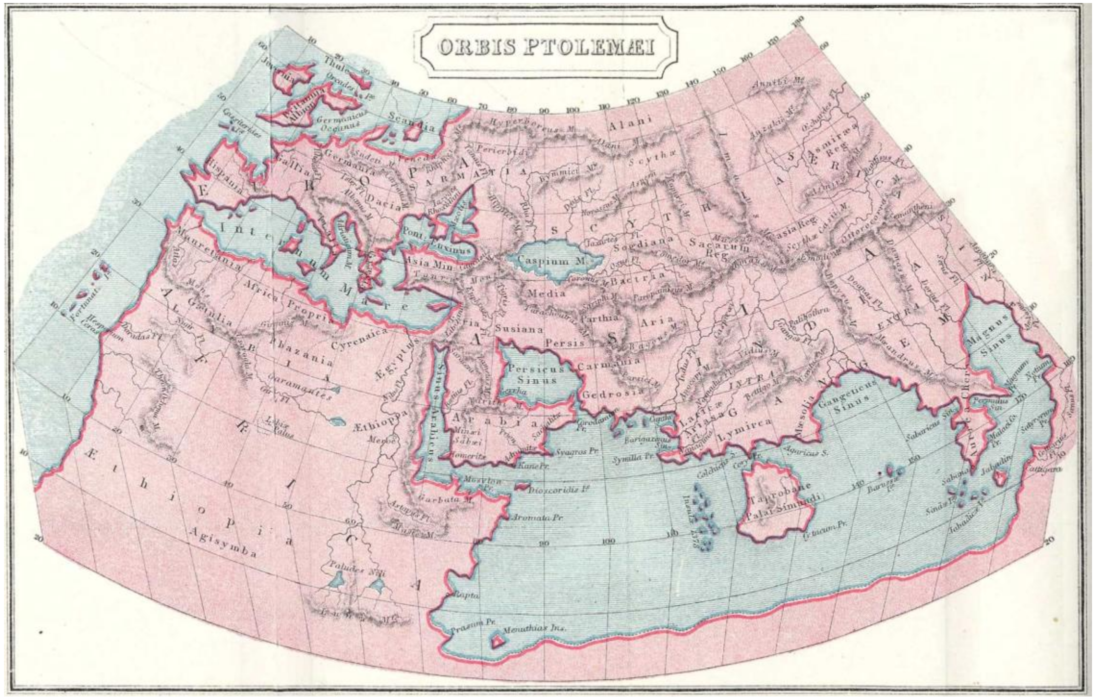
Cutting-edge technology (such as the Hubble space telescope (1990-) and WMAP (2001-2010)) has also been vital to this effort.

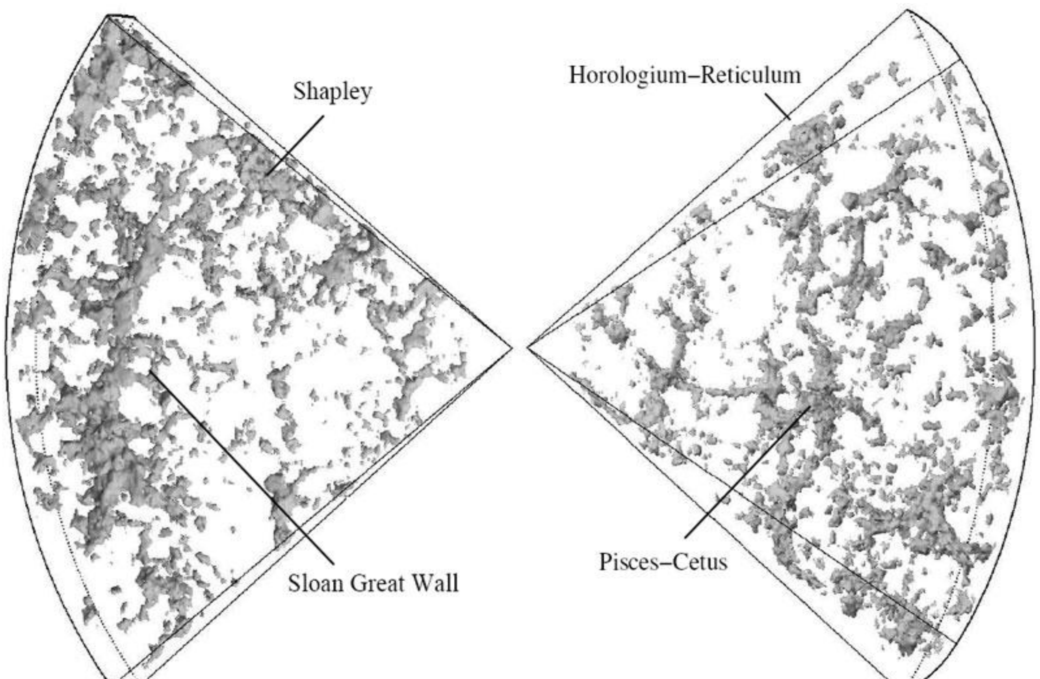
Hubble telescope, NASA

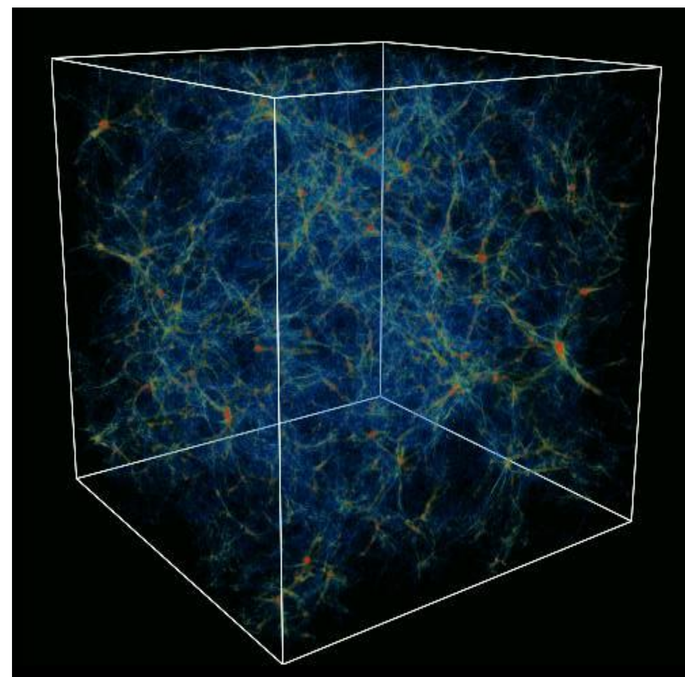
Climbing this rung of the ladder (i.e. mapping the universe at its very large scales) is still a very active area in astronomy today!



WMAP, NASA


ORBIS PTOLEMÆI


Shapley
Horologium–Reticulum
Sloan Great Wall
Pisces–Cetus



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- 174-175: Refined Hubble Constant Narrows Possible Explanations for Dark Energy – NASA/ESA/ A. Riess (STScl/JHU)
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- 185: Spinning Black Holes and MCG-6-30-15 – XMM-Newton/ESA/NASA
- 186: Hubble Space Telescope – NASA
- 187: WMAP leaving Earth/Moon Orbit for L2 - NASA
- 188: Atlas Of Ancient And Classical Geography, J. M. Dent And Sons, 1912, Map 26;
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